

COMPARING STANDARD REGRESSION AND MULTILEVEL REGRESSION MODEL FOR HIERARCHICAL DATA IN HIGH SCHOOL GRADE AVERAGE POINT (GPA)

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Abstract

This paper compares the performance of multilevel regression model and traditional regression model of hierarchical data. Multilevel regression has been used to describe an analytical approach that allows the simultaneous examination of the effects of group-level and individual-level variables on individual level outcomes. The aim of this paper is to introduce the multilevel regression model that allow to explicitly incorporate the hierarchical nature of the data into the analysis, to incorporate variables measured at different levels of hierarchy, and to examine how regression relationships vary across clusters, and compare their performance with traditional regression model on a hierarchically dataset. The comparison is based on two main criteria: the bias of the estimated regression coefficients and the size of testing significance of each regression coefficient. The results suggest that the underestimation of standard error in standard regression artificially increases the significance of hypothesis tests, and the multilevel regression models had a better model fit than standard regression.

A multilevel model of this type is provided by many computer packages, including MLwin 2.36.

Some key words: Hierarchical data; multilevel linear model; Iterative generalized least square; three level regression models.

Introduction

The individuals are influenced by the social groups or contexts to which they belong, and that those groups are in turn influenced by the individuals who make up that group. The individuals and the social groups are conceptualized as a hierarchical system of individuals nested within groups, with individuals and groups defined at separate levels of this hierarchical system. Naturally, such systems can be observed at different hierarchical levels, and variables may be defined at each level. The research into the relationships between variables characterizing individuals and variables characterizing groups are generally referred to as multilevel research (Hox 2010).

Traditional statistical methods ignore the correlation of outcomes within clusters and tend to underestimate standard errors. This artificially increases the significance of hypothesis tests, increasing the risk of falsely concluding that significant associations exist. Additionally, they do not allow one to incorporate characteristics measured at different levels of the hierarchy (Austin 2001). One assumption of the single-level multiple regression models are that the measured units are independent. Specifically, we assume that the residuals are uncorrelated with one another. If data are grouped and we have not taken account of group effects in our regression model, the independence assumption will not hold.

Fit a single-level model and ignore structure substantively you would not measure the importance of context. Technically, your standard errors would be too small, leading to incorrect inferences (concluding a high risk of Type I error). One way to allow for grouping is to include a set of dummy variables for groups as explanatory variables in the model. A model with dummy variables for groups is called a fixed effects model but, there are problems with adopting this approach when the number of groups is large. If the number of groups is large, there will be a large number of additional parameters to estimate. The effects of group-level predictors cannot be estimated simultaneously with group residuals. Fit a single level model with group level predictors unable to assess the degree of between group variations. Multilevel models enables researchers to investigate the nature of between group variability, and the effects of group-level characteristics on individual outcomes, corrects standard errors and estimate the between-group variance (Raudenbush et al. 2002)

In multilevel research, the data structure in the population is hierarchical, and the sample data are a sample from this hierarchical population. Thus, in educational research, the population consists of schools and pupils within these schools, and the sampling procedure often proceeds in two stages: First, we take a sample of schools, and next we take a sample of students within each school. Of course, in real research one may have a convenience sample at either level, or one may decide not to sample students but to study all available students in the sample of schools. Nevertheless, one should keep firmly in mind that the central statistical model in multilevel analysis is one of successive sampling from each level of a hierarchical population (Auda et al. 2012).

We look at the relationship between an outcome or response variable which is the score achieved by students in an examination and a predictor or explanatory variable obtained by the same students. In the past it would have been necessary to decide whether to carry out this analysis at school level or at student level. Both of these single-level analyses are unsatisfactory. In a school-level or aggregate analysis, the mean exam score be calculated for each school. Ordinary regression would then be used to estimate a relationship between these aggregate variables. The main problem here is that it is far from clear how to interpret any relationship found. Any causal interpretation must include students, and information about these has been discarded. In practice it is possible to find a wide variety of models, each fitting the data equally well, but giving widely different estimates. In student-level analysis an average relationship between the scores would be estimated using data for all students. The variation between schools could be modeled by incorporating separate terms for each school. This procedure is inefficient, and inadequate for the purpose of generalization. It is inefficient because it involves estimating many times more coefficients than the multilevel procedure; and because it does not treat schools as a random sample it provides no useful quantification of the variation among schools in the population more generally. By focusing attention on the levels of hierarchy in the population, multilevel modeling enables the researcher to understand where and how effects are occurring.

