



# Analysis, Performance Improvement and Control Design of Heliostat System Based on Fractional Order PID Controller

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## Abstract

The purpose of this study to improve the heliostat sun tracking system behavior by using various control system are used to tune the system. Proportional Integral Derivative controller (PID) and the Fractional order PID (FOPID) controller are used to improve the pervious result, to improve the dynamic response of the system. Analysis and control system applied are used to provide to check the precision of heliostat tracking system angle by tracing the sun's rays. The FOPID uses proportional integral derivative controller which are provide more immense merits over the conventional PID controller like straightforward design, improved set point tracking, strong disturbance dismissal, and increased capacity to manage model linear or nonlinear in case real time applications. Mainly heliostat system has vital role in Concentrated solar power (CSP) system, especially when evaluate the efficiency of overall system. CSP technology uses a heliostat system to concentrate the sun's beam energy using mirrors to a solar receiver which convert it into heat energy, subsequently delivered to a conventional power cycle. High precision solar tracking systems are used to improve the effectiveness of energy conversion, Heliostat accuracy mainly relies on the speed to reach system's acceptance angle with minimum error.

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## 1. INTRODUCTION

With almost 300 years of history, concept of fractional order use derivatives and integrate technique from calculus is a well-known branch of mathematics that extends traditional integer calculus to arbitrary orders. L'Hospital and Leibniz created the first theory of fractional order derivative under the seventeenth-century premise [3]. Real time problems are defined described, remodeled and controlled are more completely precisely in fractional order technique than the traditional integral order methods.

in last six decades Conventional PID are consider the most common controller due to simplicity in implementation. However, there are guarantees of dynamic responses, large time delay and complex design in case nonlinear system, which are led to search for alternative solution to improve the PID controller resonance for our heliostat model [1].

Podlubny [3] has proposed fractional order equation for PI controller , traditional PID controller and extended the idea of another integral order PID controller to desired fractional order PID controller by use two new parameters which are indicate by  $PI^{-\lambda}$  and  $PI^{-\lambda}D^{\mu}$ . These controllers have two new parameters first one integrator order ( $\lambda^{-}$ ) and second of one differentiator coefficient order ( $-\mu$ ) and these two another parameter provides an added (DOF) degree of freedom to increase and enhancement the performance of controller, which make Fractional Order PID better than in performance comparing to conventional PID Controller. Padula [4] are demonstrated the benefits of the Fractional order (FOPID) controller over existing integer order controllers like PID through experimental to overcome the disturbance, our target in paper to minimize the integer absolute error to enhance the system resonance and sensitivity.

Industrial controllers are used to optimize whether in case a system transient or steady state characteristics designed according to several set of parameters like rise time, overshoot and settling time in case using time domain, or gain margin, phase margin and maximum sensitivity in case using frequency domain [5], [6].

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On the other hand, the fractional order PID (FOPID) controller revised or updated version from the conventional PID controller [3] using derivatives -integral Calculus. New research is using non-integer order derivative [7],[8]. are encouraging indicators that energy efficiency and building decarbonization are altering investment choices [3].

## 2. MATHEMATICAL MODELLING OF FRACTIONAL ORDER FOPID CONTROLLER

The fractional calculus (FC) propagates the operations of differentiation and integration of the non-integer orders  $d^n y/t^n$ , where n are represent the non-integral order. The generalized equation which are illustrate by using the calculus differentiation and integration for non-integer order operator  $D_t^\alpha$  in continuous domain are given by the equation

$$aD_t^\alpha = \begin{cases} \frac{d^\alpha}{dt^\alpha} & R(\alpha) > 0 \\ 1 & R(\alpha) = 0 \\ \int_a^t (d\tau)^{-\alpha} & R(\alpha) < 0 \end{cases}$$

where  $a$  &  $t$  is refer to the integration lower and upper limits  $\alpha$  are represent the fractional differ- entiation or integration order (derivation are represent by the positive sign and Integration are represent by the negative sign of  $\alpha$ ).

