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The Role of Sigma-Delta ADCs and Zero-Forcing Estimator in Massive MIMO Channel Estimation

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ABSTRACT

In the presented work, we delve into the complexities of improving channel estimates for an uplink large multiple-input multiple-output (MIMO) system. In order to enhance the system's overall accuracy and effective resolution, the base station (BS) is fitted with 1-bit spatial sigma-delta (ΣΔ) analog-to-digital converters (ADCs). Our study presents two proposed algorithms to estimate the channel. For the first algorithm, we compute a multipath channel that is specified by angle steering, specifically angles of arrival (AoAs), and path gains. This is accomplished by uplink pilots that can aid in lessening interference that occurs caused by multipath channels when users transmit a signal to the BS. We believe that appropriately setting the quantization voltage level of the (ΣΔ) quantizer is critical for this approach to produce the best results. We present a technique for optimizing channel estimation performance when using a zero-forcing (ZF) estimator with the quantized signal from (ΣΔ) ADC in the second procedure. According to the results of numerical simulations, the suggested channel estimation algorithms outperform existing standard methods. The first algorithm demonstrates significant improvements in channel estimation accuracy over existing techniques, with a notable increment in the signal-to-noise ratio (SNR) and a decrease in the normalized mean square error (NMSE) rate. where the second algorithm shows the ideal result, which is zero error for different values of SNR.

Keywords: Massive MIMO, channel estimation, sigma-delta ADC (ΣΔ), Zero Forcing (ZF), Angle of Arrival (AoA).

I. INTRODUCTION

In the realm of modern wireless communication, there is a never-ending search for increased data rates, better spectrum efficiency, and improved system performance. Massive multiple-input multiple-output (MIMO) systems are one amazing technology that has attracted a lot of interest in this pursuit. In massive MIMO, a base station (BS) is supplied with a large antenna array with potentially hundreds of antenna elements. The large array provides excess freedom, allowing many users' equipment (UEs) to be served with the same time-frequency resource [1].

The purpose of these many antennas is to increase the capacity and efficiency of wireless communication through beamforming, spatial multiplexing, and spatial processing. Where signal propagation is highly directional, massive MIMO can be particularly beneficial for beamforming and steering across various frequency bands, including millimeter-wave (MM-wave) signals, to specific users [2, 3].

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The signal can be narrowly focused through the use of multi-user beamforming, delivering a notable upgrade in energy efficiency. The narrower beams also translate to less inter-user interference; thus, the system is robust against both unintended interference and intentional jamming. When the BS understands the channel response of the intended UE, the rewards of having numerous antennas at the BS become apparent [4]. This knowledge enables the BS to coherently combine the received signals from all antennas. Estimation of the channel response is thus a critical feature in multi-antenna systems [5], which are all about huge MIMO systems.

As illustrated in Figure 1, uplink massive MIMO represents the broadcast signal from users to the base station, allowing both the transmitter and receiver to employ diversity and spatial multiplexing, potentially resulting in considerable gains in system capacity. The key to its success is effective channel estimation, which is a crucial operation that allows the BS to decipher the signals received from various UEs. Massive MIMO uplink systems provide great spatial resolution and throughput [6]. However, the usage of high-resolution analog-to-digital converters (ADCs) and digital-to-analog converters

(DACs) for every antenna in the array significantly increases the radio frequency (RF) complexity and power consumption of the massive MIMO systems [7]. Although low-resolution or one-bit ADCs and DACs provide the greatest simplicity and power savings, they also suffer the most significant performance loss in contrast to those that higher resolution sampling, especially for medium to high signal-to-noise ratios (SNRs) and under strongly interacting conditions [8, 9].

The 1-bit spatial Sigma-Delta (ΣΔ) ADCs combine 1-bit quantization and oversampling and are renowned for their simplicity and energy efficiency. Sigma-Delta (ΣΔ) ADCs offer high-resolution quantization of received signals using just a single bit, cutting down on power usage, lowering quantization noise, and raising channel estimate precision. With the help of this technology, the power sources of UE devices might last longer, and wireless networks' environmental impact could be diminished.