The famous estimation methods for multilevel modeling are namely expectation maximization algorithm, Fisher scoring, iterative generalized least square algorithms and the maximum likelihood methods such as full information maximum likelihood or restricted/residual maximum likelihood. The restricted/residual maximum likelihood assumes that the distributions of residuals of level-1 and level-2 are normal, and the sample size is large for both levels; the number of "groups" (level-2 units) and the group's size (number of level-1 units per group). In general, a large number of groups are more important than a large group size or a large number of individuals per group (Hox 2010).

Before conducting a multilevel model analysis, a researcher must decide on several aspects, including which predictors are to be included in the analysis, if any. Second, the researcher must decide whether parameter values will be fixed or random. Fixed parameters are composed of a constant over all the groups, whereas a random parameter has a different value for each of the groups.

Data: A questionnaire has been administered on a random three-stage cluster sampling of 1543 students and 43 schools with 83 class of high school in Dakahlia Governorate where Dakahlia Governorate has 98482 secondary students and 173 schools with 4564 class. The sample has a hierarchical structure. Each student was nested within one and only one class, and each class was nested within one and only one school. One data set is the student- level characteristic such as absence rate, intelligent level and GPA in past semester. The second data set is the class level characteristics such as teacher age, teacher experience, teacher educational qualification (specialization), teacher salary and the weakly teaching hours. The last data set is the school level characteristics such as the existence of appropriate library, the existence of appropriate laboratory and the existence of appropriate yard.

Methods:

We specify three regression models that will be used in our analysis of grade point average (GPA). The first is a single level regression model that ignores potential nesting of students within classes and/or schools, the second is a two – level regression model that assumes that students are nested within classes, and the third is a three – level regression model that assumes that students are nested within classes that are nested within schools.

Random intercept model

The simplest multilevel model with a single explanatory variable is

$$y_{ij} = \beta_0 + \beta_1 x_{ij} + u_j + e_{ij}$$

The intercept for a given group j is $\beta_0 + u_j$, i.e. it will be higher or lower than the overall intercept by an amount u_j . A multilevel model can be thought of as consisting of two components: a fixed part which specifies the relationship between the mean of y and explanatory variables, $\beta_0 + \beta_1 x_{ij}$, and a random part that contains the level 1 and 2 residuals

$$u_j + e_{ij}$$

the model is sometimes written in the form of two equations as

$$y_{ij} = \beta_{0j} + \beta_1 x_{ij} + e_{ij}$$

$$\beta_{0j} = \beta_0 + u_j$$

Random slope model

A random intercept model assumes that the relationship between y and x is the same for each group, i.e. the slope β_1 is fixed across groups. We can relax this constraint by allowing the slope to vary randomly across groups, leading to a random slope

$$y_{ij} = \beta_0 + \beta_1 x_{ij} + u_{0j} + u_{1j} x_{ij} + e_{ij}$$

This can also be written as:

$$y_{ij} = \beta_{0j} + \beta_{1j}x_{ij} + e_{ij}$$

$$\beta_{0j} = \beta_0 + u_{0j}$$

$$\beta_{1j} = \beta_1 + u_{1j}$$

The random effects u_{0j} and u_{1j} are assumed to follow normal distributions with zero means, variances $\sigma_{u_0}^2$ and $\sigma_{u_1}^2$ respectively, and covariance $\sigma_{u_0u_1}$. Now the slope of the average regression line is β_1 (sometimes called the grand mean slope) and the slope of the line for group j is $\beta_1 + u_{1j}$. The covariance $\sigma_{u_0u_1}$ is the covariance between the group intercepts and slopes.

In the analysis of GPA values we have included the explanatory variables defined at the lowest level of the hierarchical structure, student characteristics such as intelligent and gender. One particular benefit of multilevel modeling, however, is the ability to explore the effects of group-level variables while simultaneously allowing for the possibility that y may be influenced by unmeasured group factors. Variables defined at level 2 are often called contextual variables and their effects on an individual's y -value are called contextual effects. If contextual effects are of interest, it is particularly important to use a multilevel modeling approach because the standard errors of coefficients of level 2 variables may be severely underestimated when a single-level model is used.

