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SOME NEUTROSOPHIC COMPACT SPACES VIA GRILLS

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Abstract. The aim of this paper is to introduce a type of compactness, defined in terms of a grill G in a neutrosophic topological space, call it neutrosophic αG - compact space with the help of neutrosophic α - open cover and neutrosophic G - cover. We would investigate some of its basic properties and characterization theorems in neutrosophic topological space with a grill G. We would define the notion of neutrosophic α - quasi Hausdorff closed space. Then we would establish the relation between neutrosophic αG - compact and neutrosophic α - quasi H - closed spaces. We would also introduce and study neutrosophic αG - compact set relative to a neutrosophic topological space X with a grill G. Lastly we would define countably neutrosophic αG - compact space and investigate some of its basic properties and characterization theorems in a neutrosophic topological space X with a grill G.

1. Introduction

We are facing real life problems due to uncertainty in our everyday life. In order to solve such problems due to uncertainty, Zadeh [28] introduced the notion of fuzzy set adding membership value. Subsequently, Atanassov [1] introduced the notion of intuitionistic fuzzy set associating with membership and non - membership values. Still it was insufficient to solve all real life problems due to uncertainty. Thereafter, Smarandache [25] introduced the notion of neutrosophic set where each element had three associated defining functions, namely the membership function (T), the non - membership function (F) and the indeterminacy function (I) defined on the universe of discourse X. These three functions are completely independent. Smarandache [26] further investigated on the applications of the neutrosophic theory. Thereafter, the notion of neutrosophic topological space was first introduced by Salama and Alblowi [22], followed by Salama and Alblowi [23].

The brilliant notion of a grill was initiated by Choquet [4]. Subsequently, it turned out to be a very convenient tool for various topological and neutrosophic topological investigations. The notion of compactness in topological space via grills was introduced by Roy and Mukherjee [20]. The notion of grill in neutrosophic topological space and neutrosophic minimal space was introduced by Pal et al. [16]. Pal and Dhar [15] introduced the notion of compactness in neutrosophic minimal

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space. Karthik and Arockiarani [11,12] investigated on weak compactness by grills in general topology. Azzamet et al. [3] introduced and studied on neutrosophic - compact space and countability neutrosophic - compact space in general topology. Besides them, many researchers [5,6,11,14,17,19,27] investigated in neutrosophic topological space. Such works motivated us to reveal this research article. We shall make sections of the article as follows. The next section briefly states on known definitions and results related to neutrosophic set and neutrosophic topological space. In section 3, we introduce the notion of neutrosophic G - compact space. We also investigate some basic properties and theorems of this space. Section 4 reveals the relation between grill and α - quasi H - closed neutrosophic space. Section 5 focuses on the introduction of neutrosophic αG - compact set relative to a neutrosophic topological space X with a grill G. Lastly in section 6, we introduce and study the notion of countably neutrosophic αG - compact space.

2. Preliminaries

We recall here some basic definitions and results which are relevant for this article.

Definition 2.1. [4] A collection G of non - empty subsets of a set X is called a grill if

(i) $A \in G$ and $A \subseteq B \subseteq X$ implies that $B \in G$ and
(ii) $A \cup B \in G$ $(A, B \subseteq X)$ implies that $A \in G$ or $B \in G$

Definition 2.2. [20] Let G be a grill on a topological space (X, τ). A cover $\{U_\alpha : \alpha \in \Lambda\}$ of X is said to be a G - cover if there exists a finite subset Λ_0 of Λ such that $X \setminus \cup_{\alpha \in \Lambda_0} U_\alpha \notin G$.