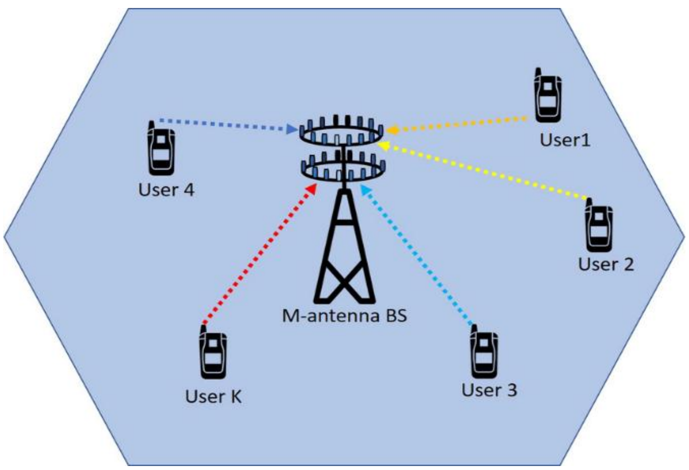


Fig.1: uplink Massive MIMO (Multiuser).

The block diagram of the sigma-delta (ΣΔ) ADC shown in Figure (2), which consists of three main units as:

1- antialiasing filter

An antialiasing filter (AAF) band limits the analog input signal to prevent aliasing during its subsequent sampling. Oversampling can considerably relax the attenuation requirements of the AAF, so that smoother transition bands are usually sufficient, compared to Nyquist-rate ADCs.

2- sigma delta modulator

the variation between the input data and the 1-bit quantized output, i.e., the quantization noise, is fed back in time by adding it to the input at the next time instance. With the quantization noise being pushed to higher temporal frequencies, this operation shapes the noise, it consists of:

- **sampling and holding (S/H) circuit:**

The Sample and Hold circuit prevent the input voltage from changing during the time it takes to convert the sampled voltage to a digital value. It essentially "holds" the voltage constant during the conversion process.

- **forward path:**
  - H(z) is the loop filter.

The loop filter is a part of the feedback loop in the ΣΔ modulator. It is responsible for shaping the noise and controlling the stability of the system. The loop filter in a ΣΔ ADC is a critical component that shapes the noise spectrum, maintains stability, and contributes to the overall performance of the ADC. Its design is a balance between noise shaping and stability considerations.

- ADC quantizer.

The quantizer, positioned at the output of the ΣΔ modulator, quantizes this high-frequency bitstream into a digital signal with a higher resolution. The goal is to recover the original analog signal with better precision. The ΣΔ ADC works by oversampling the input signal at a much higher rate than the Nyquist rate. This oversampling allows for noise shaping, moving quantization noise away from the signal band of interest. ΣΔ ADCs often use 1-bit quantizers at the output. Despite being a low-resolution representation, the oversampling and noise shaping result in effective high-resolution performance.

- **feedback loop is single bit DAC that is inherently linear.**
- **subtractor to shape for quantization noise.**

3- Decimator

It consists of:

- **Digital filter:**

the decimation filter uses a high-selectivity digital filter to sharply remove the out-of-band spectral content of the (ΣΔ) output and thus most of the shaped quantization noise. This indicates that for a low-pass signal, the effective quantization noise is negligible, and it would be as if the signal were quantized by a high-resolution quantizer.

- **Downsampler:**

After the decimation filter, the signal is often at a much higher rate than needed for further processing or transmission. Downsampling, or down-sampling, is the process of reducing the sample rate of a signal [10].

This traditional architecture, which uses a straightforward 1-bit quantizer with feedback to boost the effective resolution of time-domain signals, has recently been extended to the spatial domain. To learn more about the ADCs discussed in [11], read on.

Massive MIMO's spatial multiplexing requires a 1-bit spatial (ΣΔ) quantizer, oversampling, and feedback in the spatial domain, i.e., across antennas. The antenna elements of an array are spaced less than half a wavelength apart to conduct spatial oversampling. Each antenna's quantization noise is transmitted back along with the input for the antenna after it [12]. At lower spatial frequencies in a spatial quantizer, the quantization noise is decreased as if it were coming from a higher-resolution quantizer. When using spatial quantizers, one-bit quantized signals should have the quantization voltage level properly selected, which considerably enhances the interference performance [13].

Conversely, angle steering makes use of the enormous antenna arrays in the BS's spatial diversity. By precisely calculating the angles of arrival (AoAs) of UE signals, the BS can steer its reception beam toward the desired directions, improving signal-to-noise ratios and mitigating interference [14]. This approach enhances the overarching reliability and capacity of the communication link. Additionally, angle steering allows the quantization noise to be reduced for signals coming around the steering angle by adding phase changes to the quantization noise prior to feedback [15].

