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Improving Handling Performance of Four-Wheel Steering Vehicles Using LQR Controller

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The present investigation studies the lateral stability of vehicles by using a two-degree-of-freedom bicycle model which is implemented based on MATLAB/Simulink. The proposed model considered the driver model as an expert system to mitigate the vehicle's lateral deviation. Rear-wheel steering is incorporated into the typical front-steering vehicle model to represent a Four-Wheel Steering (4WS) system that improves lateral vehicle stability. The linear optimal control theory (LQR) is employed to determine the rear wheel steering angle as the control action to minimize the vehicle lateral responses of the vehicle, such as body sideslip angle, lateral deviation, lateral acceleration, and yaw rate. To validate the effectiveness of the proposed controller, the Particle Swarm Optimization (PSO) technique is designed to obtain the optimal gain of the LQR. Three scenarios are utilized to evaluate the proposed model. First, at a lateral deviation of 2.5 m; second, at a front steering wheel angle of 4 degrees; and third, activating both previous scenarios together. The vehicular lateral responses are represented in the time domain and root mean square values. Significant improvements are observed in the lateral stability of the vehicle whereas active four-wheel steering is employed, particularly in terms of lateral acceleration, lateral deviation, and yaw rate compared to the typical 2WS vehicle.

1. Introduction

Vehicle collisions contribute to approximately 1.2 million fatalities globally and a much higher number of injuries every year [1]. Significant factors contributing to this challenge include ineffective safety systems, human factors, and driving conditions[2, 3]. The previous several decades have seen extensive studies on driver assistance technologies such as Anti-Lock Braking Systems, Lane Keeping Assistance, and Four-Wheel Drive[4]. The 4WS is an advanced control method that enhances the steering characteristics. Compared to conventional 2WS, 4WS operates independently to direct the front and rear axles through curves based on the vehicle's state. Consequently, vehicle stability and active safety may be improved through four-wheel steering [5-9]. Most previous studies investigated how the 4WS system affected lateral stability in terms of yaw velocity, lateral acceleration, and lateral deviation; however, it did not explore how driver behavior affected the stability performance of the vehicle. A novel control strategy was introduced by Li et al. [10] for four-wheel steering vehicles that provide robustness and effective decoupling even when the steering velocity fluctuates. Liu, et al. [11] developed a PID controller to improve vehicle stability using a 4WS system while the parameters of the PID controller were selected according to the trial-and-error method. Tan et al. [11] investigated Sliding Mode Control (SMC) for active 4WS systems to enhance vehicle stability; unfortunately, SMC causes oscillations at high frequencies. Yin et al. [12] suggested that the 4WS vehicle fitted with the robust mu-synthesis controller has enhanced maneuverability and resistance to interruptions. However, there is a constraint in the validation circumstances. Ozatay et al. [13] evaluated the vehicle handling characteristics based on a Fuzzy Logic Control system using a 3 DOF model where the influence of the human driver was neglected. A 4WS vehicle dynamics model using two degrees of freedom was built using MATLAB/Simulink by Zhang et al. [14] to enhance handling stability. At low and high speeds, three optimal controls for the 4WS system are proposed for monitoring the yaw rate and centroid cornering angle. The potential benefits of 4WS systems reduce at higher speeds, hence most studies optimize control parameters for varied driving circumstances while developing controllers.

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Jing and Xiao [15] provided an LQR optimal control technique based on state feedback, which is integrated with forklift and steering need characteristics to improve mobility and handling stability. Park et al. [16] designed an LQR Controller for essential driving route tracking. To satisfy the system cost function, controller gains are acquired via trial and error. Lu et al. [9] used a genetic algorithm and a novel fitness function to generate an offline table for R and Q matrices based on maximum feedback error to operate 4WS electric vehicles using an adaptive LQR controller. An LQR controller, as suggested by Du et al. [17], employs a variational optimization approach to ascertain the optimal rear steering angle. In this study, a 2-DOF bicycle-vehicle model of the 4WS system is proposed to improve vehicle lateral stability under various driving conditions while taking the driver's responses into account. An LQR optimization control algorithm utilizing state feedback is suggested and implemented for four-wheel steering. The Particle Swarm Optimization (PSO) algorithm is designed to obtain the optimal gain of the LQR. To verify the effectiveness of the LQR optimization control technique, results from simulations for corresponding control, two-wheel steering control, and LQR optimum control are compared. Simulation findings indicate that selecting the appropriate weighted coefficient matrix for LQR can improve vehicle handling stability. The following sections represent this paper's organization. We described the human driver model in Section 2. Section 3 displays the vehicle system modeling. Section 4 illustrates the PSO-LQR controller to derive active four-wheel steering. Section 5 presents the results and discussion. And finally, Section 6 presents the conclusion.

