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Multiple oscillating layers of a double-perturbed interface with self-gravitate.

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Abstract

The multiple oscillating layers of a double perturbed interface under self-gravitating has been investigated for all the perturbation modes. This type of research may be found to examine oscillating on multiple layers with self-gravitating force. The stability criterion is constructed analytically explained and confirmed these results with the numerical computations. The governing equations (equation of motion and equation of continuous) are obtained, providing that the boundary conditions are appropriate. the fundamental equations are resolved, non-singular solutions are found using the proper boundary circumstances, also, derived the total second order differential equation. The difference between these two states, stable and unstable, relies on the value of densities. In this point, the gravitational instability of the current model, which forms the basis of this work, will be decreased, The streaming is unstable. The gravitationally stable and unstable zones are discovered and graphically displayed. The triple fluid layers' weight force and densities ratios contribute significantly to the unstable nature of the current model.

Keywords: oscillating, double-perturbed interface and Self-gravitate

1. Introduction

One of the first to demonstrate the self-gravitating instability of a static infinite homogeneous media was Jeans [1]. He discovered that the medium becomes unstable for all wavenumber perturbations below the critical point. $k^* = \sqrt{2\pi G\rho}/c$. The modification in Jeans criterion was made by several investigators e.g. Chandrasekhar and Fermi [2], Simon [3]. They include the effect of various parameters rotation, Chandrasekhar [4]. Also Elazab [5]and Radwan and Elazab [7] ,[8], Radwan , Elazab and Z.M. Ismail [10] , D. Pawlus[13], also [11],[14]. Build on our earlier research on the stability of two superposed layers. A layer of uniformly density ρ_2 gravitational fluid sandwiched between two self-gravitational fluid layers of varied density $\rho_1 and\ \rho_3$. The various strata are referred to as regions 1, 2, and 3 in figure (1). The fluid of density ρ_1 is in the region 1 with $-\infty < z \leq 0$.The fluid of density ρ_2 is in the region 2 with $0 < z \leq d$, and the fluid of density ρ_3 is in the region 3 with $d < z \leq \infty$.

2. The Governing Equation

The fluids are regarded as non-viscous and incompressible. These fluids are all affected by their own gravitational pull, weight force, and kinetic pressure. Use Cartesian coordinates (x,y,z), with the z=0 axis located along the interface between regions 1 and 2 fluid plane.