Definition 2.3. [22] Let X be an universal set. A neutrosophic set (NS, in short) A in X is a set contains triplet having truthness, falseness and indeterminacy membership values that can be characterized independently, denoted by T_A, F_A, I_A respectively in $[0,1]$. The neutrosophic set is denoted as follows:

$A = \{(x, T_A(x), F_A(x), I_A(x)) : x \in X \text{ and } T_A(x), F_A(x), I_A(x) \in [0,1]\}$ with the condition

$$0 \leq T_A(x) + F_A(x) + I_A(x) \leq 3.$$

The null and full NSs on a non - empty set X are denoted by 0_N and 1_N respectively, defined as follows:

Definition 2.4. [22] The neutrosophic sets 0_N and 1_N in X are represented as follows:

(i) $0_N = \{< x, 0, 0, 1 >: x \in X\}$.
(ii) $0_N = \{< x, 0, 1, 1 >: x \in X\}$.
(iii) $0_N = \{< x, 0, 1, 0 >: x \in X\}$.
(iv) $0_N = \{< x, 0, 0, 0 >: x \in X\}$.
(v) $1_N = \{< x, 1, 0, 0 >: x \in X\}$.
(vi) $1_N = \{< x, 1, 0, 1 >: x \in X\}$.

(vii) $1_N = \{< x, 1, 1, 0 >: x \in X\}$.
(viii) $1_N = \{< x, 1, 1, 1 >: x \in X\}$.

Clearly, $0_N \subseteq 1_N$.We have, for any neutrosophic set $A, 0_N \subseteq A \subseteq 1_N$.

Definition 2.5. [22] Let X be a non - empty set and T be the collection of neutrosophic subsets of X. Then T is said to be a neutrosophic topology (in short NT) on X if the following properties holds:

(i) $0_N, 1_N \in T$.
(ii) $U_1, U_2 \in T \Rightarrow U_1 \cap U_2 \in T$.
(iii) $\cup_{i \in \Delta u_i \in T}$, for every $\{u_i : i \in \Delta\} \subseteq T$.

Then (X, T) is called a neutrosophic topological space (in short NTS) over X. The members of T are called neutrosophic open sets (in short NOS). A neutrosophic set D is called neutrosophic closed set (in short NCS) if and only if D^c is a neutrosophic open set.

Definition 2.6. [22] Let (X, T) be a NTS and U be a NS in X. Then the neutrosophic interior (in short N_{int}) and neutrosophic closure (in short N_{cl}) of U are defined by
$N_{int}(U) = \cup\{E : E \text{ is a NOS in } X \text{ and } E \subseteq U\}$,
$N_{cl}(U) = \cap\{F : F \text{ is a NCS in } X \text{ and } U \subseteq F\}$.

Remark 2.7. [22] Clearly $N_{int}(U)$ is the largest neutrosophic open set over X which is contained in U and $N_{cl}(U)$ is the smallest neutrosophic closed set over X which contains U.

Proposition 2.8. [22] For any neutrosophic set B in (X, T), we have

(i) $N_{int}(B^c) = (N_{cl}(B))^c$.
(ii) $N_{cl}(B^c) = (N_{int}(B))^c$.

Definition 2.9. [2] A neutrosophic subset A of a neutrosophic topological space (X, T) is said to be neutrosophic α - open set (briefly Nα - OS) if $A \subseteq N_{int}(N_{cl}(N_{int}(A)))$. The complement of a N$\alpha$ - OS is called a α - closed set (briefly Nα - CS) in (X, T). The family of all Nα - OS (resp. Nα - CS) of X is denoted by Nα - O(X)(resp. Nα - C(X)).

Definition 2.10. [16] Let X be a non - empty set. A subcollection G (not containing 0_N) of P(X) is called a grill on a neutrosophic set on X if G satisfies the following conditions:

(i) $A \in G$ and $A \subseteq B$ implies $B \in G$.
(ii) $A, B \subseteq X$ and $A \cup B \in G$ implies that $A \in G$ or $B \in G$.