2. Model of Human Driver

The pneumatic tires of the vehicles cannot be maintained on the intended path during the different driving conditions because of some external perturbations such as road roughness. To keep the vehicle on the desired trajectory, it is controlled by an external driving device, which is the human driver during this study. The driver provides a feedback controller to generate steering to allow the vehicle to correct the deviation of the path to return to the desired path. The output of the driver is considered as the front tire steering angle based on preview control by rating the vehicle previewed error (deviation) of the desired trajectory. The equation of the human driver model can be shown as the following [18].

$$t_r\,\dot{\delta}_f(t) + \delta_f(t) = -k_d\,[\eta(t) + \frac{L}{V_x}\,\dot{\eta}] \tag{1}$$

Where t_r represents the time delay required to obtain the driver's response to any change in the driving conditions. There are three major time delay intervals. The first period, response delay time, allows the driver to collect environmental and vehicle data. The second delay is neuromuscular, which delivers information to muscles. Execution or time delay is the third period for vehicle control. The time delay utilized to simulate the human driver model is the total of response, neuromuscular, and performance delays. δ_f represents the front wheel steering angle, k_d represents the constant factor, $\eta(t)$ represents the deviation of the vehicle from the intended trajectory from the desired path, L represents the preview distance which is taken as a function of the vehicle longitudinal speed where $L = 0.03\,v_x^2$, and V_x represents the longitudinal forward speed of the vehicle. The driver model improves simulation results since a human being attempts to maintain the vehicle on the specified trajectory when external forces occur.

3. Vehicle System Modeling

In this study, a model of two degrees of freedom is derived, which consists of lateral motion and yaw rate motion. To simplify the model the following assumptions are taken into consideration [19]:

1. The surface of the road is level and flat.
2. Aerodynamic forces are neglected compared to tire forces.
3. The longitudinal speed of the vehicle is constant.
4. The structure of the vehicle is assumed to be a single rigid body.
5. Right and left tire forces can be summed and located at the center of the axle.

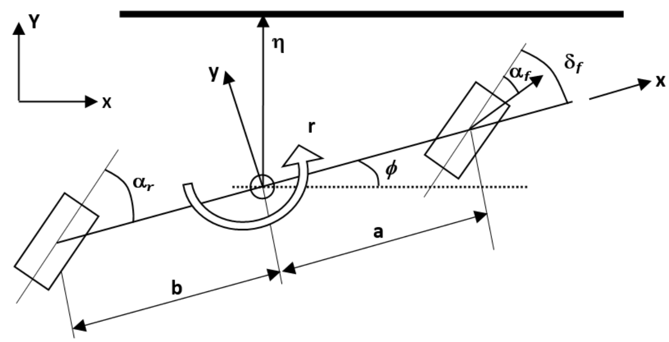


Figure 1: Bicycle Model with 2DOF.

The lateral and yaw motion equations are as follows.

$$M\,\ddot{\eta} = F_{yf} + F_{yr} \tag{2}$$

$$I_z\,\dot{r} = a * F_{yf} - b * F_{yr} \tag{3}$$

Where F_{yf} and F_{yr} are the front and rear tire lateral force respectively, $\ddot{\eta}$ is defined as the body's lateral acceleration, $\dot{r}$is known as the body yaw acceleration, and r is defined as the yaw rate of the vehicle.

Using the variable η and r as assuming the steering angle is small, the front (α_f) and rear (α_r) side slip angles can be defined as the following equations.

$$\alpha_f = \delta_f + \phi - \frac{\dot{\eta} + a\,r}{V_x} \tag{4}$$

$$\alpha_r = \phi - \frac{\dot{\eta} - b\,r}{V_x} \tag{5}$$

The difference between the 4WS and 2WS systems in the rear side slip angle (α_r) is that the rear steering angle is added to the equation in the 4WS system as follows.

$$\alpha_r = \delta_r + \phi - \frac{\dot{\eta} - b\,r}{V_x} \tag{6}$$

By neglecting the camber angle, the longitudinal force, and leaving the friction coefficient between the road and the tires. The lateral force F_y depends on the side slip angle (α) as well as the wheel vertical load (F_z). So, the lateral force acting on the tires is a function of the side slip angle (α) and the cornering stiffness. The front and rear lateral force can be defined as follows.

$$F_{yf} = C_{\alpha f}\ (\alpha_f)\ \alpha_f \tag{7}$$

$$F_{yr} = C_{\alpha r}\ (\alpha_r)\ \alpha_r \tag{8}$$

The equations of lateral and yaw motions including the human driver model can be rewritten and expressed in state space and first order. The state space equations are as follows.

$$\dot{X} = A\,x + B\,u \tag{9}$$

Where x = $[\ \eta \quad \dot{\eta} \quad \phi \quad r \quad \delta_f\ ]^{T}$ is the state variables, $u = zero$ for 2WS and $u = \delta_r$ for 4WS system.