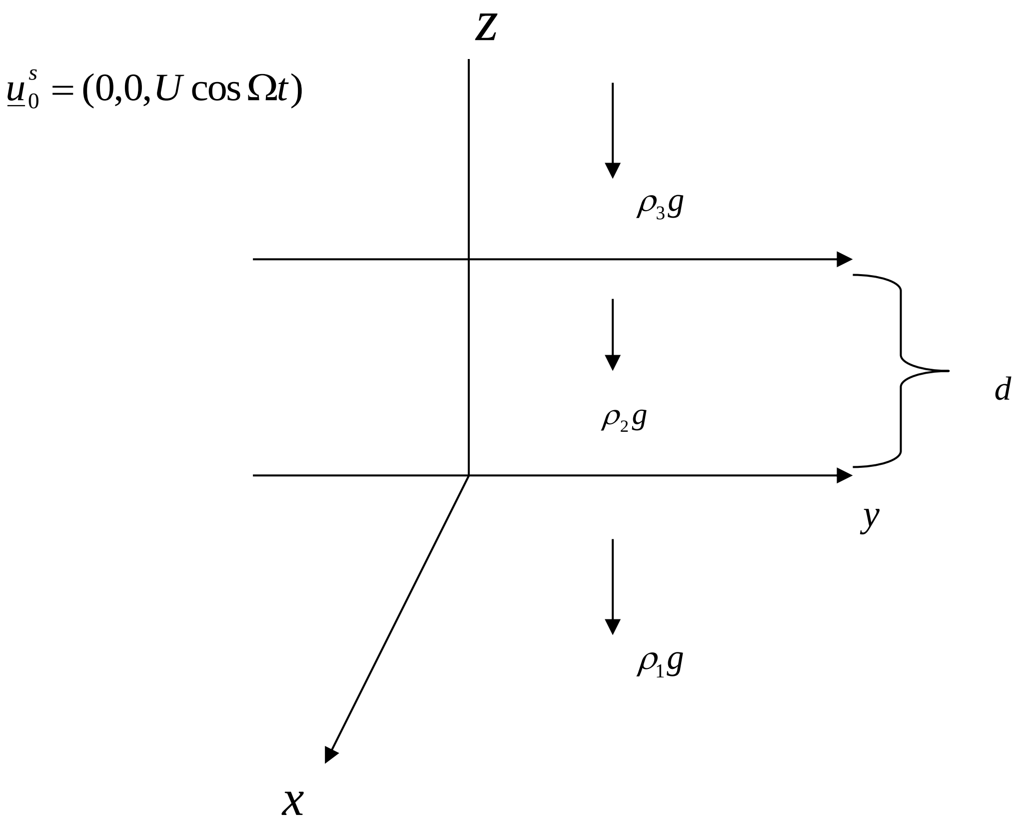

Fig.1 Sketch of Triple Superposed Fluids

The fundamental equations can be expressed as follows:

$$\rho^s\left(\frac{\partial \underline{u}^s}{\partial t}+(\underline{u}^s\cdot\nabla)\underline{u}^s\right)=-\nabla p^s-\rho^s\nabla V^s-\rho^s g e_z \quad (1)$$

$$\nabla^2 V^s = 4\pi G\rho^s \quad (2)$$

$$\nabla.\underline{u}^s = 0 \quad (3)$$

where, s stands for 1, 2 and 3 . Here ρ , $\underline{u}$ and p are the fluid mass density, velocity vector and kinetic pressure, V is the gravitational potential and G is the gravitational constant.

3. State of Equilibrium

In this situation, index 0 describes the physical quantities. The assumption make that the fluids flow quickly $\underline{u}_0^s=(0,0,U\cos\Omega t)$. Equations (1) through (3) contain the gravitodynamic equations for such condition.

$$\rho^s\frac{d\underline{u}_0^s}{dt}=-\nabla p_0^s-\rho^s\nabla V_0^s-\rho^s g e_z \quad (4)$$

$$\nabla^2 V_0^s = 4\pi G\rho^s \quad (5)$$

$$\nabla.\underline{u}_0^s = 0 \quad (6)$$

Each region solves equations (4) through (6), and the necessary boundary conditions are then applied at the interfaces of z=0 and z=d. The non-singular formula for the kinetic pressures and self-gravitating potentials in the three regions are

$$V_0^I = 2\pi G\rho_1 z^2 + c_1 z + c_2 \quad (7)$$

$$V_0^{II} = 2\pi G\rho_2 z^2 + c_1 z + c_2 \quad (8)$$

$$V_0^{III} = 2\pi G\rho_3 z^2 + c_1 z + 4\pi G d(\rho_2-\rho_3)z + c_2 - 2\pi G d^2(\rho_2-\rho_3) \quad (9)$$

$$p_0^I = A_1 - \rho^I[2\pi G\rho_1 z^2 + c_1 z + c_2] - \rho_1 g z \quad (10)$$

$$p_0^{II} = A_2 - \rho^{II}[2\pi G\rho_2 z^2 + c_1 z + c_2] - \rho_2 g z \quad (11)$$

$$p_0^{III} = A_3 - \rho^{III}[2\pi G\rho_3 z^2 + c_1 z + 4\pi G d(\rho_2-\rho_3)z] \\ + c_2 - 2\pi G d^2(\rho_2-\rho_3)] - \rho_3 g z \quad (12)$$

Where c_2 is an arbitrary constant while c_1 is a parameter a length unit in this problem.

4. Perturbed State

The different physical quantities of the present model could be written as (S=1, 2and 3) for a slight divergence from the unperturbed condition.

$$\underline{u}^s = (0,0,U\cos\Omega t) + \varepsilon(t)\underline{u}_1^s + \ldots \qquad (13)$$

$$p^s = p_0^s + \varepsilon(t)p_1^s + \ldots \qquad (14)$$

$$V^s = V_0^s + \varepsilon(t)V_1^s + \ldots \qquad (15)$$

Here, a quantity with index 0 refers to the undisturbed condition, while a quantity with index 1 refers to a modest increment of this variable. The amplitude of perturbation $\varepsilon(t)$, is given by

$$\varepsilon(t) = \varepsilon_0 \exp(\sigma t), \qquad (16)$$

where $\varepsilon_0(=\varepsilon(t)$ at $t=0)$ is the initial amplitude of perturbation and σ is the growth rate. By inserting the expansions(13),(14) and(15) into equations (1),(2) and (3) yield

$$\rho^s\left(\frac{\partial}{\partial t} + \underline{u}_0^s.\nabla\right)\underline{u}_1^s = -\nabla p_1^s \qquad (17)$$