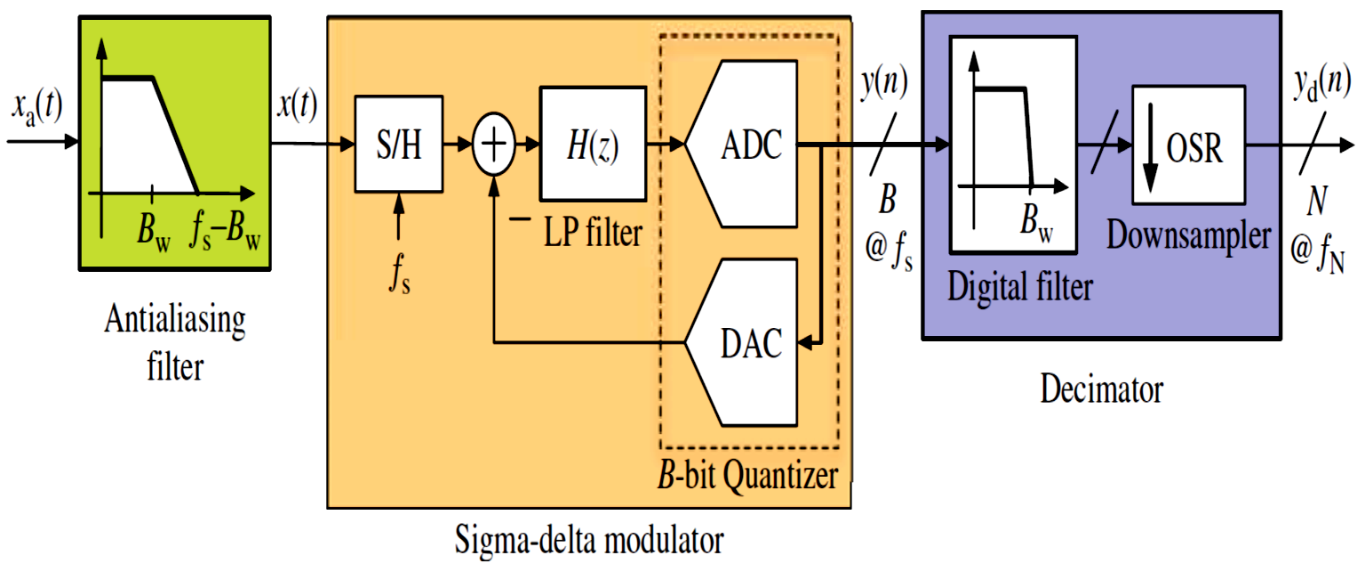


Fig.2: block diagram of sigma-delta ADC

With angle steering, signals within a spatially focused area of interest that possess a particular width and are centered around any specified angle can have a greater effective resolution [16].

The fusion of 1-bit spatial (ΣΔ) ADCs and angle steering in Uplink Massive MIMO heralds a new era in wireless communication. It combines the benefits of energy-efficient signal processing with the spatial selectivity of advanced beamforming techniques, promising to unlock the full potential of Uplink Massive MIMO systems. Due to the nonlinearity and noise that the low-resolution ADCs introduce, channel estimation with coarsely quantized data is difficult. The so-called Bussgang decomposition is typically used to construct an analogous linear model to the conventional 1-bit quantizer, and a linear minimum mean squared error (LMMSE) channel estimator was proposed for 1-bit massive MIMO systems. Bussgang decomposition is a method for decomposing the intricate channel response into simpler parts. into two separate components: a deterministic component (linear) and a random component (nonlinear) [17, 18].

Hence, the LMMSE estimator is founded on a linear combination that was selected to reduce the mean square error (MSE) between the channel that was estimated and the channel that was observed. The Zero Forcing (ZF) estimator is an approach used in signal processing and communication systems, particularly in channel estimation and equalization [19].

Its primary aim is to eliminate or "force to zero" the interference between transmitted signals in a multi-user or multi-antenna system. These estimators are used to linearize low-resolution quantizers and influence energy and spectrum efficiency [20]. They have also been extended to spatial (ΣΔ) quantizers for massive MIMO, as illustrated in outcomes of the simulation. In practice, Bussgang is not appropriate for the (ΣΔ) quantizer because the presence of error feedback to adjacent antennas leads to a linear model but is inconsistent with the corresponding hardware implementation.