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & -\left(\frac{C_{\alpha f}+C_{\alpha r}}{M\,V_x}\right) & \left(\frac{C_{\alpha f}+C_{\alpha r}}{M}\right) & -\left(\frac{a\,C_{\alpha f}-b\,C_{\alpha r}}{M\,V_x}\right) & \frac{C_{\alpha f}}{M} \\ 0 & 0 & 0 & 1 & 0 \\ 0 & -\left(\frac{a\,C_{\alpha f}-b\,C_{\alpha r}}{I_z\,V_x}\right) & \left(\frac{a\,C_{\alpha f}-b\,C_{\alpha r}}{I_z}\right) & -\left(\frac{a^2\,C_{\alpha f}+b^2\,C_{\alpha r}}{I_z\,V_x}\right) & \frac{a\,C_{\alpha f}}{I_z} \\ -\frac{K_d}{t_r} & -\frac{K_d\,L}{V_x\,t_r} & 0 & 0 & -\frac{1}{t_r} \end{bmatrix} \tag{10}$$

$$B = zero \tag{11}$$

$$y = C\,x + D\,u \tag{12}$$

Where y is defined as a linear combination of state space variables, it is also known as the output vector.

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & \frac{1}{V_x} & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & -\left(\frac{C_{\alpha f}+C_{\alpha r}}{M\,V_x}\right) & \left(\frac{C_{\alpha f}+C_{\alpha r}}{M}\right) & -\left(\frac{a\,C_{\alpha f}-b\,C_{\alpha r}}{M\,V_x}\right) & \frac{C_{\alpha f}}{M} \end{bmatrix} \tag{13}$$

$$D = zero \tag{14}$$

The vehicle parameters and human response utilized during the simulation were obtained from the reference[18].

Table 1:Vehicle Model Parameters

Symbol	Meaning	Value	Units
M	Vehicle mass	1892	kg
b	Distance from the C.G to the rear axle	1.42	m
a	Distance from the C.G to the front axle	1.34	m
l	Wheelbase	2.76	m
t_r	Time delay	0.15	s
K_d	The gain factor of steering	0.015	Rad/m
$C_{\alpha f}$	Cornering stiffness of the front tire	1401.6	N/deg
$C_{\alpha r}$	Cornering stiffness of the rear tire	1362	N/deg
I_z	Moment of inertia for yaw motion	3270	$kg.m^2$

4. Design of Rear Wheel Controller

The Linear Quadratic Regulator method (LQR) will be used for minimizing a performance index with the closed-loop dynamic system of the 4WS system.

$$Minimize\ J = \int_{t_o}^{t_f} \left[y^T Q\,y + u^T R_1\,u + \dot{U}^T R_2\,\dot{u} \right] dt \tag{15}$$

$$Subject\ \dot{x} = Ax + Bu \tag{16}$$

Where J and $\dot{X}$ are defined as the performance index. Q, R1, and R2 are weighting functions of the control which help in determining which parameters have more effect on the vehicle stability during the critical driving conditions and need to be reduced first and quickly than the other parameters.

$$y_e = C_e X + D_e U \tag{17}$$

While y_e is defined as the output vector or error vector that the controller aims to minimize.

$$C_e = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & -V_x & 0 & 0 \end{bmatrix}, D_e = zero \tag{18}$$

$$X_c = [x \quad u]^T \tag{19}$$

$$u_c = \dot{u} \tag{20}$$

Equation 15 is transformed into the standard form of the LQR controller using equations 19 and 20.

$$Minimize\ J = \int_{t_o}^{t_f} [x_c{}^T Q_c\,x_c + u_c^T\,R\,u_c]\,dt \tag{21}$$

$$\text{Subject } \dot{x_c} = A_c x_c + B_c u_c \tag{22}$$

The optimal control input u_c is obtained as follows.

$$u_c(t) = -R^{-1} B^T P\,x_c(t) \tag{23}$$

The values of matrix P are generated from the solution of the steady-state Riccati equation using the PSO algorithm.