$$P_1^s = p_1^s + \rho^s V_1^s \qquad (18)$$

$$\nabla.\underline{u}_1^s = 0 \qquad (19)$$

$$\nabla^2 V_1^s = 0 \qquad (20)$$

4.1. Fourier Analysis

Based on the linear perturbation technique which has been used for stability problems, the fluctuating parts $\underline{u}_1^s, p_1^s and \Phi_1^s$ may be given as:

$$Q_1^s(x,y,z,t) = Q_1^s(z)\ \ \varepsilon(t)\exp(i(k_x x + k_y y + k_z z)) \qquad (21)$$

where k_x , k_y ,and k_z denote the wave numbers along the x, y, and z axes, respectively, and $Q^s{}_1$ (z) is a function that exclusively affects z. Based on the space-time dependence (16) and (21), the linearized perturbation equations (17)--(20) are solved and the finite solution of the velocities, pressures and gravitational potentials in the perturbation state are given by :-

$$\underline{u}_1^I(x,y,z,t) = \frac{-kL_1\exp(kz + i(k_x x + k_y y + k_z z) + \sigma t)}{\rho_1(\sigma + ik_z U\cos\Omega t)} \qquad (22)$$

$$\underline{u}_1^{II}(x,y,z,t) = \frac{(kL_2 \exp(-kz) - kH_2 \exp(kz)) \exp(i(k_x x + k_y y + k_z z) + \sigma t)}{\rho_2(\sigma + ik_z U \cos \Omega t)} \quad (23)$$

$$\underline{u}_1^{III}(x,y,z,t) = \frac{kL_3 \exp(-kz + \sigma t + i(k_x x + k_y y + k_z z))}{\rho_3(\sigma + ik_z U \cos \Omega t)} \quad (24)$$

$$p_1^{I}(x,y,z,t) = L_1 \exp(i(k_x x + k_y y + k_z z)) + kz + \sigma t) \quad (25)$$

$$p_1^{II}(x,y,z,t) = (L_2 \exp(-kz) + H_2 \exp(kz)) \exp(i(k_x x + k_y y + k_z z) + \sigma t)$$

$$p_1^{III}(x,y,z,t) = L_3 \exp(i(k_x x + k_y y + k_z z) - kz + \sigma t)$$

(26)
(27)

$$\Phi_1^{I}(x,y,z,t) = M_1 \exp(i(k_x x + k_y y + k_z z) + kz + \sigma t)$$

(28)

$$\Phi_1^{II}(x,y,z,t) = (M_2 \exp(kz) + F_2 \exp(-kz)) \exp(i(k_x x + k_y y + k_z z) + \sigma t)$$

$$\Phi_1^{III}(x,y,z,t) = M_3 \exp(i(k_x x + k_y y + k_z z) - kz + \sigma t)$$

(29)

Here F_s, L_s, H_s, M_s and (s=1, 2 , 3) are constants of integration to be determined while k (= $\sqrt{k_x^{\ 2} + k_y^{\ 2} + k_z^2}$) is the net wave number . (30)

4.2. Stability Criterion

4.2.1. Kinematic State

The velocity of the double perturbed interfaces at z =0 and z = d must be compatible with the normal components of the velocities $\underline{u}^s$ (s=1, 2, and 3), which also need to be continuous. The skewed interfaces are provided by

$$z^{I,II} = 0 + \varepsilon(t) \exp(i(k_x x + k_y y + k_z z))$$

$$z^{II,III} = d + \varepsilon(t) \exp(i(k_x x + k_y y + k_z z)) \quad (31)$$

(32)

Consequently, we have

$$L_1 = -\frac{\rho_1}{k}(\sigma + ik_z U \cos\Omega t)^2 \tag{33}$$

$$L_2 = \frac{\rho_2}{k}\frac{(\sigma + ik_z U \cos\Omega t)^2(\exp(kd) - 1)}{(\exp(kd) - \exp(-kd))} \tag{34}$$

$$H_2 = \frac{\rho_2}{k}\frac{(\sigma + ik_z U \cos\Omega t)^2(\exp(-kd) - 1)}{(\exp(kd) - \exp(-kd))} \tag{35}$$

$$L_3 = \frac{\rho_3}{k}(\sigma + ik_z U \cos\Omega t)^2 \exp(kd) \tag{36}$$

4.2.2. Self-gravitating condition

The self-gravitating potentials ($V^s = V_0^s + \varepsilon(t)V_1^s$)and their derivatives must be continuous across the interfaces (31) and (32) at the initial positions z =0 and $z = d$ .These conditions yield