The derivative method are used in case using fractional order calculus are represented using Grünwald–Letnikov derivative Eq. (1) . The derivative are extended in fractional calculus that allows one to take the derivative a non-integer number of times as shown in Eqs. (3) and (4)

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \tag{1}$$

$$f''(x) = \lim_{h \rightarrow 0} \frac{f'(x+h) - f'(x)}{h} \tag{2}$$

Back Substitute by using Eq. (1) in Eq. (2)

$$f''(x) = \lim_{h_1 \rightarrow 0} \frac{\lim_{h_2 \rightarrow 0} \frac{f(x+h_1+h_2) - f(x+h_1)}{h_2} - \lim_{h_2 \rightarrow 0} \frac{f(x+h_2) - f(x)}{h_2}}{h_1} \tag{3}$$

Assuming that the (h) is converge synchronously.

$$= \lim_{h \rightarrow 0} \frac{f(x+2h) - 2f(x+h) + f(x)}{h^2} \tag{4}$$

Using Mean value theorem,

$$f^{(n)}(x) = \lim_{h \rightarrow 0} \frac{\sum_{0 \leq m \leq n} (-1)^m \binom{n}{m} f(x + (n-m)h)}{h^n} \tag{5}$$

Removing the limitation that n be a positive integer, it is reasonable to define:

$$\mathbb{D}^q f(x) = \lim_{h \rightarrow 0} \frac{1}{h^q} \sum_{0 \leq m \leq \infty} (-1)^m \binom{q}{m} f(x + (q-m)h) \tag{6}$$

Grünwald–Letnikov derivative equation can define and simplified by.

$$\Delta_h^q f(x) = \sum_{0 \leq m \leq \infty} (-1)^m \binom{q}{m} f(x + (q-m)h) \tag{7}$$

The Grünwald–Letnikov equation can simplified

$$aD_t^\alpha f(t) = \lim_{h \rightarrow 0} \frac{1}{h^\alpha} \sum_{m=0}^{\frac{t-a}{h}} (-1)^m \binom{q}{m} f(x - mh) \tag{8}$$

### 3. CONTROLLER DESIGN FOR HELIOSTAT SYSTEM

Our study the behavior of the system, used two controllers PID controller and Fractional Order PID. Conventional PID controller are produced a control signal that can dynamically control and minimize the error or the difference between the output and the desired setpoint of a certain system. PID controller we can express with Eq. (9) and Fig.1

$$u(t) = k_p e(t) + k_i \int_0^t e(t) dt + k_d \frac{de(t)}{dt} \quad (9)$$

where  $k_p$ ,  $k_i$  and  $k_d$  are parameters which represent the controller proportional gain , integral gain , and derivative gain, respectively and signal  $e(t)$  gives the error at different instances of time.

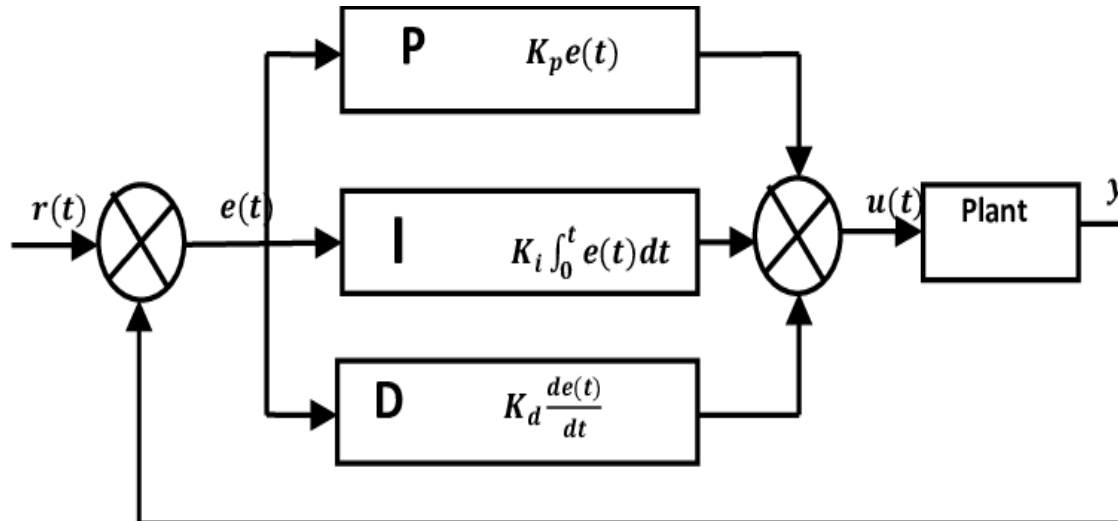


Fig.1 Schematic diagram of PID Controller include system.