$$P\,A_c + A_c^T P - P\,B_c\,R^{-1} B_c^T\,P + Q_c = 0 \tag{24}$$

The input of control is

$$u_c = -K\,x_c = -K_1\,x - K_2\,u \tag{25}$$

The optimal equation for the system state is

$$\dot{x_c} = A_f x_c + B_f u_c \tag{26}$$

$$\text{Where } A_f = \begin{bmatrix} A & B \\ -K_1 & -K_2 \end{bmatrix}, B_f = Zero$$

The output equation of the controller is

$$y = C_f x_c + D_f u_c \tag{27}$$

Where $C_f = [C \quad D]$, $D_f = Zero$

The LQR controller is known as a state feedback controller. To evaluate the accurate values of controller gains, the Particle Swarm Optimization (PSO) algorithm was utilized. PSO solves the online handling optimization problem to find the best Q and R matrices regarding state errors and physical constraints. The PSO uses the lateral deviation in test scenario 1, the front steering angle in test scenario 2, and both the lateral deviation and front steering angle in test scenario 3 as a feedback for the active LQR 4WS controller to change the LQR controller action continuously according to different driving conditions. The PSO works with the cost function to track the error of the output and determine the physical importance of improving the vehicle's lateral stability. The block diagram of the rear wheel controller is provided in Figure 2.

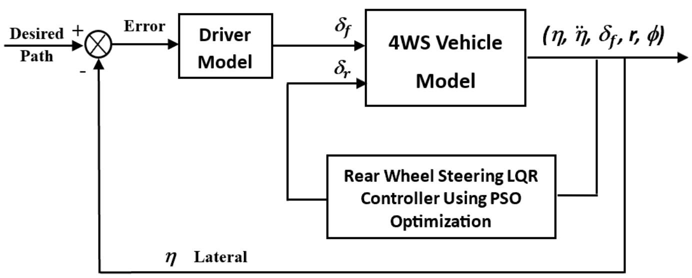


Figure 2: Rear wheel steering controller block diagram.

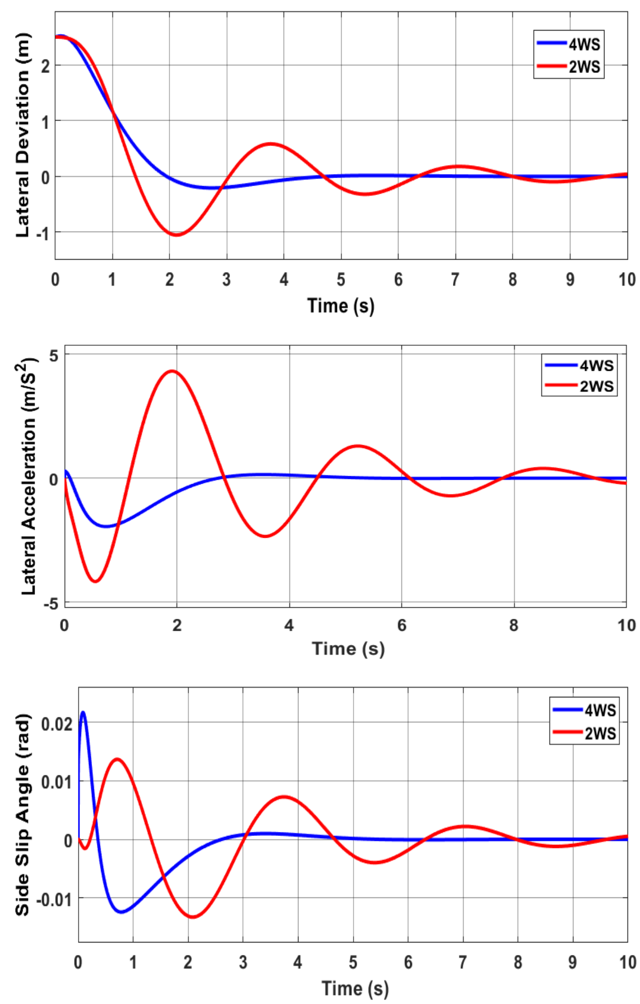
5. Results & Discussions

In this study, a comparative analysis of the vehicle handling characteristics of the conventional 2WS and 4WS LQR controls across various scenarios was conducted using MATLAB Simulink 2018. The model parameters used in the proposed assessments are shown in The vehicle parameters and human response utilized during the simulation were obtained from the reference[18].