$$M_1 = \frac{2\pi G}{k}((\rho_2 - \rho_1) - (\rho_2 - \rho_3)\exp(-kd)) \tag{37}$$

$$M_2 = \frac{2\pi G}{k}(\rho_3 - \rho_2)\exp(-kd) \tag{38}$$

$$F_2 = \frac{2\pi G}{k}(\rho_2 - \rho_1) \tag{39}$$

$$M_3 = \frac{2\pi G}{k}(\rho_2 - \rho_1) + \frac{2\pi G}{k}(\rho_3 - \rho_2)\exp(kd) \tag{40}$$

4.2.3. pressure condition

At z =0 and z =d, the pressure must be constant across the double perturbed interfaces (31) and (32). The following is a list of possible conditions.At z =0, It have from equation (18)

$$p_1^I - p_1^{II} = \exp(i(k_x x + k_y y + k_z z) + \sigma t)\left(\frac{\partial p_0^{II}}{\partial z} - \frac{\partial p_0^I}{\partial z}\right) \tag{41}$$

At $z = d$, we have

$$p_1^{II} - p_1^{III} = \exp(i(k_x x + k_y y + k_z z) + \sigma t)\left(\frac{\partial p_0^{III}}{\partial z} - \frac{\partial p_0^{II}}{\partial z}\right) \tag{42}$$

By utilizing the conditions (41) and (42) and matching the results we finally obtain the dispersion relation

$$\frac{(\sigma + ik_z U\cos\Omega t)^2}{4\pi G\rho_2} = \frac{\frac{kg}{4\pi G\rho_2}(\frac{\rho_3-\rho_1}{\rho_2})\sinh q}{2(\cosh q - 1) + (\frac{\rho_1+\rho_3}{\rho_2})\sinh q} - \frac{(1-\frac{\rho_1}{\rho_2})(1-\frac{\rho_3}{\rho_2})\exp(-q)\sinh q}{(\frac{\rho_1+\rho_3}{\rho_2})\sinh q + 2(\cosh q - 1)}$$

$$+ \frac{\sinh q[(1-\frac{\rho_1}{\rho_2})^2 + (\frac{\rho_3}{\rho_2}-1)^2]}{2(\frac{\rho_1+\rho_3}{\rho_2})\sinh q + 4(\cosh q - 1)} + \frac{q(\frac{\rho_3}{\rho_2}-1)\sinh q}{(\frac{\rho_1+\rho_3}{\rho_2})\sinh q + 2(\cosh q - 1)}$$

$$+ \frac{\frac{kc_1}{4\pi G\rho_2}(\frac{\rho_3-\rho_1}{\rho_2})\sinh q}{(\frac{\rho_1+\rho_3}{\rho_2})\sinh q + 2(\cosh q - 1)} \quad (43)$$

where q =kd is dimensionless net wavenumber.

5. General Discussions.

The dispersion relation of multiple oscillating layers of double perturbed interfaces with self-gravitate is described by the relation (43). It has the most informational instability in relation to the current issue. Since the present relation is somewhat more general, some limiting cases reported or not yet recovered, it relates the growth rate σ with the densities ρ_1, ρ_2 and ρ_3 of the various fluids and of the fluid layers, the net wavenumber q, The necessary criterion could be obtained by applying the dispersion relation's (43) following simplifications.

(i) $\rho_2 = 0, \rho_3 = 0, U = 0$ (44)

Semi –infinite layer

(ii) $\rho_2 = 0, d = 0, U = 0$ (45)

Two semi –infinite layers

(iii) $\rho_3 \neq 0, \rho_1 = 0, U = 0$ (46)

Unreal model

(iv) $\rho_2 = 0, U = 0, d \neq 0, \rho_1 = \rho_3$ (47)

Gas layer , say , immersed in an infinite medium

(v) $\rho_1 \neq 0, \rho_3 \neq 0, \rho_2 = 0, U = 0, d \neq 0$ (48)

Gas layer ,say, sandwiched between two different layers

(vi) The above cases (i)---(v),but with streaming fluid layers with velocity

$\underline{u}_0 = (0,0,U\cos\Omega t)$ in the initial state (49)