- contributions

- Consequently, using 1-bit ADCs and angular models for massive MIMO channel estimation, we seek to achieve optimal channel estimation through the methods proposed in this research.
- We propose the use of Zero Forcing (ZF) with Sigma-Delta ADC (ΣΔ) and an angular model in massive MIMO systems. This combination involves combining two key techniques to enhance communication performance. ZF aims to eliminate interference and improve signal quality [21], while Sigma-Delta ADC provides quantization with high resolution and low power consumption.
- Combining ZF with Sigma-Delta ADC helps enhance the overall communication performance of massive MIMO systems. It reduces interference and ensures that the quantized digital signal retains high accuracy.
- In addition, this combination can make the system more robust in challenging wireless environments where interference is an extreme concern.
- In the present study, we explore the ideal channel estimate for a huge MIMO system coupled with a first-order spatial ADC and angle steering.
- Our goal is to find the most effective method for precise channel estimate by using zero-force receiver estimators.

The paperwork is structured in the manner described below: provided in Section (II) is the methodology of the proposed system; Section (III) discusses the simulation results; and Section (IV) presents the conclusion of the paper.

II. METHODOLOGY OF THE PROPOSED SYSTEM:

Two channel estimation algorithms are presented in this section in an effort to boost performance. The first algorithm estimates multipath channels in uplink Massive MIMO (MU-MIMO) by leveraging angles of arrival (AoA) with first-order 1-bit spatial (ΣΔ) quantizers and careful selection of quantization voltage level. The second algorithm applies a zero-forcing estimator with a 1-bit spatial (ΣΔ) quantizer output. Then, we will talk about how well the two suggested algorithms perform.

1. Channel estimation with AoA and First-order 1-bit spatial ΣΔ ADC:

We consider an uplink massive MIMO system with K single-antenna user terminals and a BS provided with a huge number of antennas Nr, where (Nr > K) and a 1-bit spatial (ΣΔ) quantizer. During the training period, the BS has a ULA with Nr antennas and receives and processes uplink training pilots from K users that transmit their pilot sequences of length N simultaneously.

To estimate AoAs at BS, the multi-antenna UE transmits pilot symbols to the BS in an omnidirectional manner [16]. The recommended omnidirectional transmission assures that enough power reaches the BS across all the pathways because the angles of departure AoDs are unknown. It also gives us the option to select an appropriate quantization voltage level, which is necessary for a path gain estimate.

Let us collect the uplink training pilots in $S = [s(1), s(2) \dots, s(T)]$, T represents the length of the pilot. Without sacrificing generality, let us assume that the columns of the pilot matrix have a unit norm. Before quantization, the signal that the BS received is:

$$X = \sqrt{P}\, H\, S + N \tag{1}$$

Where P is the power of transmit at the UE and uplink SNR of the system, H implies the MIMO channel matrix, and N is additive noise.

The channel between K_{th} UEs and BS with L_k paths is given by:

$$h_K = \frac{1}{\sqrt{L_K}} A_{BS}(\theta_K)\alpha_K \tag{2}$$

Assuming that (L_K) represents the paths connecting UE and BS,$A_{BS}(\theta_K) = [a_{BS}(\theta_K, 1), \dots, a_{BS}(\theta_K, L_K)] \in \mathbb{C}^{N_r \times L_k}$ where the superscript denotes the array manifold at the BS , θ_k represents the collection of AoAs of paths from the K_{th} UE at BS, and α_k is the complex path gain.

then the overall channel matrix for the MU-MIMO system is:

$$H = [h_1 \quad h_2 \quad \dots h_k]\,, \qquad \in \mathbb{C}^{Nr \times K}$$

The received signal at the base station from eq (1) is then quantized using an Nr-channel 1-bit spatial ΣΔ ADC to obtain Y. Prior to quantizing the signal, the quantization voltage level must be selected, as it plays a crucial role for assessing the channel estimation and beamforming performance of a massive MIMO system with 1-bit spatial ΣΔ ADCs.

$Y = [y(1), y(2), \dots y(T)]$ which is given by:

$$Y = \sqrt{\frac{P}{L_k}}\, A_{BS}(\theta_k)\, diag(\alpha_k)1 + n_{k,t} \tag{3}$$

Here, $n_{k,t}$ denotes the K_{th} column of $NS^H(t)$. Therefore, we can use (3) to evaluate channel parameters (θ_k, α_k) for each user.

2. Impact of Zero forcing estimator with 1-bit spatial ΣΔ ADC:

In this work, we utilize the output of the ΣΔ quantizer signal for the zero-forcing estimator. Furthermore, it has been demonstrated that the ZF can accomplish perfect detection even when using one-bit ADCs, considering the base station's number of antennas increases towards infinity. To obtain the ZF estimate, we multiply the received signal by the pseudo-inverse of the channel matrix H.