Table *1* throughout the simulations. In addition, the velocity of the vehicle remained constant at 22 m/s. The rear wheel steering angle is the control action that has been derived using a Linear Quadratic Regulator controller, which minimizes all state variables. The Particle Swarm Optimization technique was used to improve the controller's scenarios to achieve the best control of the desired results. The vehicle's lateral stability is evaluated using lateral deviation, lateral acceleration, side slip angle, front steering angle, and yaw rate response. Regarding the evaluation of the vehicle's stability, three scenarios are used to assess the suggested model: adjusting the lateral deviation to 2.5 meters, setting the front steering wheel angle to 4 degrees, and activating both situations simultaneously. The contact area between the tire and the road was assumed to be constant during the study. The obtained vehicle handling responses are represented in the time domain and RMS values. Notably, the rear steer angle generated by the rear steering controller contributes to the vehicle's response, in addition to the driver's steering commands to the steering wheel.

5.1. Scenario 1(initial lateral deviation equal to 2.5 m)

Through the adaptability of the large disturbances, the LQR control method is used to get the optimal rear steering angle using full-state feedback. To compare the performance of the proposed controller described in the study with that of the conventional 2WS, the initial lateral deviation has been set at 2.5 m. Figure 3 displays the findings for the vehicle's handling and stability responses in the time domain, while Table 2 provides their corresponding RMS values. The responses include lateral acceleration, sideslip angles, yaw rate, front/rear steering angles, and lateral deviation. As shown in the Figure, the steady-state value of vehicle handling responses was approximately achieved 5 seconds earlier compared to the conventional 2WS system. Table 2 shows that adopting an active rear-wheel steering system results in considerable improvements in RMS values over a traditional 2WS system. According to the steering angle response shown in Figure, the steering wheel angle is largely reduced when using optimal 4WS. These results show a decrease in the amount of effort that the driver must make, which is a desired outcome.



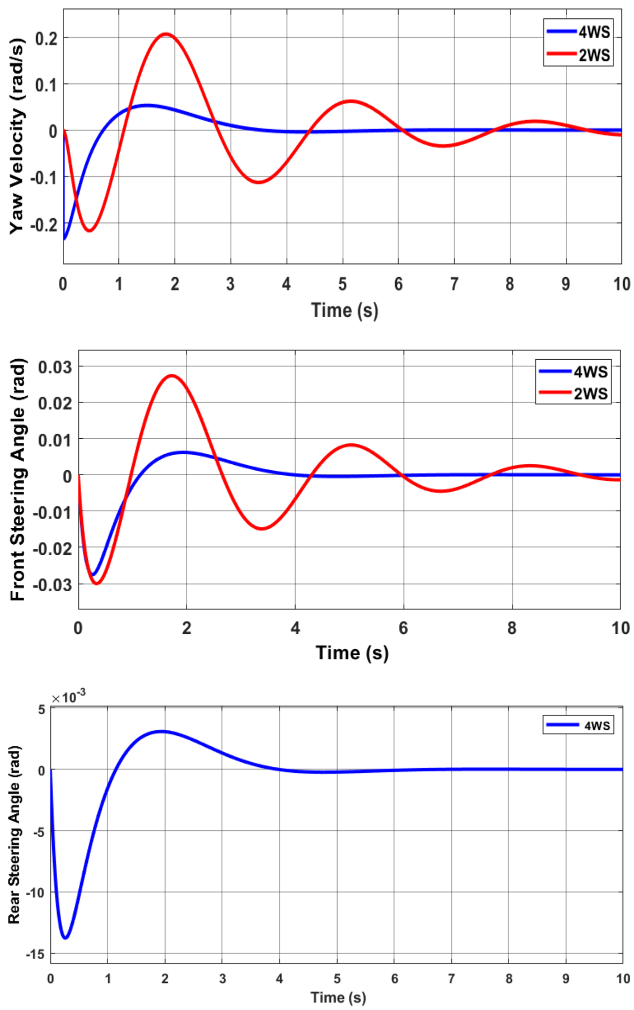


Figure 3 Comparison of vehicle handling performance between conventional 2WS and Optimal 4WS systems with respect to initial lateral deviation 2.5 m.