6. Numerical Discussion

Conducting the analytical discussion of relation (43) in order to distinguish between the stable and unstable domains and their characteristics is difficult. It is possible to construct the dispersion relation (43) to manage such discussions mathematically. In the dimensionless form illustrated below, the latter is expressed:

$$\frac{(\sigma+ik_zU\cos\Omega t)^2}{4\pi G\rho_2}=D+E+F \tag{50}$$

$$D=\frac{g^*(b-a)\sinh q}{2(\cosh q-1)+(a+b)\sinh q}-\frac{(1-a)(1-b)\exp(-q)\sinh q}{(a+b)\sinh q+2(\cosh q-1)}$$

+

$$E=\frac{\sinh q[(1-a)^2+(1-b)^2]}{2(a+b)\sinh q+4(\cosh q-1)}+\frac{q(b-1)\sinh q}{(a+b)\sinh q+2(\cosh q-1)}$$

+

$$F=\frac{c^*(b-a)\sinh q}{(a+b)\sinh q+2(\cosh q-1)}$$

with

$$a=\frac{\rho_1}{\rho_2},b=\frac{\rho_3}{\rho_2},\ q=kd\ ,g^*=\frac{kg}{4\pi G\rho_2},c^*=\frac{kc_1}{4\pi G\rho_2} \tag{51}$$

and inserted in a computer. The numerical analysis of the relation (50) together with (51) have been carried out for the different values (a,b)=(0, 0.1) , (0.1 , 0.2) , (0.2 , 0.3) , (2 , 0.4) , (6, 0.3) , (7 , 0.1)for several values of the speed

U* =0 , 0.2 , 0.7 , and 0.9 .The numerical data are collected and presented graphically (see figs .(2) to (6)).

6.1. Non streaming case

In such a case the fluid is considered to be stationary in the initial state. The numerical data for different values of $a=\frac{\rho_1}{\rho_2},b=\frac{\rho_3}{\rho_2},$ are given as follows .

1 - For (a,b))=(0, 0.1) , (0.1 , 0.2) , (0.2 , 0.3) , (2 , 0.4) , (6, 0.3) , (7 , 0.1)
And $C^*=1$, corresponding to $g^*=0,U^*=0$. The unstable domains have been discovered to be $0<q<1.0463$, $0<q<0.9465$, $0<q<0.8466$, $0<q<1.1498$, $0<q<1.8496$, and $0<q<2.1496$,
while the stable domains are $1.0463<q<\infty$, $0.9465<q<\infty$, $0.8466<q<\infty$, $1.1498<q<\infty$, $1.8496<q<\infty$, and $2.1496<q<\infty$ see fig (2).

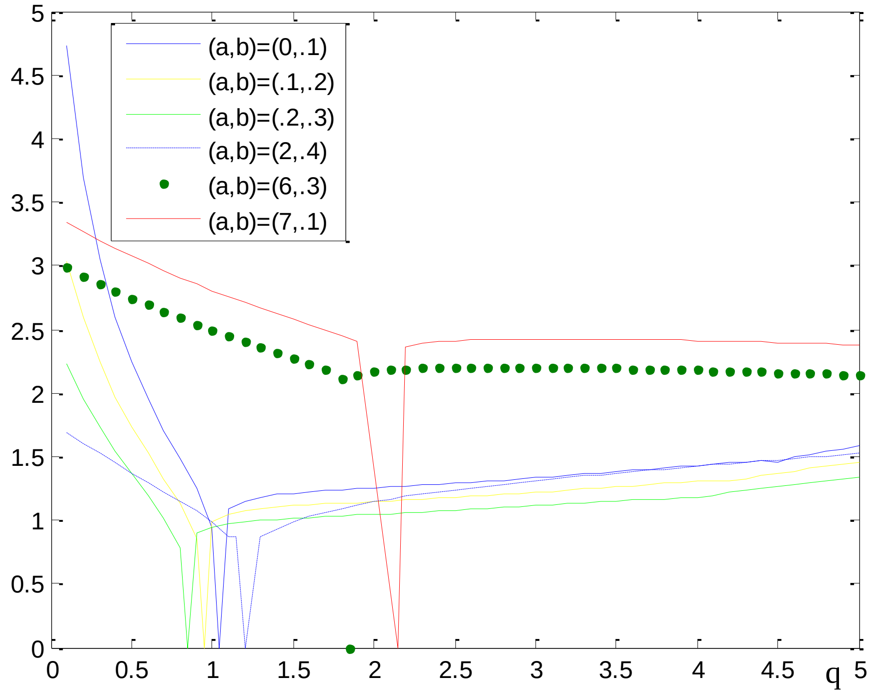

Fig.2. $C^*=1$, $g^*=0, U^*=0$, (a,b))=(0, 0.1) , (0.1 , 0.2) , (0.2 , 0.3) , (2 , 0.4) , (6, 0.3) , (7 , 0.1)and

$$\sigma^* = \frac{\sigma}{\sqrt{4\pi G\rho_2}}$$

2- For (a,b))=(0, 0.1) , (0.1 , 0.2) , (0.2 , 0.3) , (2 , 0.4) , (6, 0.3) , (7 , 0.1)