Definition 2.11. [18] Let G be a grill on a neutrosophic topological space (X, T). A cover $\{U_\alpha : \alpha \in \Lambda\}$ of X is said to be a neutrosophic G - cover if there exists a finite subset Λ_0 of Λ such that $X \setminus \cup_{\alpha \in \Lambda_0} U_\alpha \notin G$.

Definition 2.12. [18] Let G be a grill on a neutrosophic topological space (X, T). Then (X, T) is said to be neutrosophic compact with respect to the grill G or simply neutrosophic G - compact if every open cover of X is a neutrosophic G - cover.

3. NEUTROSOPHIC αG - COMPACT SPACE

In this section, we introduce and study neutrosophic αG - compact space in the neutrosophic topological space with a grill.

Definition 3.1 Let G be a grill on a neutrosophictopological space (X, T). Then (X, T) is said to be neutrosophic αG - compact space if every neutrosophic α - open cover of X is a neutrosophic G - cover.

Remark 3.2. Every neutrosophic α - compact space (X, T) is clearly neutrosophic αG - compact for any grill G on X.

Proposition 3.3. Let $G = P(X) - 0_N$. Then neutrosophic αG - compactness of a neutrosophic space (X, T) reduces to the neutrosophic compactness and neutrosophic α - compactness of (X, T).

Proof. First, we want to show that every neutrosophic α - compact space (X, T) is neutrosophic compact space. Let $\{U_\beta : \beta \in \Lambda\}$ be any neutrosophic open cover of X. Since every neutrosophic open set is neutrosophic α - open, then $\{U_\beta : \beta \in \Lambda\}$ is neutrosophic α - open cover of X. Since (X, T) is neutrosophic α - compact space, then there exists a finite subcover $\{U_\beta : \beta \in \Lambda_0\}$ of X and hence (X, T) is a compact space. Now let (X, T) be a neutrosophic αG - compact, where $G = P(X) - 0_N$. Let $\{U_\beta : \beta \in \Lambda\}$ be any neutrosophic α - open cover of X. Since (X, T) is neutrosophic αG - compact space, then there exists a finite subset Λ_0 of Λ such that $X - \cup_{(\beta \in \Lambda_0)} U_\beta \notin G$. Since $G = P(X) - 0_N$, then $X - \cup_{(\beta \in \Lambda_0)} = 0_N$.Then $\{U_\beta : \beta \in \Lambda_0\}$ is a finite neutrosophic sub cover of X. So (X, T) is a neutrosophic α - compact space and hence is a neutrosophic compact space.

Proposition 3.4. Let $G = P(X) - 0_N$ be a grill on a neutrosophic space (X, T) and the neutrosophic space (X, T_G) is a neutrosophic αG - compact. Then (X, T) is a neutrosophic α - compact space and neutrosophic compact (as $T \subseteq T_G$) and hence is neutrosophic αG - compact.

Proof. Let (X, T_G) be a neutrosophic αG - compact space, where $G = P(X) - 0_N$. Now we want to show that (X, T) is a neutrosophic compact space. Let $\{U_\beta : \beta \in \Lambda\}$ be any neutrosophic $T\alpha$ - open cover of X. Since $T \subseteq T_G$, then $\{U_\beta : \beta \in \Lambda\}$ is a $T_G\alpha$ - open cover of X. Since (X, T_G) is a neutrosophic αG - compact space, then there exists a neutrosophic finite subset Λ_0 of Λ such that$X - \cup_{(\beta \in \Lambda_0)} U_\beta \notin G$. Since $G = P(X) - 0_N$, then $X - \cup_{(\beta \in \Lambda_0)} U_\beta = 0_N$. Then $\{U_\beta : \beta \in \Lambda_0\}$ is a neutrosophic finite subcover of X. So (X, T) is a neutrosophic α - compact space and hence a neutrosophic compact space.

Remark 3.5. Every neutrosophic αG - compact space (X, T) is clearly a neutrosophic G - compact for any grill G on X.