From equations (2, and 3) can calculate X_{ZF} as:

$$X_{ZF} = H^{\dagger}\ Y \tag{4}$$

Where, $H^{\dagger}$ denotes the pseudo inverse matrix of **H**, from (3) Y is the output of ΣΔ, and using the algebraic formula of pseudo-inverse matrix **H**, the ZF estimate can be expressed as,

$$X_{ZF} = (H^H\, H)^{-1}\, H\, Y \qquad (5)$$

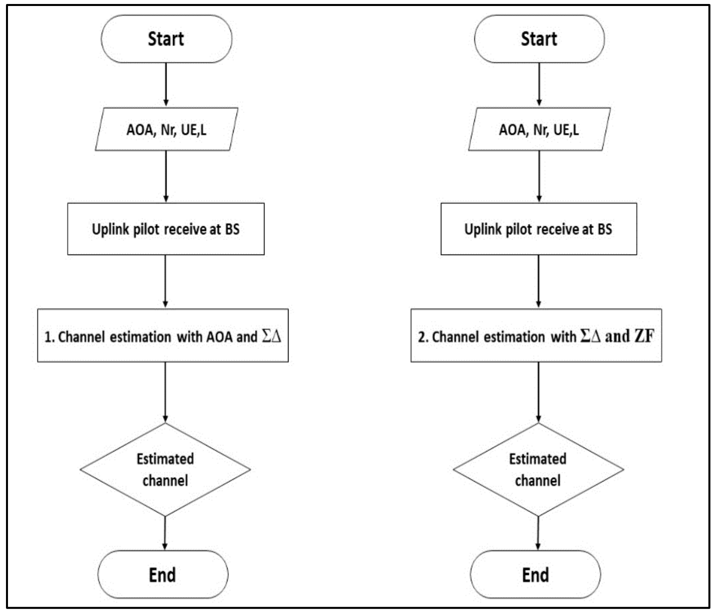
In massive MIMO, combining sigma-delta ADC and zero forcing of received signals yields impressive results and provides an approach to optimal channel estimation, as explained in the section below.

3. Performance

Through the simulation results presented in the next section, we assess the effectiveness of the recommended channel estimation algorithms, measured in terms of normalized mean squared error (NMSE) with varying signal-to-noise ratio (SNR) values. Because of its capacity to evaluate the precision and reliability of the estimated channel reaction in relation to the genuine channel reaction, NMSE was chosen as the performance metric. It promotes a quantitative measure of how well the estimated channel matches the genuine channel. NMSE is calculated by finding the mean square error (MSE) between the predicted and true values and then normalizing it by dividing it by the variance of the true values. A lower NMSE value indicates a better match between the estimated channel response and the true channel response. Conversely, a high NMSE value indicates significant deviations from the actual channel, indicating poorer estimation accuracy.

SNR, on the other hand, quantifies the relationship between the desired signal strength and any interference or background noise. It depicts the signal intensity with respect to the system noise. A lower NMSE is often associated with a greater SNR. This is due to the fact that a greater SNR implies that the signal is more dominant than the noise, resulting in more accurate estimation or prediction and, as a result, an improvement in overall spectral efficiency and channel estimation performance. Lower noise levels permit the mathematical model to capture the underlying signal properties more efficiently, resulting in less inaccuracy among the anticipated and true values.

The flow chart of methodology of the proposed system can be drawn as:



SIMULATION RESULTS

In this section, we present the results from several numerical simulations using MATLAB software to demonstrate the effectiveness of the channel estimation algorithms developed for the Massive MIMO technique using 1-bit spatial (ΣΔ) ADCs. We compare various algorithms based on their normalized mean square error (NMSE) and signal-to-noise ratio (SNR). We consider a MU-MIMO setup with K users = 16 and a BS with Nr = {128,256} spaced d = 1/8 wavelength apart. For each user, we utilize a multipath channel L_k = 3, T = 1, and calculate NMSE for each path using 500 independent channel realizations.