A level 2 explanatory variable can be included in a multilevel model in exactly the same way as a level 1 variable. For example, if we have a level 1 variable x_{1ij} and a level 2 variable x_{2j} the random intercept model becomes

$$y_{ij} = \beta_0 + \beta_1 x_{1ij} + \beta_2 x_{2j} + u_j + e_{ij}.$$

Contextual variables may come from a number of sources. Data may be collected at level 2, e.g. community surveys in which key figures in the community are interviewed. Contextual data may also derive from level 1 data that is aggregated to form level 2 variables. These data may come from an external source, e.g. a Census, or the same source as the level 1 data to be analyzed.

Cross-level interactions

As in multiple regressions, we can allow for the possibility that the effect of one explanatory variable on y depends on the value of another explanatory variable. Recall that such effects are called interaction effects and are represented in a model by including the product of the interacting variables as explanatory variables. Interactions can also be included in a multilevel model and these can be between any pair (or larger set) of variables, regardless of the level at which they are defined. An interaction between a level 1 variable and a level 2 variable is known as a cross-level interaction.

A multilevel model for group effects

The simplest possible regression model is a model for the mean of dependent variable y with no explanatory variables

$$Y_{ijk} = \beta_0 + e_{ijk} + u_{jk} + v_k$$

Where y_{ijk} is the value of y for the i th student ($i=1, \dots, 1543$) in class ($j=1 \dots 83$) in school ($k=1, \dots, 34$), β_0 is the mean of GPA in all schools, and e_{ijk} is the 'residual' for the i th student, u_{jk} the effect of class j , and v_k the effect of school k .

In a three level we split the residual into three components, corresponding to the three levels in the data structure. We denote the group-level residuals, by u_{jk} , v_k and the individual residuals by e_{ijk} .

The variance partition coefficient (VPC) measures the proportion of total variance that is due to differences between groups. In the three levels the residuals split into three components, corresponding to the three levels in the data structure:

$$VPC = \frac{\sigma_v^2}{\sigma_v^2 + \sigma_u^2 + \sigma_e^2}$$

The VPC ranges from 0 (no group differences) to 1 (no within-group Differences).

Testing for group effects (LR test)

We can test the null hypothesis that there are no group differences, $H_0: \sigma_u^2=0$, by comparing models in a likelihood ratio test. The likelihood ratio test statistic is calculated as:

Suppose model I is nested within model II

$$2 \times \text{Log} \left(\frac{L_{II}}{L_I} \right) = 2 \times (\text{Log} L_{II} - \text{Log} L_I) \sim \chi_q^2$$

Where: L_I and L_{II} are the values of the single-level and multilevel models respectively, q is the number of additional parameters in model II, $-2\text{Log} L$ is called the deviance.

LR tests with halved P-value (one tailed P-value) for tests of variance and covariance parameters is recommended (Snijders et al. 2012)

Results and discussion

All regression models were estimated using the statistical software package MLwin Release 2.36. The single, two, and three level regression models were all estimated as linear mixed effects models. Model fit indices, variance parameter, and regression coefficient estimates for all three models are reported.

The null model is written as:

$$Z_{ijk} = \beta_{0ijk} + v_k + u_{jk} + e_{ijk}$$

Output from MLwin software showing model fit statistics and parameter estimates for a null model:

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O. Equation
Zijk ~ N(AB, Ω)
Zijk = β0ijkCons
β0ijk = 71.519(3.016) + vjk + ujk + eijk
[vjk] ~ N(0, Ωv) : Ωv = [299.383(74.915)]
[ujk] ~ N(0, Ωu) : Ωu = [10.692(4.658)]
[eijk] ~ N(0, Ωe) : Ωe = [201.321(7.447)]
-2*loglikelihood(IGLS Deviance) = 12737.044(1543 of 1543 cases in use)

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The only coefficient in the fixed part of the model is the intercept and this is estimated to be 71.54, with a standard error or 3.016. Thus, the student score, the mean student is predicted to score 71.54 out of 100. The z-ratio for this parameter estimate

$$z = \frac{71.54}{3.016} = 23.72$$

The deviance statistic $D = 12737$, the difference in deviances between two nested models gives the likelihood ratio test statistic for comparing the fit of the two models; students are clustered by classrooms and schools.