Theorem 3.6. Let G be a grill on a neutrosophic topological space (X, T). Then (X, T_G) is a neutrosophic αG - compact if (X, T) is a neutrosophic αG - compact.

Proof. Let $\{U_\beta : \beta \in \Lambda\}$ be a basic $T_G\alpha$ - open cover of X. Then by definition of a base for T_G, we get for each $\beta \in \Lambda$, $U_\beta = V_\beta \beta - H_\beta$, where $V_\beta \in T$ and $H_\beta \notin G$. Then $\{V_\beta : \beta \in \Lambda\}$ is a neutrosophic T - open cover of X. Since every neutrosophic open set is a neutrosophic α - open, then $\{V_\beta : \beta \in \Lambda\}$ is a $T\alpha$ - open cover of X. Since (X, T) is a neutrosophic αG - compact, then there exists a neutrosophic finite subset Λ_0 of Λ such that $X - \cup_{\beta \in \Lambda_0} V_\beta \notin G$. Then $X - \cup_{\beta \in \Lambda_0} U_\beta = X - \cup_{\beta \in \Lambda_0}(V_\beta - H_\beta) \subseteq X - (\cup_{\beta \in \Lambda_0} V_\beta - \cup_{\beta \in \Lambda_0} H_\beta) \subseteq (X - \cup_{\beta \in \Lambda_0} V_\beta) \cup (\cup_{\beta \in \Lambda_0} H_\beta) \notin G$. So (X, T_G) is a neutrosophic αG - compact.

Remark 3.7. Let G be a grill on a neutrosophic topological space (X, T). Then (X, T_G) is a neutrosophic G - compact if (X, T) is a neutrosophic αG - compact.

4. GRILLS AND COMPACTNESS OF HAUSDORFF SPACES

In this section, we establish the relation between grill and α - quasi H - closed.

Definition 4.1. A neutrosophic topological space (X, T) is said to be a neutrosophic α - quasi H - closed (NαQHC, in short) if for every neutrosophic α - open cover U of X, there is a neutrosophic finite subcollection U_0 of U such that $X = \cup\{cl(u) : u \in U_0\}$. A neutrosophic Hausdorff α - quasi H - closed space is denoted by Nα - QHC.

Proposition 4.2. Let G be a grill on a neutrosophic topological space (X, T) such that $T - 0_N \subseteq G$. If (X, T) is a neutrosophic αG - compact then (X, T) is Nα-QHC.

Proof. Let $\{U_\beta : \beta \in \Lambda\}$ be any neutrosophic α - open cover of X. Since (X, T) is a neutrosophic αG - compact, then there exists a neutrosophic finite subset Λ_0 of Λ such that $X - \cup_{\beta \in \Lambda_0} U_\beta \notin G$. Then $\text{int}(X - \cup_{\beta \in \Lambda_0} U_\beta) = 0_N$. For otherwise, $\text{int}(X - \cup_{\beta \in \Lambda_0} U_\beta) \in T - 0_N \subseteq G$ and hence $X - \cup_{\beta \in \Lambda_0} U_\beta \in G$, a contradiction. It follows that $X - cl(X - \cup_{\beta \in \Lambda_0} U_\beta)^c = 0_N$. Hence $X = \cup_{\beta \in \Lambda_0} cl(U_\beta)$ and (X, T) is a Nα-QHC space.

Definition 4.3. Let (X, T) be a neutrosophic topological space and G be a grill on X. Then the space X is said to be a neutrosophic αG - regular if for any neutrosophic α - closed set F in X with a neutrosophic point $x_{r,s,t}$ where $x_{r,s,t} \notin F$, there exists disjoint neutrosophic α - open sets U and V such that the neutrosophic point $x_{r,s,t} \in U$ and $F - V \notin G$.

Proposition 4.4. Let G be a grill on a Hausdorff space (X, T). If (X, T) is a neutrosophic αG - compact then it is a neutrosophic αG - regular.