And $C^*=1$, corresponding to $g^*=0.5.1, U^*=0$. The unstable domains have been discovered to be $0< q <1.0462$, $0< q <9463$, $0< q <0.8465$, $0< q <1.1495$, $0< q <0.2487$, and $0< q < 0.5492$

while the stable domains are $1.0462< q <\infty$, $0.9463< q <\infty$, $0.8465< q <\infty$, $1.1495< q <\infty$, $0.2487< q <\infty$, and $0.5492< q <\infty$ .see fig (3)

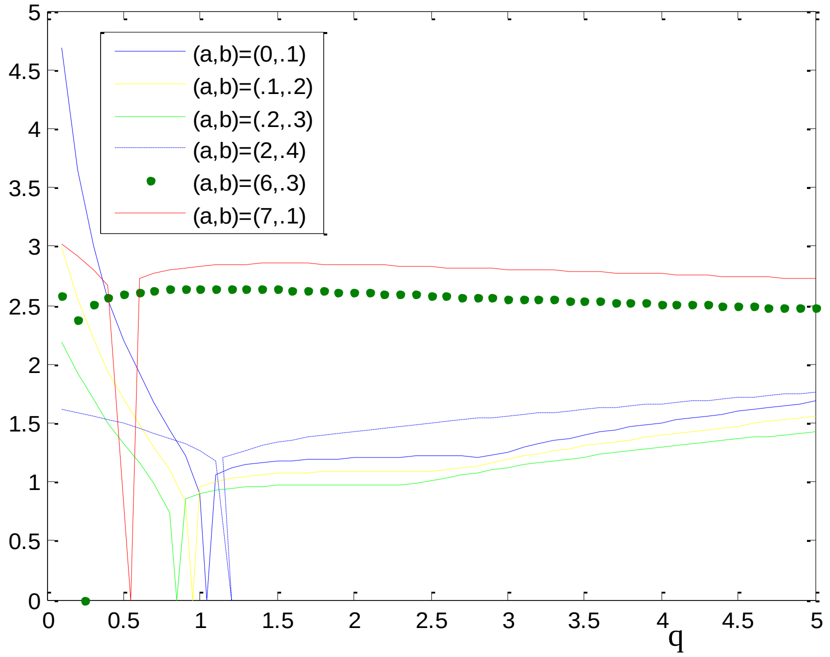

Fig.3. C^*=1, g^*=0.5.1,U^*=0, (a,b))=(0, 0.1) , (0.1 , 0.2) , (0.2 , 0.3) , (2 , 0.4) , (6, 0.3) , (7 , 0.1)and

$$\sigma^* = \frac{\sigma}{\sqrt{4\pi G\rho_2}}$$

6.2. Streaming case

1- For (a,b))=(0, 0.1) , (0.1 , 0.2) , (0.2 , 0.3) , (2 , 0.4) , (6, 0.3) , (7 , 0.1)

And C^*=1, corresponding to g^*=1,U^*=0.2 .The unstable domains have been discovered to be 0< q <1.2050, 0< q <0.9469, 0< q <0.8471,and 0< q <1.1497

while the stable domains1.2050< $q < \infty$, 0.9469< $q < \infty$, 0.8471< $q < \infty$, 1.1497< $q < \infty$,and stable along two value of a,b=(6, 0.3) , (7 , 0.1) in 0< $q < \infty$ .see fig (4).