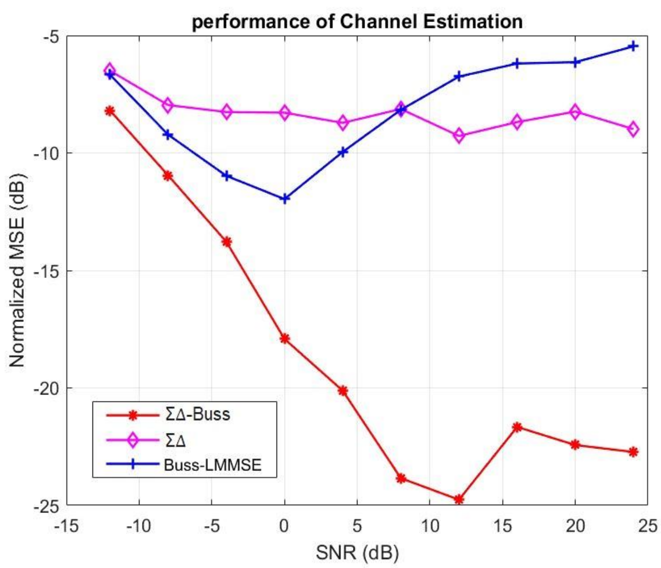
First, we determine the effectiveness of channel estimation using 1-bit spatial with angle steering. For Bartlett beamforming, we use the search grid A = {-90°, -89°, ……,89°, 90°} and AoA is drawn uniformly at random from sectors [45°] and [60°] with a minimum spacing of {20°, 10°} respectively.

We contrast the performance of the showed approach for channel estimation, dubbed as "ΣΔ angular" with the following techniques in MU-MIMO channel estimation:

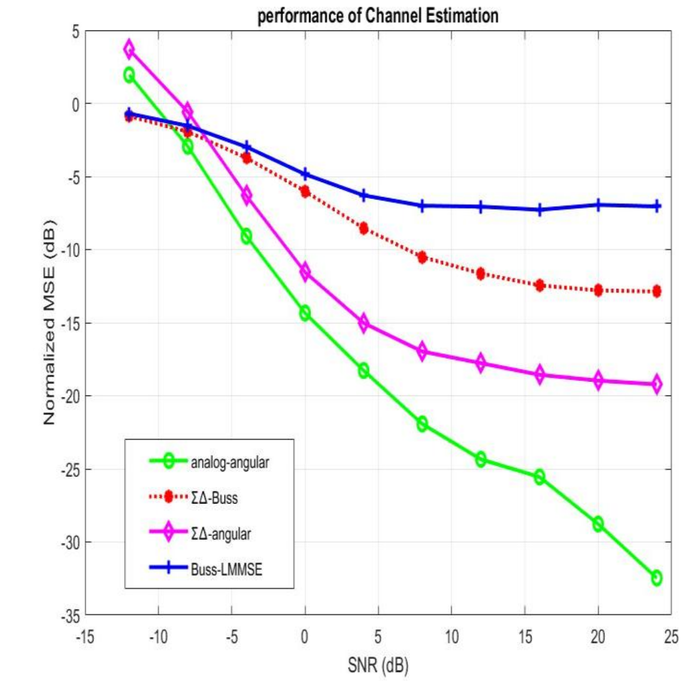
(1) Channel estimation using 1-bit spatial ΣΔ ADC with the Bussgang decomposft5ition dubbed as "ΣΔ Buss".
(2) Channel estimation with Bussgang decomposition followed by computing LMMSE dubbed as "Buss LMMSE".
(3) Channel estimation with proposed angle without quantization dubbed as "analog angular".

Hence, "analog angular" relates to preserving the phase information (angle) of the received signals, which is crucial for accurately estimating and quantizing 1-bit magnitude. This technique offers greater precision and enhances performance but comes with hardware complexity and processing load. We use "analog angular" as a benchmark for the proposed method "ΣΔ angular" to demonstrate the loss generated by 1-bit spatial ΣΔ quantization. Figure 3 displays the NMSE outcomes for several techniques. It is clear that the recommended approach is effective and performs more effectively in terms of NMSE than existing methods, namely "Buss LMMSE" and "ΣΔ Buss".

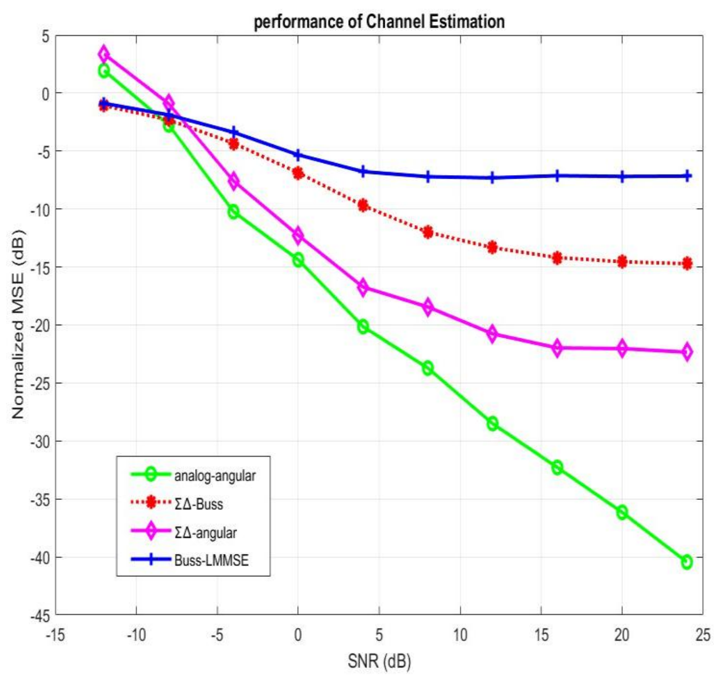
Fig. (3) shows the outcomes for channel estimation in (a) multipath L = 3, K = 16, Nr = 128, and without angle estimation. (b) same as (a), but using angle estimation with (AoA = 45°, AoA grid = [-70, 70], and minimum space = 20°). (c) using angle estimation with (AoA = 60°, AoA grid [-90, 90], and minimum space = 10°). (d) multipath L = 3, K = 16, Nr = 256, and using angle estimation with (AoA = 60°, AoA = [-90,90], and minimum space 10°).