Proof. Let F be any neutrosophic α - closed subset of X and a neutrosophic point $x_{r,s,t}$ where $x_{r,s,t} \notin F$. Since X is a neutrosophic Hausdorff space, then for each neutrosophic point $y_{r,s,t}$ where $y_{r,s,t} \notin F$, there exists two disjoint neutrosophic open sets U_x and V_y such that $x_{r,s,t} \in U_x$ and $y_{r,s,t} \in V_y$. Thus $\{V_y : y_{r,s,t} \in F\} \cup \{X - F\}$ is a neutrosophic α - open cover of X. Since (X, T) is a neutrosophic αG - compact, then there exists finitely many neutrosophic points $y_{1_{r,s,t}}, y_{2_{r,s,t}}, ..., y_{n_{r,s,t}} \in F$ such that $X - [(\bigcup_{i=1}^{n} V_{y_i}) \cup (X - F)] \notin G$. Let $G = X - \bigcup_{i=1}^{n} clV_{y_{i_{r,s,t}}}$ and $H = \bigcup_{i=1}^{n} V_{y_{i_{r,s,t}}}$. Then G and H are disjoint non - empty neutrosophic α - open sets in X such that for a neutrosophic point $x_{r,s,t}$ where $x_{r,s,t} \in G$ (since $x_{r,s,t} \notin clV_{y_{i_{r,s,t}}}$ for all $i = 1, 2, 3, ..., n$) and $F - H = F \cup [X - \bigcup_{i=1}^{n} V_{y_{i_{r,s,t}}}] = X - [(\bigcup_{i=1}^{n} V_{y_{i_{r,s,t}}}) \cup (X - F)] \notin G$. So (X, T) is a neutrosophic αG - regular.

Corollary 4.5. Let G be a grill on a neutrosophic Hausdorff space (X, T) such that $T - 0_N \subseteq G$. If (X, T) is a neutrosophic αG - compact then it is a NαHC and neutrosophic αG - regular.

Proof. Direct consequence of Propositions 4.2.and 4.4.

5. Neutrosophic αG - Compact Sets Relative To A Space

In this section, we present and investigate neutrosophic αG - compact sets relative to a space.

Definition 5.1. Let G be a grill on a neutrosophic topological space (X, T). A subset A of the space (X, T) is said to be neutrosophic αG - compact relative to X if for every cover $\{U_\lambda : \lambda \in \Lambda\}$ of A by neutrosophic α - open sets of X, there exists a finite subset Λ_0 of Λ such that $A \setminus \cup_{\lambda \in \Lambda_0} U_\lambda \notin G$.

Theorem 5.2. The following are equivalent for a subset A of a neutrosophic topological space (X, T):

(a) A is neutrosophic αG - compact relative to X, where the grill $G = P(X) - 0_N$.
(b) A is neutrosophic α - compact relative to X.

Proof. $(a) \Rightarrow (b)$. Let $\{U_\lambda : \lambda \in \Lambda\}$ be a cover of A by neutrosophic α - open sets of X . Since A is neutrosophic αG - compact relative to X, then there exists a finite subset Λ_0 of Λ such that $A \setminus \cup_{\lambda\in\Lambda_0} U_\lambda \notin G$. Since $G = P(X) - 0_N$, then $A \setminus \cup_{\lambda\in\Lambda_0} U_\lambda = 0_N$. Then $A = \cup_{\lambda\in\Lambda_0} U_\lambda$. Thus A is neutrosophic α - compact relative to X .
$(b) \Rightarrow (a)$. Let $\{U_\lambda : \lambda \in \Lambda\}$ be a neutrosophic cover of A by neutrosophic α - open sets of X. Since A is neutrosophic α - compact relative to X, then there exists a finite neutrosophic subcover $\{U_\lambda : \lambda \in \Lambda_0\}$ of A (i.e., $A \subseteq \cup_{\lambda\in\Lambda_0} U_\lambda$). Then $A \setminus \cup_{\lambda\in\Lambda_0} U_\lambda = 0_N \notin G$. Thus A is neutrosophic αG - compact relative to X.