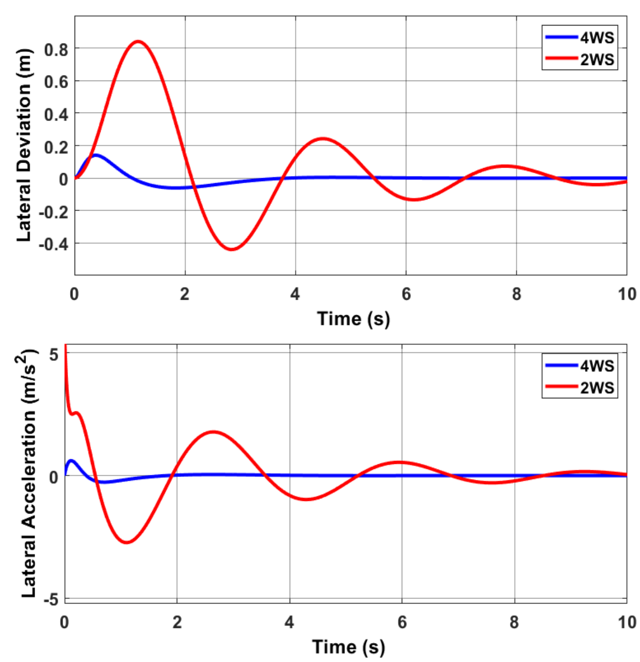
Table 2 RMS values of vehicle handling performance for the 2WS and the 4WS system at an initial lateral deviation of 2.5 m

Parameter	2WS	4WS	Improvement (%)
Lateral acceleration	1.776	0.6309	64.48
Lateral deviation	0.7814	0.6816	12.77
Front steering angle	0.01151	0.006194	46.19
Side slip angle	0.005555	0.004617	16.89
Yaw velocity	0.08616	0.03924	54.46
Rear steering angle	N/A	0.006194	N/A

5.2. Scenario 2(initial front steering angle equal to 4°)

In this study, a second scenario is in which a vehicle travels at a constant speed of 22 m/s while the steering angle is kept at 4 degrees. The rapid response characteristic of the steady state of all handling performance can be observed clearly in the optimum 4WS in almost every one of the sub-figures that are included in Figure 4. In addition, the overshoot of the responses, except for the side slip angle response, has been successfully reduced. On the contrary, the response of the 2WS vehicle demonstrates a much greater overshoot and a longer settling time. The RMS values of the vehicle handling indices under a 4° front steering angle input are displayed in Table 3. The table also displays the percentage improvements gained by the 4WS technique over the 2WS system. As can be seen in the Figure, the 4WS controller has resulted in a reduction of about 79.69% in lateral deviation and 80.37% in lateral acceleration. According to the side slip angle response given in the Figure, 4WS exhibits a larger overshoot value with a lower settling time instead of the 2WS system. This contrasts with the 2WS vehicle, which has a longer settling time and less overshoot than the other vehicle.

Also, Figure 4 illustrates how the 4WS system lowers the yaw velocity, increasing the vehicle's stability along its vertical axis. The Figure illustrates the 32% improvement in the front steering. So, the work required to steer the vehicle can be compared from the front steering angle with the time. Notably, the curve fluctuations of the front steering angle decrease with time which indicates that the work required to direct the vehicle to its desired direction will also decrease. Additionally, the Figure indicates a rapid decrease in the rear steering angle value, indicating that the 4WS requires less energy.



