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Combined Heat and Power Economic Dispatch Solution by Equilibrium Optimizer

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Abstract Due to its great economic benefit, the demand for utilizing combined heat and power (CHP) systems has increased. Many facilities and buildings need to be supplied with both energy and heat. Usually, CHP systems are used with other systems that only produce power or heat. The economic dispatch (ED) problem for the CHP systems, which seek to minimize the cost of fuel, is highly challenging. One of the most significant algorithms for solving the ED problem in CHP systems is the Equilibrium Optimizer (EO). This work examined the effectiveness of the EO in solving the ED in CHP systems. In this paper, the EO was applied to many different studied systems, taking into account the effect of losses and the effect of valve point in these different systems. The effectiveness of the EO would be compared to other algorithms applied in some of the previous works. The proposed algorithm (EO) has high-quality solutions and superior performance compared to some solutions presented recently.

Keywords: Combined Heat and Power; Economic Dispatch; Equilibrium Optimizer; Non-convex Optimization.

1 Introduction

Cogeneration systems, such as combined heat and power (CHP) systems, are the best way to produce two types of usable energy from a single fuel source[1]. In CHP systems, heat (steam) is the primary energy form, while electricity is the secondary energy source. CHP systems are usually used where there is a need for heat besides electrical power. To ensure meeting the required energy efficiencies and environmental requirements, CHP systems are often placed close to buildings where heat is needed.

The CHP Economic Dispatch (CHPED) issue aims to meet all the system heat and power demands at the minimum fuel cost while satisfying all the constraints. There are many algorithms that were used to get the optimal solution to the CHPED problem[2].

In this study, to solve the CHPED issue, equilibrium optimization (EO) was used. Different scales of test systems were used to evaluate the effectiveness of the suggested method. By comparing the results of the EO with some other previous techniques, the EO proved to be a reliable optimization method. Findings indicated that the EO can find the best solution to the CHPED problem.

2 Problem Formulation

The CHPED issue faced many challenges, such as the system having dual demands (heat and power) and the dependency of the capacity of the generated power on the capacity of the generated heat in the cogeneration units.

In the CHPED problem, there are three types of units: power-only, heat-only, and combined heat and power units. The objective function of CHPED is to minimize the operation cost of the system while satisfying the system constraints.

Many algorithms were used to solve the CHPED problem, such as: genetic algorithm (GA)[3], harmony search (HS)[4], Differential Evolution (DE)[5], improved ant colony search (IACS) [6], evolutionary programming (EP)[7], improved genetic algorithm with multiplier

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updating (IGA-MU)[8], firefly algorithm (FA)[9], a self-adaptive real-coded genetic algorithm (SARGA)[10], exchange market algorithm (EMA)[11], time varying acceleration coefficients particle swarm optimization (TVAC-PSO)[12], Direct solution [13] , integrated genetic - tabu search algorithm (G-ATS)[14], real coded genetic algorithm (RCGA)[15],Artificial Immune System (AIS) [16], Bee Colony Optimization (BCO)[17], Lagrangian relaxation(LR)[18], benders decomposition (BD)[19], crisscross optimization algorithm(COA)[20], stochastic fractal search (SFS)[21], grasshopper optimization algorithm (GOA)[22], adaptive cuckoo search with differential evolution mutation (ACS-DEM)[23], group search optimization (GSO)[24], wild goats algorithm (WGA)[25], particle swarm optimization (PSO)[26], invasive weed optimization (IWO)[27], marine predators algorithm (MPOA)[28], artificial bee colony (ABC)[29], hybrid heap-based and jellyfish search algorithm (HBJSA)[30], real coded genetic algorithm with improved Mühlenbein mutation (RCGA-IMM)[15], weighted vertices-based optimizer (WVO)[31], etc.