PID controller calculate the control signal  $u(t)$  by adjusting gradually the error signal  $e(t)$  , each part from controller is improve the system by certain value. The distinctive characteristic of the PID controller is the ability influence to system by use the three control terms of proportional, integral and derivative to controller to provide the optimal and accurate control. Derivative part is adding gain (N) as filter, to maintain output as consistently [10].

### 4. FRACTIONAL ORDER PID (FOPID) CONTROLLER

Fractional order Calculus equation is used to implement and design Fractional order controller by addition parameters to increase degree of Freedom of Controller (DOF) [11]. Fractional orders are expressed in time domain are express using Eq. (10)

$$u_{FOPID}(t) = k_p e(t) + k_i \frac{d^{-\lambda} e(t)}{dt^{\lambda}} + k_d \frac{d^{\mu} e(t)}{dt^{\mu}}$$

Also, we can express as

$$u_{FOPID}(t) = k_p e(t) + k_i \mathcal{D}^{-\lambda} e(t) + k_d \mathcal{D}^{\mu} e(t) \quad (10)$$

where  $k_p$ ,  $k_i$  and  $k_d$  are parameters which represent the controller proportional gain, integral gain , and derivative gain, respectively and signal  $e(t)$  gives the error at different instances of time. *Two parameters are added* ( $\lambda$ ) and ( $\mu$ ) , which adding two degrees of freedom to improve the performance of our controller , we can express controller in s domain using Laplace transform in Eq. (10) to get Eq. (11) and Fig.2.

$$u_{FOPID}(s) = (k_p + k_i \frac{1}{s^{\lambda}} + k_d s^{\mu}) E(s) \quad (11)$$

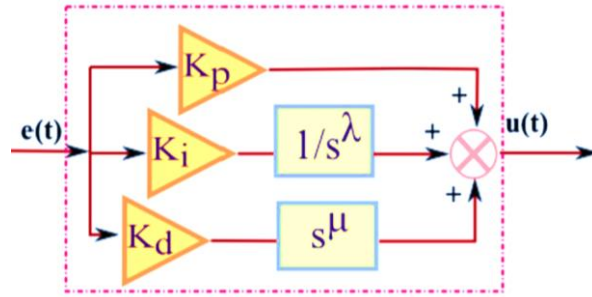


Fig.2 Schematic diagram of FOPID Controller.

5. RESULTS AND DISCUSSION

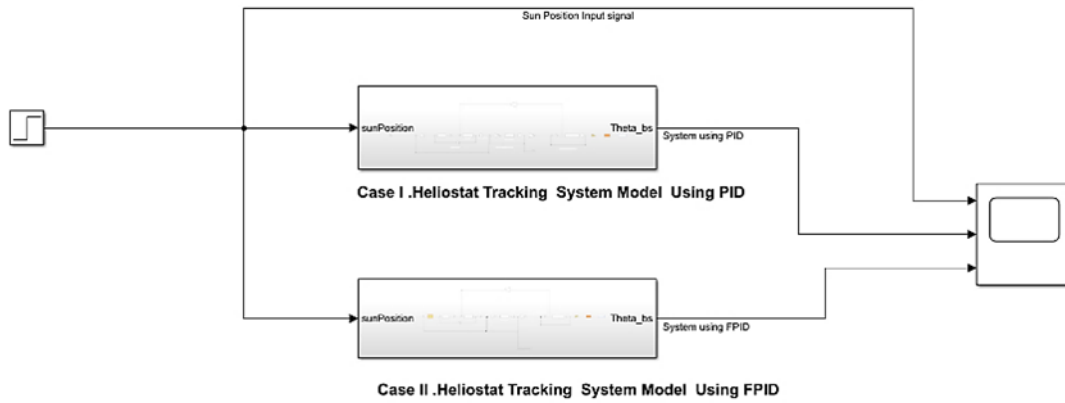


Fig.3 Simulink result by Applying step input signal.