Proposition 5.3. Let G be a grill on a neutrosophic topological space (X, T). If A_i, $i = 1, 2$ are neutrosophic αG - compact subsets relative to a neutrosophic space (X, T), then $A_1 \cup A_2$ is neutrosophic αG - compact relative to X.

Proof. Let $\{U_\lambda : \lambda \in \Lambda\}$ be a neutrosophic cover of $A_1 \cup A_2$ by neutrosophic α - open sets of X. Then it is a neutrosophic α - open cover of A_i for $i = 1, 2$ Since A_i is neutrosophic αG - compact relative to X, then there exists a finite subset Λ_1 of Λ such that $A_1 \setminus \cup_{\lambda\in\Lambda_1} U_\lambda \notin G$ and there exists a finite subset Λ_2 of Λ such that $A_2 \setminus \cup_{\lambda\in\Lambda_2} U_\lambda \notin G$. Since $(A_1 \setminus \cup_{\lambda\in\Lambda_1} U_\lambda) \cup (A_2 \setminus \cup_{\lambda\in\Lambda_2} U_\lambda) \supseteq (A_1 \cup A_2) \setminus \cup_{\lambda_1\cup\lambda_2} U_\lambda \notin G$. Then there exists a finite subset $\Lambda_1 \cup \Lambda_2$ of Λ such that $(A_1 \cup A_2) \setminus \cup_{\lambda_1\cup\lambda_2} U_\lambda \notin G$. Thus $A_1 \cup A_2$ is a neutrosophic αG - compact relative to X.

Theorem 5.4. Let (X, T) be a neutrosophic topological space with a grill G on X. If A is neutrosophic αG - compact relative to X, then $(A, T/A)$ is G/A - compact.

Proof. Let$\{(U_\lambda \cap A) : \lambda \in \Lambda\}$ be neutrosophic T/A - open cover of A, where $U_\lambda \in T$ for each $\lambda \in \Lambda$. Now $\{U_\lambda : \lambda \in \Lambda\}$ is a neutrosophic cover of A by neutrosophic α - open subsets of X. Since A is neutrosophic αG - compact relative to X, then there exists a finite subset Λ_0 of Λ such that $A \setminus \cup_{\lambda\in\Lambda_0} U_\lambda \notin G$. Thus $A \cap [A \setminus \cup_{\lambda\in\Lambda_0} U_\lambda] \notin A \cap G$. Since $A \setminus \cup_{\lambda\in\Lambda_0} (U_\lambda \cap A) = A \setminus \cup_{\lambda\in\Lambda_0} U_\lambda = A \cap [A \setminus \cup_{\lambda\in\Lambda_0} U_\lambda] \notin A \cap G$, then there exists a finite subset Λ_0 of Λ such that $A \setminus \cup_{\lambda\in\Lambda_0} (U_\lambda \cap A) \notin G/A$. Thus $(A, T/A)$ is G/A - compact.

6. Countably Neutrosophic αG - Compact Space

In this section, we focus on the properties of countably neutrosophic αG - compact space.

Definition 6.1. Let G be a grill on a neutrosophic topological space (X, T). A neutrosophic topological space (X, T) is said to be countably neutrosophic αG - compact if for every countable neutrosophic α - open cover $\{U_n : n \in N\}$ of X there exists a finite subset N_0 of N such that $X \setminus U_{U\in N_0} U_n \notin G$, where N denotes the set of positive integers.

Proposition 6.2. Let G be a grill on a neutrosophic topological space (X, T). If the neutrosophic space (X, T) is countably neutrosophic αG - compact, then for any countable family $\{f_n : n \in N\}$ of neutrosophic α - closed sets of X such that $\cap\{f_n : n \in N\} = 0_N$, there exists a finite subset N_0 of N such that $\cap\{f_n : n \in N_0\} \notin G$.