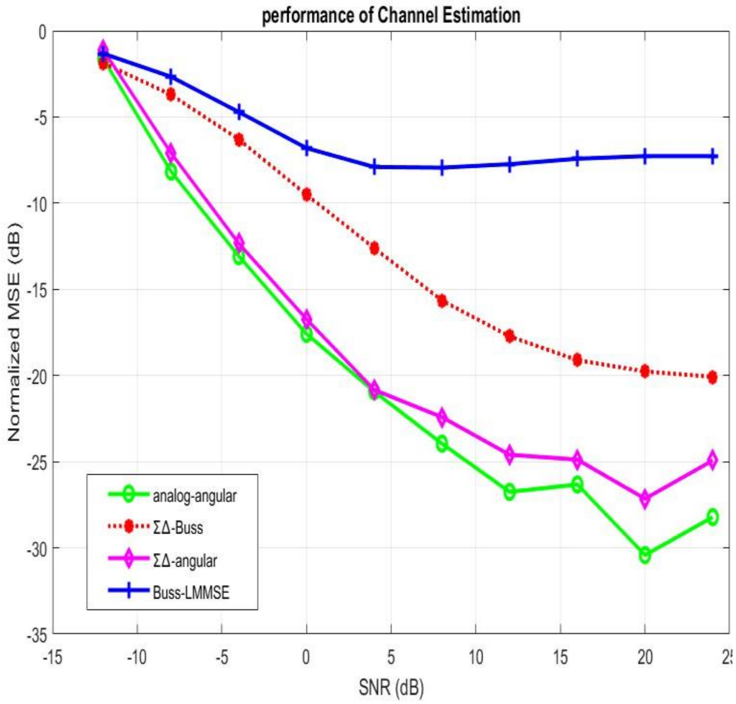
3(a)



3(b)



3(c)



3(d)

Fig (3) performance of channel estimation in (a), (b), (c), and (d).

- In the displayed results of channel estimation without angle steering, as depicted in figure 3(a), it is clear that the NMSE of "ΣΔ" remains almost constant in the rate range of {-5 to -10}. However, the NMSE of "ΣΔ Buss" experiences a decrease at high SNR levels, while the achievement of "Buss LMMSE" is deemed unsatisfactory.
- The impact of AoA on the improvement of channel estimation performance is illustrated through Figures 3(b) and 3(c). It is apparent that, at low-to-medium SNRs, the outcomes for "ΣΔ angular" is comparable to that of "analog angular". This confirms the developed theory that 1-bit spatial ΣΔ ADCs possess a higher effective resolution, which can be employed for parametric estimation.
- At higher Signal-to-Noise Ratios (SNRs), the Normalized Mean Square Error (NMSE) outcomes for both "ΣΔ Buss" and "ΣΔ angular" methods decreases due to an inevitable increase in quantization noise that occurs at high SNRs. However, even at high SNRs, the proposed "ΣΔ angular" method outperforms the "ΣΔ Buss" method as SNRs increase.
- To illustrate this, Fig. 3(d) shows what occurs when a set of antenna Base Stations (BS) is increased to 256. This leads to an improvement in the efficiency of the "ΣΔ angular" method, making it similar to the "analog angular" method while significantly outperforming existing techniques.

The effectiveness of adaptive channel estimation techniques ("analog angular" and "ΣΔ angular") is superior to that of non-parametric rival techniques ("ΣΔ Buss" and "Buss LMMSE"). This is due to the parametric techniques are able to make use of the structure in the angular channel model, resulting in more accurate estimations. Therefore, these techniques are more reliable and efficient in predicting channel characteristics.