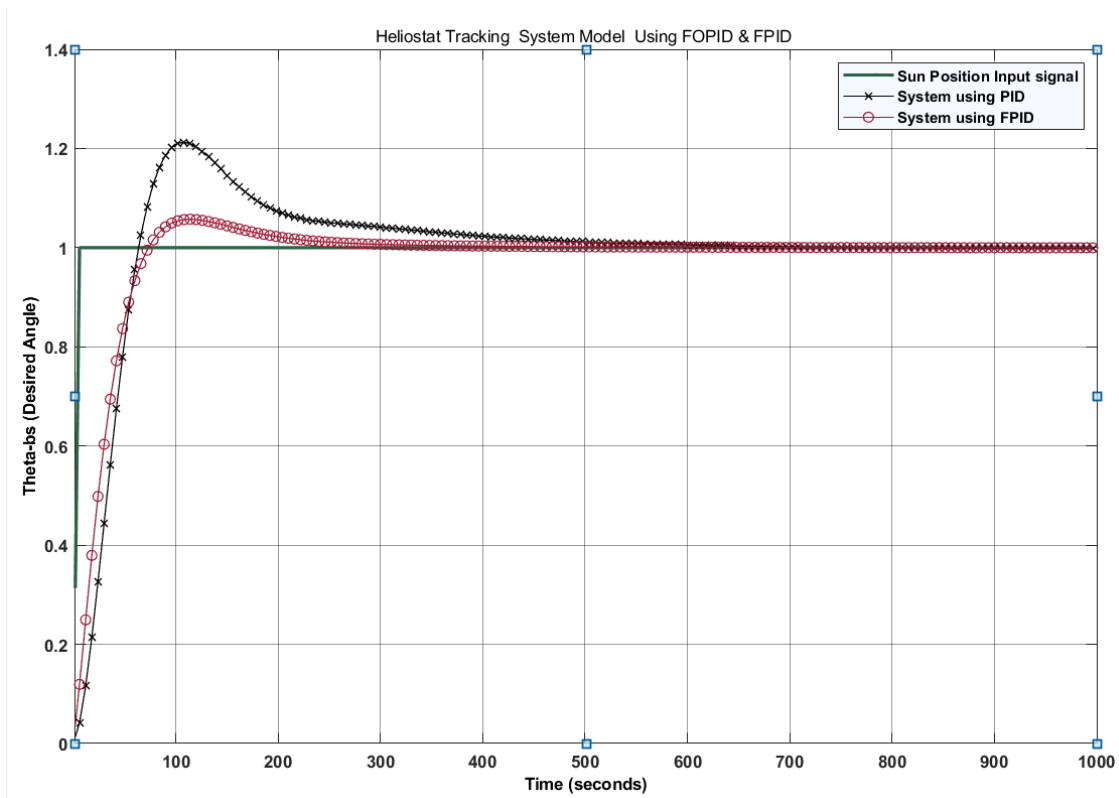


Fig.4 Simulation result using step Function.

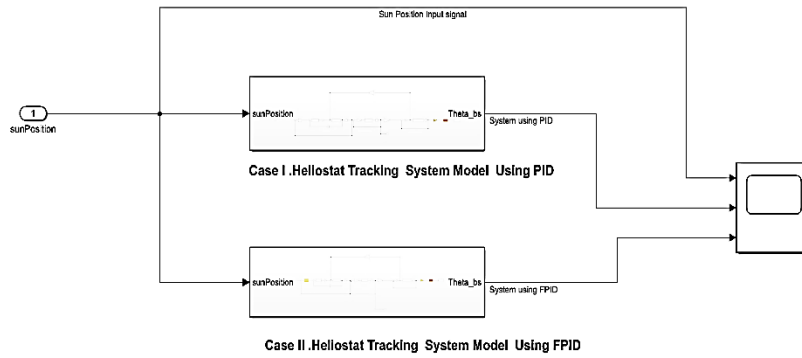


Fig.5 Simulink result by Applying Real input signal sun position for specific period (12 Hour)