Proof. Let $\{f_n : n \in N\}$ be a countable family of neutrosophic α - closed sets of X such that $\cap\{f_n : n \in N\} = 0_N$. Then $\{X\backslash f_n : n \in N\}$ is a countable neutrosophic α - open cover of X. Then there exists a finite subset N_0 of N such that $X\backslash \cup_{n\in N_0} (X\backslash f_n) \notin G$. This leads to $\cap_{n\in N_0}[X\backslash(X\backslash f_n)] \notin G$ (i.e, $\cap_{n\in N_0} f_n \notin G$).

Proposition 6.3. If (X, T) is a countably neautrosophic αG - compact, G_1 and G_2 are two grills on X such that $G_1 \supseteq G_2$ then (X, T) is a countably neutrosophic α - compact.

Proof. Let $\{U_n : n \in N\}$ be a countable neutrosophic α - open cover of X . Since (X, T) is a countably neutrosophic αG - compact, then there exists a finite subset N_0 of N such that $X\backslash \cup_{n\in N_0} U_n \notin G_1$. Since $G_1 \supseteq G_2$, thus $X\backslash \cup_{n\in N_0} U_n \notin G_2$. Thus (X, T) is a countably neutrosophic αG_2 - compact.

Theorem 6.4. If $G = P(X) - 0_N$ is the grill on the neautrosophic space (X, T) , then the following are equivalent:

(a) The neutrosophic space (X, T) is countably neutrosophic α - compact.
(b) The neutrosophic space (X, T) is countably neautrosophic αG - compact.

Proof. $(a) \Rightarrow (b)$. Let $\{U_n : n \in N\}$ be a countable neutrosophic α - open cover of X. By (a), there exists a finite neutrosophic subcover $\{U_n : n \in N_0\}$ of X. Then $X\backslash \bigcup_{n\in N_0} U_n = 0_N \notin G$. Thus (X, T) is countably neutrosophic αG - compact.
$(b) \Rightarrow (a)$. Let $\{U_n : n \in N\}$ be a countable neautrosophic α - open cover of X. By (b), there exists a finite subset N_0 of N such that $X\backslash \bigcup_{n\in N_0} U_n \notin G$. Since $G = P(X) - 0_N$, then $X\backslash \bigcup_{n\in N_0} U_n = 0_N$. This means that $\{U_n : n \in N_0\}$ is a finite neutrosophic subcover of X. Thus (X, T) is countably neutrosophic α - compact.

Proposition 6.5. If (X, T) is countably neutrosophic α - compact, then (X, T) is countably neutrosophic αG - compact, where G is a grill on X.

Proof. Obvious.

Definition 6.6. A neutrosophic space (X, T) is called neutrosophic α - Lindel¨ of if and only if every neutrosophic α - open cover of X has a countable subcover.

Theorem 6.7. If (X, T) is a countably neutrosophic αG - compact and neutrosophic α - Lindel¨ of space, then (X, T) is a neutrosophic αG - compact, where G is a grill on (X, T) .

Proof. Let $\{U_\lambda : \lambda \in \Lambda\}$ be a neutrosophic α - open cover of X. Since (X, T) is neutrosophic α - Lindel¨ of, there exists a countable neutrosophic subset Λ_1 of Λ such that $X = \cup_{\lambda\in\Lambda_1} U_\lambda$. But (X, T, G) is countably neutrosophic αG - compact and hence there exists a finite subset λ_0 of λ_1 such that $X\backslash \cup_{\lambda\in\Lambda_1} U_\lambda \notin G$. Thus (X, T) is a neutrosophic αG - compact.