Second, we show the result of combining the zero-forcing estimator with the output from the sigma-delta (ΣΔ) ADC in Figure 4. After conducting the initial algorithm simulation, our primary objective is to attain the most favorable outcome by improving channel estimation while minimizing the margin of error and ultimately achieving the optimal scenario. In order to achieve better performance, we employ the use of ZF to eliminate interference or errors and further improve performance by using ZF after quantizing the received signal. Upon analyzing the results in Fig. 4, it is obvious that the “ΣΔ ZF” channel estimator has the most favorable performance, with zero errors across all values of SNR. This indicates that it is an ideal case with no deviations from the expected outcome.

We can summary this comparison in table (1) at the end of our paper.

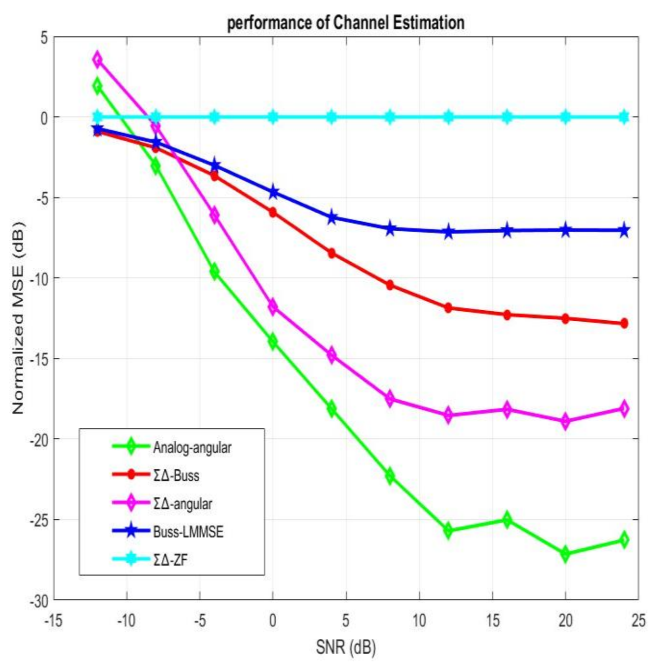


Fig. (4) Impact of ZF estimator on the performance of channel estimation.

III. CONCLUSIONS

We have recently developed two innovative approaches to enhance the performance of massive Multiple-Input Multiple-Output (MIMO) systems, which employ a large antenna array at the base station for simultaneous transmission and reception of signals to multiple users. The first method employs an angular model with spatial sigma-delta (ΣΔ) Analog-to-Digital Converters (ADC), demonstrating a significant impact on improving channel estimation and reducing errors caused by interference, fading, and noise.

The second approach involves the utilization of a Zero-Forcing (ZF) estimator with the quantized signal from the spatial (ΣΔ) ADC. This technique linearizes the output of the 1-bit spatial (ΣΔ) ADC, effectively eliminating interference. Our research indicates that these proposed strategies outperform conventional methods by a significant margin, achieving an optimal scenario for channel estimation.

The primary goal of our investigation is to enhance channel estimation, leading to improvements in spectral efficiency, energy efficiency, and the throughput of massive MIMO. Simultaneously, we aim to reduce computational and hardware complexity. These methods provide potential solutions to common challenges in wireless communication. We believe further research and exploration will help identify the most suitable and low-cost techniques and parameters, paving the way for practical implementation and advancements in this field.

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An analytical comparison table.

Method	Performance metric	Parameters	Results	Notes
Channel estimation with ΣΔ and AOA	- **NMSE** - **SNR**	L=3, Nr=128, K=16, AoA = 45°, AoA grid = [-70, 70], and mini space = 20°	NMSE drops with an increase in SNR "nalog-angular" outperform on "ΣΔ-angular"	**- Impact of AOA on the improvement of channel estimation performance.** **- ΣΔ leads to improvement in accuracy and efficiency**
		L=3, Nr=128, K=16, AoA = 60°, AoA grid = [-90, 90], and mini space = 10°	NMSE drops faster with an increase in SNR	
		L=3, Nr=256, K=16, AoA = 60°, AoA grid = [-90, 90], and mini space = 10°	Improvement in "ΣΔ-angular"	
Channel estimation with ΣΔ and ZF		L=3, Nr=128, K=16 AoA = 60°, AoA grid = [-90, 90], and mini space = 10°	Zero error across all values of SNR	**ΣΔ with ZF eliminate interference or errors**

Table (1)