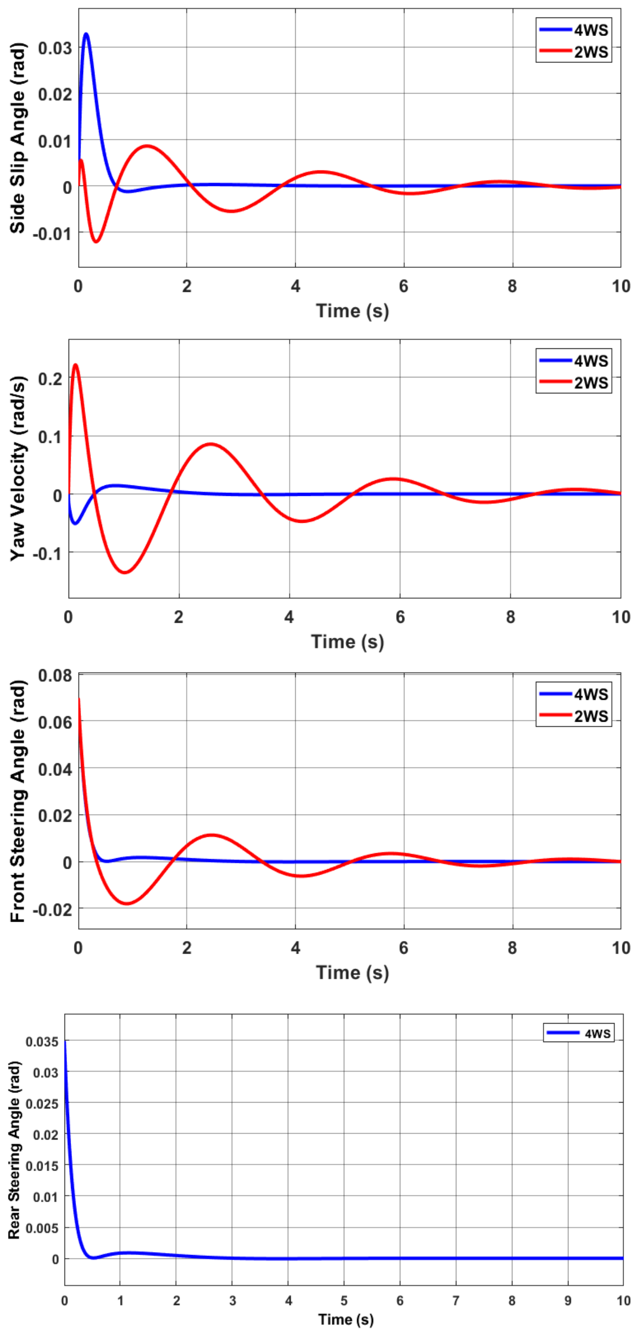


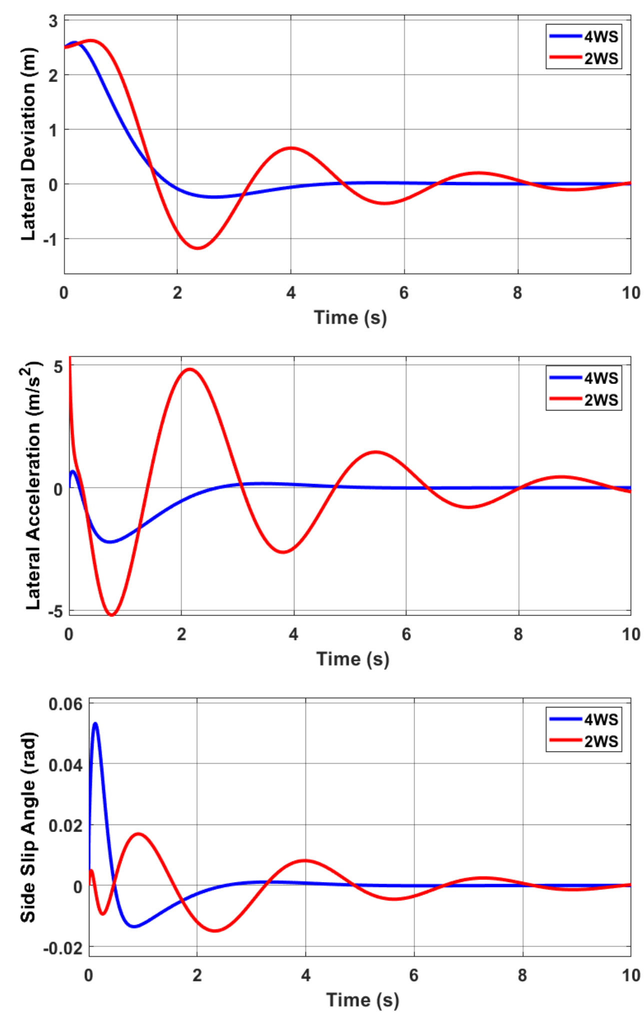
Figure 4 Comparison of vehicle handling performance between conventional 2WS and Optimal 4WS systems with respect to steering angle 4°.

Table 3 RMS values of vehicle handling performance for the 2WS and the 4WS system at steering angle 4°.

Parameter	2WS	4WS	Improvement (%)
Lateral acceleration	0.5092	0.1034	79.69
Lateral deviation	0.1836	0.03604	80.37
Front steering angle	0.008435	0.005732	32.05
Side slip angle	0.007572	0.005075	32.98
Yaw velocity	0.03587	0.008347	76.73
Rear steering angle	N/A	0.006194	N/A

5.3. Scenario 3(initial lateral deviation 2.5 m and front steering angle equal to 4°)

To evaluate the reliability of the presented vehicle control, the lateral deviation has been set to 2.5 meters, and the wheel steer angle was carried out at 4 degrees. In Figure 5, a comparison is presented between the transient handling responses illustrated by the 2WS and 4WS systems. A comparison is conducted between the behaviors of the 2WS vehicle and those of the 4WS system, which is found to be both quicker and more adaptive. A further noteworthy point is that the rising time of the 4WS vehicle is substantially less than that of its equivalent, the 2WS vehicle. According to the data provided in the table, the 4WS controller has succeeded in a decrease in lateral acceleration and lateral deviation by 24.29% and 67.15% respectively. The RMS values of the vehicle handling indices under a 4° front steering angle and lateral deviation 2.5 m input are displayed in Table *3* 4.