The school level VPC (variance partition coefficient) is calculated as

$$VPCv = \frac{\sigma_v^2}{\sigma_v^2 + \sigma_u^2 + \sigma_e^2} = 0.59$$

The class level VPC is calculated as

$$VPCu = \frac{\sigma_u^2}{\sigma_v^2 + \sigma_u^2 + \sigma_e^2} = 0.02$$

The student level VPC is calculated as

$$VPCe = \frac{\sigma_e^2}{\sigma_v^2 + \sigma_u^2 + \sigma_e^2} = 0.39$$

59% of variation in GPA scores lies between schools, 2% lies within schools between classes and 39% lies within classes between students. Thus the most variation was in schools.

Adding student level predictor variables

In student-level analysis, the exploratory analyses extract important variables are intelligent and sex. We will model the effects of these interventions by including binary indicator variables for sex, GPA in past semester and intelligent score. The model is written as:

$$Z_{ijk} = \beta_{0ijk} \text{Cons} + \beta_{1ijk} \text{Int}_{ijk} + \beta_{2ijk} \text{Sex}_{ijk} + \beta_{3ijk} \text{Deg}_{ijk} + v_k + u_{jk} + e_{ijk}$$

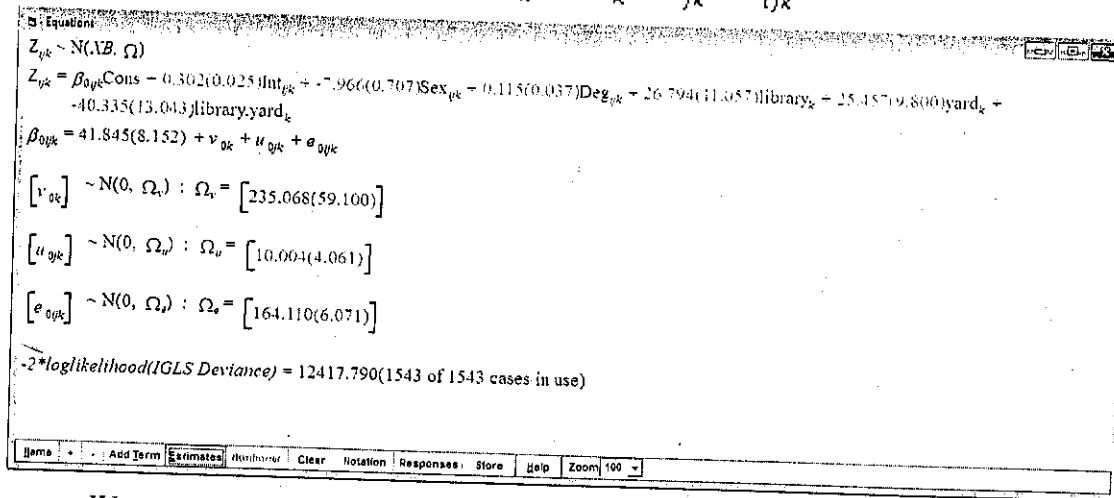
The coefficient on intelligent is 0.302, this effect is highly statistically significant with a z-ratio of 12.08 with p-value=0.0001. The coefficient on sex is -7.957, this effect is highly statistically significant with a z-ratio of 11.25 and p-value=0.0001. The coefficient on degree is 0.115, this effect is highly statistically significant with a z-ratio of 3.01 and p-value=0.0019. Adding intelligent, sex and degree reduces the school level variance and the student level variance student level variance. The decline level variance in the school level variance shows that there are large differences in students' GPA between schools.

The deviance statistic for this model is $D = 12427$. LR tests ($\chi^2 = 309.96, p < 0.001$). The LR test therefore confirms that the additional predictors significantly improve the fit of the model.

Adding school level predictor variables

In School-level analysis, the exploratory analyses extract the important variables, that the presence of appropriate school library and suitable yard, We model the effects of these two interventions by including binary indicator variables for library and yard along with their interaction. The model is written as:

$$Z_{ijk} = \beta_{0ijk} \text{Cons} + \beta_{1ijk} \text{Int}_{ijk} + \beta_{2ijk} \text{Sex}_{ijk} + \beta_{3ijk} \text{Deg}_{ijk} + \beta_{4k} \text{yard} + \beta_{5k} \text{library}_k + \beta_{6k} \text{library}_k * \text{yard}_k + v_k + u_{jk} + e_{ijk}$$



We can perform an LR test to confirm that the additional predictors significantly improve the fit of the model. The ($\chi^2 = 9.29, p < 0.001$). The LR test therefore confirms that the additional predictors significantly improve the fit of the model.