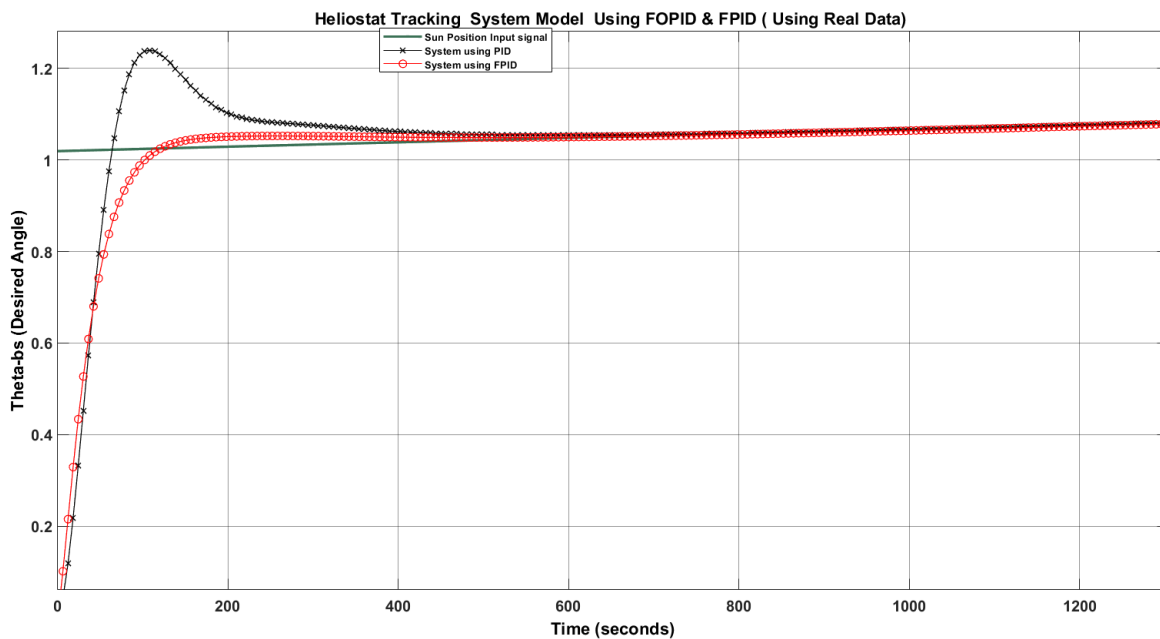


Fig.6 Simulation result by Applying Real sun position for specific period (12 Hour)

We are checking the performance of the heliostat tracking system model using two controller, traditional PID and Fractional order PID as we see in Fig [3], [5]. Comparing the overshoot in two cases by applying step function and also when apply real data for specific time (12 hours) which are consider Solar radiation period.

From two fig. [4], [6] the response of the system overshoot in case of using PID controller is higher than Fractional order PID (FOPID), but settling time for the controller is not satisfied and need to optimized and increase the speed of the system.