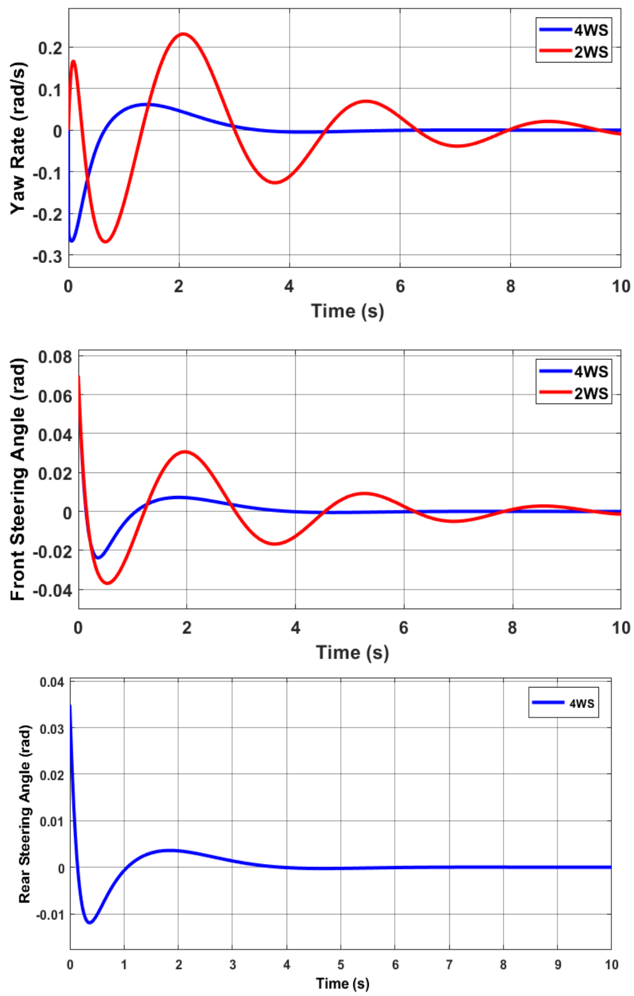


Figure 5 Comparison of vehicle handling performance between conventional 2WS and Optimal 4WS systems with respect to steering angle 4° and initial deviation 2.5m.

Table 4 RMS values of vehicle handling performance of the 2WS and the 4WS System at steering angle 4° and initial deviation 2.5m.

Parameter	2WS	4WS	Improvement percentage (%)
Lateral acceleration	2.085	0.6848	67.15
Lateral deviation	0.9335	0.7067	24.29
Front steering angle	0.01426	0.006738	52.75
Side slip angle	0.008515	0.006604	18.17
Yaw velocity	0.1027	0.04678	54. 45
Rear steering angle	N/A	0.006738	N/A

According to these results, the optimal 4WS system significantly decreased the lateral deviation. Furthermore, the vehicle's lateral acceleration is significantly reduced. As expected, the driver's effort, as shown by the root-mean-square values of the steering wheel angle, was drastically decreased. From the perspective of the driver, this implies that the ideal 4WS system makes the intended trajectory much easier to accomplish. The findings indicate that the side slip angle degraded in every scenario. Under these circumstances, the side slip motion of the vehicle will be increased because the front and rear steering angles are in the same direction. Through this comparison, the enhancement of most performance criteria including the lateral deviation, lateral acceleration, and yaw rate of the optimal 4WS system compared to 2WS has been highlighted. In Future study, the roll motion and longitudinal will be incorporated into the vehicle model to evaluate the vehicular lateral handling and rollover threshold. The effects of the nonlinear tires model are considered. The influence of suspension characteristics is considered and therefore the ride comfort will be studied.

6. Conclusions

The study focused on a vehicle handling model with two degrees of freedom (DOF) and a four-wheel steering control system, considering the driver model when running a vehicle speed of 80 km/h. A state feedback based LQR control approach for a four-wheel steering system is suggested. The controller regards all model state variables as being reduced and used for full-state feedback. PSO technique is employed to determine the most optimal gains to track and improve handling performance with continuously varying driving situations. Analyzed comparisons were conducted between the 2WS and 4WS vehicle handling models. Optimal four-wheel steering provides considerable enhancements compared to two-wheel steering, particularly in terms of lateral deviation, lateral acceleration, and yaw rate. As calculated RMS values, significant improvements were achieved with 70.44%, 39.14%, and 61.88%, respectively. According to the driver, in comparison with conventional 2WS, the RMS of the steering wheel angle is 46.19% reduced. Because of this, the vehicle's active four-wheel steering system greatly improves the ride's ability to be controlled and comfortable.

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