Adding class level variable

The implementation of teacher characteristics such as the experience and specialization was carried out at the classroom level. The classroom level variance provides a measure of the extent to which classrooms vary in this respect. The extents to which classrooms vary across four conditions:

- Neither experience nor Specialization
- Specialization only
- Experience only
- Both experience and Specialization

We can explore this hypothesis by estimating separate classroom level variances for each of the four conditions. The four sets of classroom effects $u_{7jk}, u_{8jk}, u_{9jk}$ and u_{10jk} are modeled as independent.

The model is written as:

$$Z_{ijk} = \beta_{0ijk} \text{Cons} + \beta_{1ijk} \text{Int}_{ijk} + \beta_{2ijk} \text{Sex}_{ijk} + \beta_{3ijk} \text{Deg}_{ijk} + \beta_{4k} \text{yard} + \beta_{5k} \text{library}_k + \beta_{6k} \text{library}_k * \text{yard}_k + v_k + u_{7jk} \text{neither} + u_{8jk} \text{exp}_k \text{only} + u_{9jk} \text{spe}_k \text{only} + u_{10jk} \text{exp}_k \text{and spe}_k + e_{ijk}$$

$$v_k \sim N(0, \sigma_u^2)$$

$$\begin{pmatrix} U_{7jk} \\ U_{8jk} \\ U_{9jk} \\ U_{10jk} \end{pmatrix} \sim N \left(\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{u7}^2 & & & \\ 0 & \sigma_{u8}^2 & & \\ 0 & 0 & \sigma_{u9}^2 & \\ 0 & 0 & 0 & \sigma_{u10}^2 \end{pmatrix} \right)$$

$$e_{ijk} \sim N(0, \sigma_e^2)$$

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$Z_{jk} \sim N(\mathbf{X}_j \mathbf{B}, \Omega)$

$Z_{jk} = \beta_{0jk} \text{Cons} + 0.301(0.024) \text{Int}_{ijk} + -7.643(0.343) \text{Sex}_{jk} + 0.115(0.037) \text{De}_{jk} + 24.786(11.124) \text{library}_{jk} - 23.686(9.765) \text{yard}_{jk} + -38.384(13.065) \text{library.yard}_{jk} + u_{7jk} \text{both}_{jk} + u_{8jk} \text{neither}_{jk} + u_{9jk} \text{exp_only}_{jk} + u_{10jk} \text{spec_only}_{jk}$

$\beta_{0jk} = 43.204(7.999) + v_{0jk} + e_{0jk}$

$[v_{0jk}] \sim N(0, \Omega_v) : \Omega_v = [238.249(34.290)]$

$\begin{bmatrix} u_{7jk} \\ u_{8jk} \\ u_{9jk} \\ u_{10jk} \end{bmatrix} \sim N(0, \Omega_u) : \Omega_u = \begin{bmatrix} 11.117(0.933) & & & \\ 0 & 16.603(13.719) & & \\ 0 & 0 & 18.373(13.900) & \\ 0 & 0 & 0 & 7.666(0.279) \end{bmatrix}$

$[e_{0jk}] \sim N(0, \Omega_e) : \Omega_e = [164.454(5.987)]$

$-2 * \log \text{likelihood} / \text{IGLS Deviance} = 12404.705(1543 \text{ of } 1543 \text{ cases in use})$

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An LR test comparing this model to one which assumes a constant class level variance across the four conditions strongly rejected the constant model ($\chi^2 = 13.34$, $P = 0.009$). Thus we find there is evidence that class level heterogeneity varies across the study condition.

Discussion:

The availability of a practical method for fitting multilevel models with many random error terms raises a number of important considerations which are counterparts and extensions to those arising in ordinary least squares model. Thus, for example, decisions are required concerning which error parameters should be included; whether there is a prior order in which they should be introduced; how one interprets the estimates; the use of residuals at different levels and so forth. There is also the general issue of how to deal with coefficients which may be treated either as fixed or random. It is to be hoped that extensive practical use of these models will provide the experience for forming sound judgments on these issues.