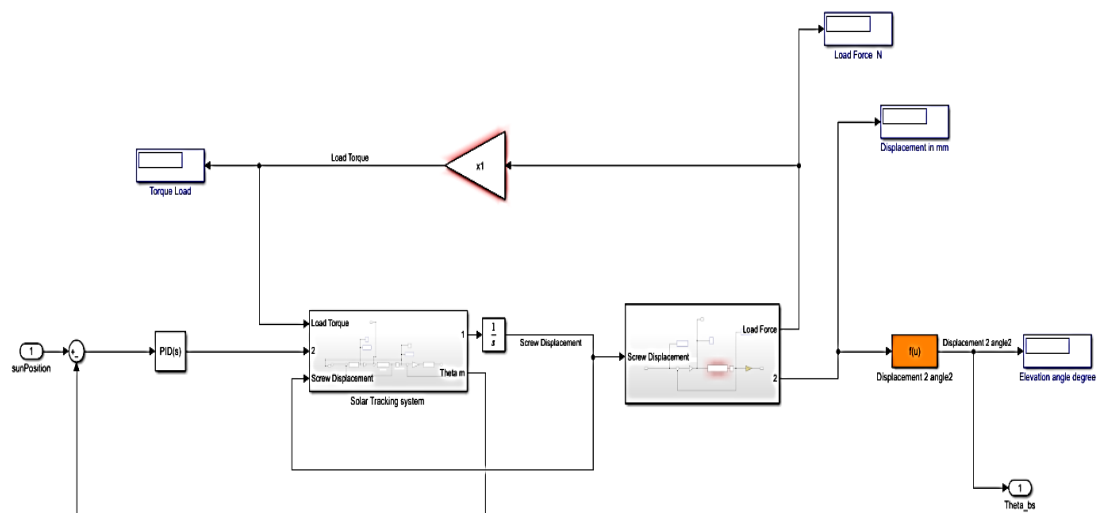


Fig. 7 Simulink Model for Heliostat system Using PID Controller

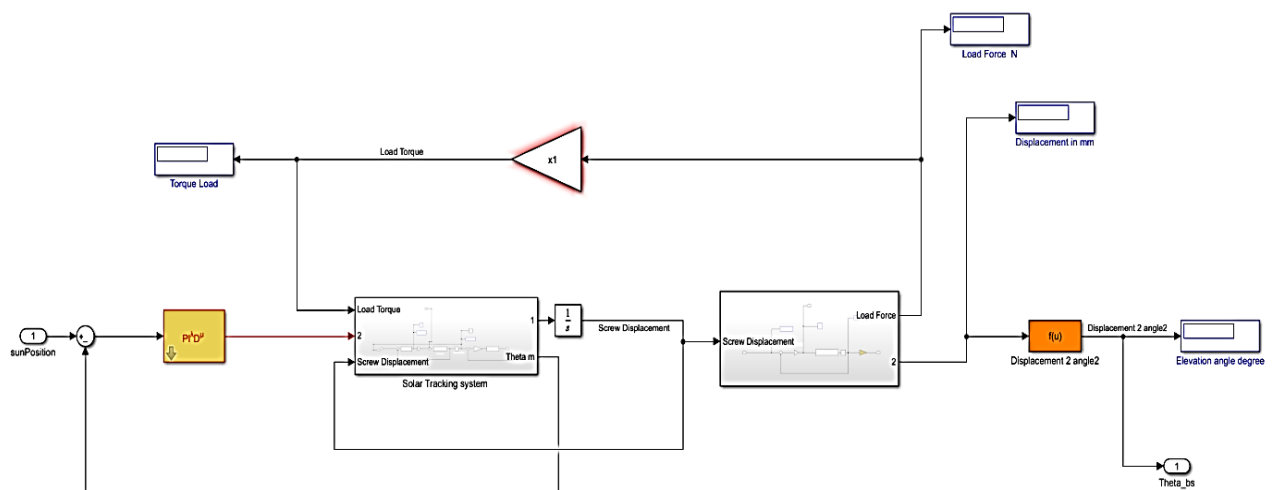


Fig. 8 Simulink Model for Heliostat system Using FOPID Controller

**PID & Fractional Order PID Controllers Parameters & Result:**

Table.1 PID & FOPID controllers' parameter

Parameters	$K_p$	$K_I$	$K_d$	$\lambda$	$\mu$	Overshoot	Rise Time
Heliostat system Using PID Controller	0.00997	1.66E-05	1.356	-	-	15.6 %	47.746 s
Heliostat system Using FOPID Controller	0.00001	0.001	0.8	0.001	0.8	0.5%	65.963 s

**6. CONCLUSION**

This Work show the effect of applying of different sample inputs in our heliostat design system, in addition to apply real data for sun position for 12 hour which are consider interval for solar radiation for heliostat can track and reflect to Central solar receiver (CSR) tower. The First part of this work aims to design heliostat system completely with control algorithm to guide the heliostat to optimal position in two directions (azimuth and

elevation angle). Future work, we can optimize the result for system speed by using optimization technique like Sooty Tern Optimization Algorithm (STOA), Gray Wolf optimization technique (GWO).

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