3 Mathematical Modeling of the CHPED Problem

3.1 Objective Function

The formulation of the CHPED problem is given in [2]. The objective function is defined as:

$$OF = \sum_{i=1}^{N_P} C_i(P_i^P) + \sum_{j=1}^{N_c} C_j(P_j^c, H_j^c) + \sum_{K=1}^{N_h} Ck(H_k^h) \quad (1)$$

Where;
$C_i(P_i^P)$: Cost Function of Power Only Unit (\$/h),
$C_j(P_j^c, H_j^c)$: Cost Function of Cogeneration Unit (\$/h),
$Ck(H_k^h)$: Cost Function of Heat Only Unit (\$/h).

$$C_i\,(P_i^P) = a_i((P_i^P)^2 + b_iP_i^P + c_i \quad (2)$$

$$C_i(P_i^P) = a_i((P_i^P)^2 + b_iP_i^P + c_i + \left|d_i\, sin(e_i\,(P_i^{Pmin} - P_i^P))\right| \quad (3)$$

Where;
$a_i,\ b_i,\ c_i$: Cost Function Coefficients of power only unit,
$d_i,\ e_i$: cost coefficients of valve-point impacts,
p_{pi}^{min} : Lower Power out of power only unit,
P_i^P : Power out of power only unit.

$$C_i(P_j^c, H_j^c) = a_j(P_j^c)^2 + b_jP_j^c + C_j + d_j(H_j^c)^2 + e_jH_j^c + f_jP_j^cH_j^c \quad (\$/h) \quad (4)$$

Where;
$a_j,\ b_j,\ C_j,\ d_j,\ e_j,\ f_j$: Cost Function Coefficients of Cogeneration Unit.

$$C_k\,(H_K^h) = a_k(H_k^h)^2 + b_kH_k^h + C_k \quad (\$/h) \quad (5)$$

Where;
$a_k,\ b_k,\ C_k$: Cost Function Coefficients of heat only unit.

3.2 Constraints

3.2.1 Power balanced Constraints:

The power generated from pure power units and combined units must meet the demand and lost capacity of lines[32].

$$\sum_{i=1}^{N_p} P_i^p + \sum_{j=1}^{N_c} P_j^c = P_d \quad (6)$$

Where;
P_d: Power demand
P_i^p: power output of only power unit.
P_j^c: power output of cogeneration units.

3.2.2 Heat balance Constraints:

The total heat produced by pure heat units and cogeneration units must meet heat demand and neglected loss heat.

$$\sum_{j=1}^{N_c} H_j^c + \sum_{K=1}^{N_h} H_k^h = Hd \quad (7)$$

Where;
Hd: Heat Demand,
H_j^c: Heat output of cogeneration units,
H_k^h: Heat output of heat only units.

3.2.3 Generation power and heat limits:

The produced electric and heat powers should be in the acceptable range for each unit:

$$P_i^{pmin} \le P_i^p \le P_i^{Pmax} \quad i = 1, 2, \ldots, Np \quad (8)$$

Where;
$P_i^{pmin},\ P_i^{Pmax}$: lower & upper output power of power only units.

$$P_j^{Cmin}(H_j^c) \le P_j^c \le P_j^{Cmax}(H_j^c) \quad j = 1,2,..,Nc \quad (9)$$

Where;
$P_j^{C_{min}}$, $P_j^{C_{max}}$:lower & upper output power of cogeneration units.

$$H_j^{Cmin}\left(P_j^c\right) \leq H_j^c \leq H_j^{Cmax}\left(H_j^c\right) \; j = 1,2,..,Nc \qquad (10)$$

Where;
$H_j^{C_{min}}$, $H_j^{C_{max}}$: lower & upper heat output of cogeneration units.

$$H_i^{h_{min}} \leq H_k^h \leq H_k^{h_{max}} \quad k = 1,2,\ldots,N_h \qquad (11)$$

Where;
$H_i^{h_{min}}$, $H_k^{h_{max}}$: lower & upper heat output of heat only units.

4 CHPED Formulation Through The EO

The EO algorithm, inspired by the control volume mass balance, is designed to estimate both dynamic and equilibrium states. EO falls into the third class of optimization algorithms, as it is derived from physical laws found in nature [33]. Inside the EO, each particle (solution) with its concentration (position) functions as a search agent. Search agents update their concentrations randomly according to the best-so-far solutions, known as equilibrium candidates, to eventually reach the state of equilibrium (optimal result). To enhance the EO algorithm's ability in exploration, exploitation, and local minima avoidance, a well-defined "generation rate" term has been proven effective. The mass-balanced equation is represented as:

$$V\frac{dx}{dt} = QX_{eq} - QX + G \qquad (12)$$

Where, V is the control volume, X is the concentration, and Q the flow rate, $V\frac{dx}{dt}$ is the rate of change of mass in the control volume, $X_{eq} = X_{pool}$ represents the concentration at an equilibrium state in which there is no generation inside the control volume, G is the mass generation rate inside the control volume.
The concentration (X) can be represented as follows:

$$X = X_{eq} + \left(X_O - X_{eq}\right) exp[-\lambda(t - t_0)] + \frac{G}{\lambda V}\left(1 - (exp[-\lambda(t - t_0)])\right) \qquad (13)$$

Where;
$\lambda = \left(\frac{Q}{V}\right)$, X_0 denotes the initial concentration, while t_0 refers to the initial start time.