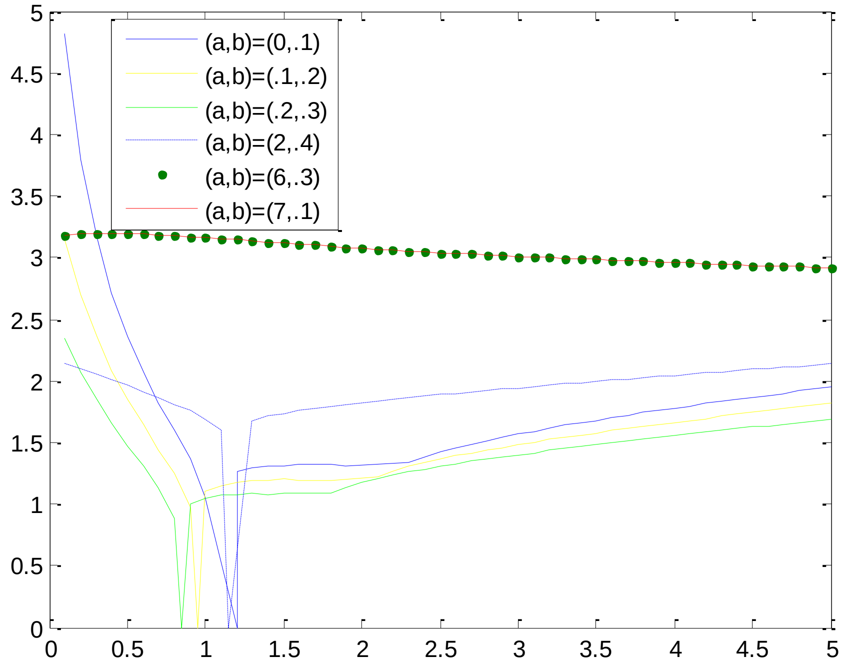

Fig .4. C^*=1, g^*=1,U^*=0.2, (a,b))=(0, 0.1) , (0.1 , 0.2) , (0.2 , 0.3) , (2 , 0.4) , (6, 0.3) , (7 , 0.1) and

$$\sigma^* = \frac{\sigma}{\sqrt{4\pi G\rho_2}}$$

2-For (a,b))=(0, 0.1) , (0.1 , 0.2) , (0.2 , 0.3) , (2 , 0.4) , (6, 0.3) , (7 , 0.1)

And C^* =1, corresponding to g^*=2,U^*=0.7. The unstable domains have been discovered to be $0<q<1.0477$, $0<q<0.9480$, $0<q<1.0485$, $0<q<1.1499$ and stable along two value of a,b=(6, 0.3) , (7 , 0.1) in $0<q<\infty$.see fig (5).

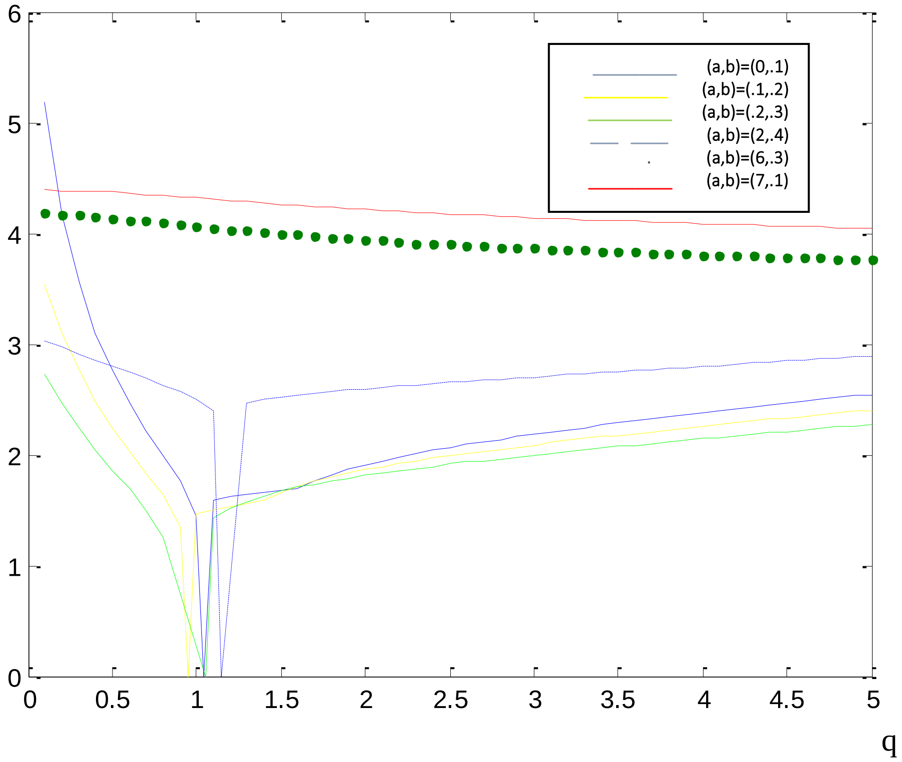

Fig.5. C^* =1, g^*=2,U^*=0.7, (a,b))=(0, 0.1) , (0.1 , 0.2) , (0.2 , 0.3) , (2 , 0.4) , (6, 0.3) , (7 , 0.1) and

$$\sigma^* = \frac{\sigma}{\sqrt{4\pi G\rho_2}}$$

3-For (a,b))=(0, 0.1) , (0.1 , 0.2) , (0.2 , 0.3) , (2 , 0.4) , (6, 0.3) , (7 , 0.1)