In this search, we have used three different model specifications to analyze GPA. One specification was the single level regression model that is typically used to analyze GPA, and the other two were multilevel regression models that recognized the hierarchical nature of the data. The two-level regression model recognized the nesting of students within classes, and the three-level regression model considered students as nested within classes and classes as nested within school.

The multilevel regression models were estimated by Iterative Generalized Least Square (IGLS). We found that multilevel regression models, in this case the three-level regression model, should be considered in analysis of GPA, as indicated by goodness of fit statistics (Max Log L).

Table 1: Regression model results: parameters estimates, variance parameters and model fit indices

	Intercept Only	Single-level	Two-level	Three-level
Parameter estimates				
Fixed Part	Coef.(s.e.)	Coef.(s.e.)	Coef.(s.e.)	Coef.(s.e.)
Intercept	71.549(3.016)	56.83(3.84)	41.84(8.15)	43.204(7.99)
IQ		0.115(0.037)	0.11(0.03)	0.115(0.037)
Int		0.302(0.025)	0.302(0.025)	0.301(0.024)
Sex		-7.95(0.707)	-7.96(0.707)	-7.64(0.343)
Library			26.79(11.05)	24.78(11.12)
Yard			25.45(9.8)	23.68(9.765)
Library*Yard			40.33(13.04)	-38.38(13.06)
Neither				-3.586(2.130)
Spec_only				000(0.000)
Exp_only				0.653(1.956)
Both				1.210(1.184)
Variance Parameters				
$var(e_{ijk})$	201.32(7.44)	164.08(6.07)	164.11(6.07)	164.45(5.98)
$var(u_{jk})$	10.69(4.65)	10.14(4.07)	10.004(4.06)	<div style="border: 1px solid black; padding: 5px; display: inline-block;"> 10.91(4.65) 18.31(14.69) 11.53(15.14) 7.655(9.181) </div>
$var(v_k)$	299.38(74.94)	284.72(71.16)	235.06(59.1)	238.24(34.29)
Model fit indices				
Deviance	12737.04	12427.19	12417.79	12404.7

Table 1 presents the parameter estimates and standard error for all models. In this table, the intercept-only model estimates the intercept as 71.54, which is simply the average across all schools, classes and students. The variance of the student level residual errors $var(e_{ijk})$ is estimated as 201.32. The variance of the class level residual errors $var(u_{jk})$ is estimated as 10.69. The variance of the school level residual errors $var(v_k)$ is estimated as 299.38. All parameter estimates are significant. The interclass correlation measure homogeneity of the observed responses within a given cluster which equal $\rho_v = 0.59$ and $\rho_{vu} = 0.61$ thus, 59% of the variance of the GPA is at school level and 61% of the variance of the GPA is at class and school level.

The second model includes student gender, degree and intelligent as explanatory variables. The regression coefficients for all three variables are significant. The variance of the student level residual errors $var(e_{ijk})$ is estimated as 164.08. The variance of the class level residual errors $var(u_{jk})$ is estimated as 10.14. The variance of the school level residual errors $var(v_k)$ is estimated as 284.72. The third model includes the effect of school characteristic by including two binary variables for existence of suitable library and suitable yard along with their interaction.

. The variance of the student level residual errors $\text{var}(e_{ijk})$ is estimated as 164.11. The variance of the class level residual errors $\text{var}(u_{jk})$ is estimated as 10.004. The variance of the school level residual errors $\text{var}(v_k)$ is estimated as 235.06.

The fourth model includes the effect of class characteristic by including four binary variables for teacher characteristics. The variance of the student level residual errors $\text{var}(e_{ijk})$ is estimated as 164.45. The variance of the class level residual errors $\text{var}(u_{jk})$ is estimated as [10.92 18.31 21.53 7.65]. The variance of the school level residual errors $\text{var}(v_k)$ is estimated as 238.24.