Theorem 6.8. Let $f : (X, T_1, G) \to (Y, T_2)$ be a neutrosophic α - irresolute surjection. If (X, T_1, G) is countably neutrosophic αG - compact, then $(Y, T_2, f(G))$ is countably neutrosophic $\alpha f(G)$ - compact.

Proof. Let $\{V_n : n \in N\}$ be a countable neutrosophic α - open cover of Y. Since f is neutrosophic α - irresolute, then $\{f^{-1}(V_n) : n \in N\}$ is a countable neutrosophic α - open cover of X and hence there exists a finite subset N_0 of N such that $X\backslash \cup_{n\in N_0} f^{-1}(V_n) \notin G$. Since f is surjective, we have $Y\backslash \cup_{n\in N_0} V_n = f[X\backslash \cup_{n\in N_0} f^{-1}(V_n)] \notin f(G)$. Thus $(Y, T_2, f(G))$ is countably neutrosophic $\alpha f(G)$ - compact.

Theorem 6.9. Let $f : (X, T_1, G) \to (Y, T_2)$ be a neutrosophic α - continuous surjection. If (X, T_1, G) is countably neutrosophic αG - compact, then $(Y, T_2, f(G))$ is countable neutrosophic $f(G)$ - compact.

Proof. Let $\{V_n : n \in N\}$ be a countable neutrosophic open cover of Y. Then $\{Y \backslash V_n : n \in N\}$ is a countable neutrosophic closed subsets of Y. Since f is neutrosophic α - continuous, then $\{f^{-1}(Y \backslash V_n) : n \in N\}$ is a countable neutrosophic α - closed subset of X. Then $\{X \backslash f^{-1}(Y \backslash V_n) : n \in N\}$ is a countable neutrosophic α - open subsets of X. Since $X \backslash f^{-1}(Y \backslash V_n) = f^{-1}(V_n)$, then $\{f^{-1}(V_n) : n \in N\}$ is a countable neutrosophic α - open cover of X. Since (X, T_1, G) is countably neutrosophic αG - compact, there exists a finite subset N_0 of N such that $X \backslash \cup_{n \in N_0} f^{-1}(V_n) \notin G$. Thus $f(X \backslash U_{n \in N_0} f^{-1}(Vn)) \notin f(G)$. This leads to $Y \backslash \cup_{n \in N_0} V_n \notin f(G)$ and hence $(Y, T_2, f(G))$ is a neutrosophic $f(G)$ - compact.

Theorem 6.10. If $f : (X, T_1) \to (Y, T_2, G)$ is neutrosophicpre α - open bijection and (Y, T_2, G) is countably neutrosophic αG - compact, then (X, T_1) is countably neutrosophic $\alpha f^{-1}(G)$ - compact.

Proof. Since f is neutrosophic pre - α - open, then $f(F)$ is α - open in (Y, T_2, G) for every neutrosophic α - open set F in (X, T_1). Since f is bijection, then $f^{-1} : (Y, T_2, G) \to (X, T_1)$ exists and a neutrosophic α - irresolute surjection. Therefore, the proof follows from the Theorem 6.9.

7. Conclusion

In this paper, we have presented some new neutrosophic compact spaces via grills. We have introduced and investigated neutrosophic αG - compact space in neutrosophic topological space. We have shown that every neutrosophic αG - compact space is neutrosophic G - compact space for any grill G on a neutrosophic topological space X. We have established the relation between grill and α - quasi H - closed space in a neutrosophic topological space. We have studied some basic properties of neutrosophic αG - compact sets relative to a neutrosophic topological space. We have also focused on the properties of countably neutrosophic αG - compact space in a neutrosophic topological space. It is expected that the work done will help in further investigation of the compactness in neutrosophic topological space. In the future, it is hoped that the notion of compactness which have been discussed here can also be extended in neutrosophic supra topological space [7], neutrosophic bi - topological space [13], neutrosophic tri - topological space [8], neutrosophic soft topological space [9], neutrosophic multiset topological space [10], etc.

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