And C^* =1, corresponding to g^*=3, U^*=0.9 . The unstable domains have been discovered to be $0<q<1.0492$, $0<q<.9456$ $0<q<0.8470$, $0<q<1.1499$

and stable along two value of a,b=(6, 0.3) , (7 , 0.1) in $0<q<\infty$.see fig (6).

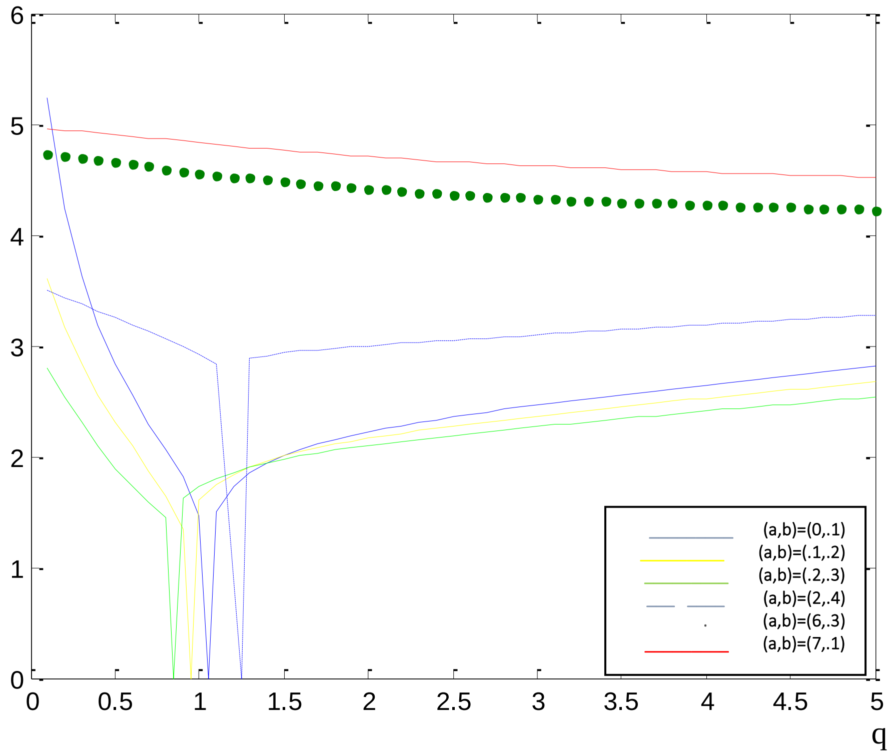

Fig.6. $C^* = 1$, $g^* = 3$, $U^* = 0.9$, (a,b))=(0, 0.1) , (0.1 , 0.2) , (0.2 , 0.3) , (2 , 0.4) , (6, 0.3) , (7 , 0.1) and

$$\sigma^* = \frac{\sigma}{\sqrt{4\pi G \rho_2}}$$

7. Conclusion

The numerical solution which has been obtained by using MATLAB may service as a tool to compared with its analytical counterpart to reach the following in which it may give rise to be in favor of the reliability of the above mentioned analytical method.

From this perspective,The figured out that the model completely stabilizes for both very long and short wavelengths with the same values of of a,b=(6, 0.3) , (7 , 0.1) ,U^*.=0.2,0.7,and 0.9 It is discovered that the unstable domains are growing with rising U values. For the same values of a,b , it is discovered that the model becomes entirely stable not only for short wavelengths but also for very long wavelengths, indicating that streaming has a destabilizing impact on the model for all short and long wavelengths.

In this study it can be discussed the oscillation effect which modified a lot of the desstabilizing with self – gravitrating force. This model's tendency toward instability is mostly determined by the weight force and densities ratios of the triple fluid layers with streaming velocity $\underline{u}_0^s = (0,0,U\cos\Omega t)$ of the triple fluids.

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