Table 2: Comparison of ordinary least square Regression and Three multilevel model: parameters estimates, parameters variance and model fit indices

	Three-level	Ordinary least square Estimation
Parameter estimates		
Fixed Part	Coef.(s.e.)	Coef.(s.e.)
Intercept	43.204(7.99)	59.58(2.98)
Deg.	0.115(0.037)	0.073(0.045)
Int	0.301(0.024)	0.29(0.03)
Sex	-7.64(0.343)	-8.36(0.87)
Library	24.78(11.12)	13.24(1.96)
Yard	23.68(9.765)	9.09(1.65)
Library*Yard	-38.38(13.06)	-16.97(2.329)
Neither	-3.586(2.130)	-27.50(1.44)
Spec only	000(0.000)	000(0.000)
Exp only	0.653(1.956)	-7.12(1.40)
Both	1.210(1.184)	2.47(1.13)
Parameters Variance		
$\text{var}(e_{ijk})$	164.45(5.98)	
$\text{var}(u_{jk})$	[16.9 18.3 21.5 7.6]	
$\text{var}(v_k)$	238.24(34.29)	
Deviance	12404.7	

Student level results close to multilevel, but estimates are more similar than standard error, underestimation of standard errors by ordinary regression analysis is expected science assumption of independence of observation is violated.

Students homogeneous within classes than schools where students within classes model, VPC=0.02 and students within school model, VPC=0.59.

Based on 3-level model, Explained variance respectively:

$$\text{Level-1 } R_1^2 = 1 - \frac{\hat{\sigma}_P^2}{\hat{\sigma}_0^2}$$

$$\text{Level-2 } R_2^2 = 1 - \frac{\hat{\sigma}_{uP}^2}{\hat{\sigma}_{u0}^2}$$

$$\text{Level-3 } R_3^2 = 1 - \frac{\hat{\sigma}_{vP}^2}{\hat{\sigma}_{v0}^2}$$

Subscript 0 refers to a model with no covariates (null model), subscript P refers to a model with P covariates (full model).

Table 3: Explained variance in three multilevel model

Level	Variance	Null model	Full model	R^2
Student	$\hat{\sigma}_v^2$	201.32	164.45	18%
Class	$\hat{\sigma}_u^2$	10.69	10	6%
School	$\hat{\sigma}_v^2$	299.38	238.24	20%

Table (4) contains model fit indices, or goodness-of-fit statistics, that allow an analyst to choose the "best" model among various candidate models. We used standard regression and three level regression model fit indices to determine which was the best one to use in our analyses of hierarchical data. The first model fit index was the value of the maximized log likelihood function (Max Log L), where higher (more positive) values of this index indicated a better model fit. We also used two penalized likelihood criterion, Akaike's Information Criterion (AIC) and Schwarz's Bayesian Information Criterion (BIC), to compare alternative models (Burnham and Anderson 2004). A smaller value of AIC and BIC indicate a better model fit. Comparing the two model fit indices for the regression models, it was clear that the multilevel regression models had a better model fit than standard regression, as the multilevel regression models had higher Max Log L and smaller AIC and BIC. This result suggested that accounting for the hierarchical structure of the data resulted in a better goodness-of-fit.

Table 4: Model Fit Indices

	Model Fit Indices	
	Multilevel regression	Standard regression
MAX Log L	-6202	-6511.6
AIC	12536.1	13041
BIC	12413.5	13089

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الملخص العربي:

يهدف البحث الي مقارنة نموذج الانحدار متعدد المستويات مع نموذج الانحدار التقليدي في تحليل البيانات الهرمية. تم استخدام الانحدار متعدد المستويات وذلك لاختبار تأثير متغيرات مستقلة مقاسه علي مستوى الطالب ومستوي الفصل ومستوى المدرسة علي مستوى الطالب الدراسي GPA كمتغير تابع.

وشملت المقارنة عنصرين أساسيين هما تحيز مقدرات نموذج الانحدار ومستوي المعنوية المقدره لمعاملات الانحدار. وتم استخدام الحزمة الإحصائية MLWIN اصدار ٢,٣٦ لإجراء التحليل.

اعتمدت الدراسة علي عينة عنقودية مكونه من ٤٣ مدرسة و٨٣ فصل و١٥٤٣ طالب من محافظة الدقهلية وكانت المحافظة بها ٩٨٤٨٢ طالب ثانوي و١٧٣ مدرسة و٤٥٦٤ فصل. وقد اثبتت الدراسة انخفاض مقدرات النموذج وانخفاض الخطأ المعياري لمقدرات نموذج الانحدار التقليدي مما ادي الي زيادة مستوي معنويه الاختبارات المستخدمة. وأكدت الدراسة ايضا بالاعتماد علي بعض مقاييس جودة التوفيق أن نموذج الانحدار متعدد المستويات أفضل في تحليل البيانات الهرمية.