The following steps describe the procedure of the EO:

Step 1: Initialization

The concentrations are initialized randomly as follows:

$$X_i^{initial} = X_{min} + rand_i\,(X_{max} - X_{min}) \quad i = 1,2,..n \qquad (14)$$

Where;
$X_i^{initial}$ refers to the initial concentration vector of the i-th particle, X_{max} refers to the maximum limit of the control variables while X_{min} is the minimum limit. $rand_i$ is a random variable within [0,1]. Then, evaluate the objective function for each concentration.

Step 2: Assigning the Equilibrium candidate's

The populations are sorting, and the four best solutions are captured and their average value to form the pool vector X_{Pool} as follows:

$$X_{avg} = \frac{X_1 + X_2 + X_3 + X_4}{4} \qquad (15)$$

$$X_{pool} = \{X_1, X_2, X_3, X_4, X_{avg}\} \qquad (16)$$

Step 3: The concentration Update

Two randomly vectors (r, λ) are generated randomly and utilized to adjust an exponential factor (F) for updating the concentrations as follows:

$$F = a_1 sign(r - 0.5)\left[e^{-\lambda t} - 1\right] \qquad (17)$$

$$t = \left(1 - \frac{T}{T_{Max}}\right)^{\left(a_2 \frac{T}{T_{Max}}\right)} \qquad (18)$$

Where;
F is the exponential term, a_1 and a_2 refer to constant

terms, which are set to be 2 and 1, respectively, to adjust the exponential factor. T_{Max} is the maximum iteration number, and T denotes the *T*-th iteration. It should be indicated out here that a_1 is utilized to controls the exploration phase of the EO and a_2 is utilized to control the exploitation process of the EO. Sign $(r - 0.5)$ controls the direction of the exploration.

Step 4: Concentration updating based on the generation rate

The generation rate is an efficient method to enhance the exploitation phase of the optimization algorithm as follows:

$$G = G_0 \, e^{-k(t-t_0)} \tag{19}$$

Where;

G_0 is the initial value, k indicates a decay constant,

Where;

r_1 and r_2 are random numbers within the range [0,1], GCP is defined as the Generation rate Control Parameter, GP denotes the generation probability to control the participation probability of concentration, which is updated by the generation rate. If GP = 1 generation rate

$$G_0 = GCP \; (X_{pool} - \lambda X) \tag{20}$$

$$GCP = \begin{cases} 0.5\, r_1 & r_2 \geq GP \\ 0 & r_2 < GP \end{cases} \tag{21}$$

will be no participate in the optimization process. If GP =0, the generation rate will participate in the process. GP = 0.5 provides an excellent balancing between exploitation and exploration phases. Referring to the previous steps, the updated equation of the EO is formulated as follows:

$$X = X_{pool} + (X - X_{pool}).F + \frac{G}{\lambda V}(1 - F) \tag{22}$$

Step 5: Adding memory saving

In this step, the obtained concentration is compared with the previous concentration, and it will be accepted if this value is enhanced. **Fig.** 1 shows the flow chart of optimal energy management.

5 Results and discussion

This should include the findings of the study including, if appropriate, results of statistical analysis which must be included either in the text or as tables and figures. For research articles this section should discuss the implications of the findings in context of existing research and highlight limitations of the study. For methodology manuscripts this section should include a discussion of any practical or operational issues involved in performing the study and any issues not covered in other sections. The test problems taken into consideration are taken from Refs. [6, 34, 35]. For the cogeneration units, implausible solutions are rendered feasible while randomly generating candidate solutions by fixing them to the closest straight line in the contour. In the case of power-only & heat-only units, infeasible candidates are moved to the nearest upper or lower limits. The equality constraints are taken care of by the use of penalty functions augmenting the objective function. Simulations were conducted in MATLAB R2015a.

5.1 Tested System 1

A tested system of four units is taken to illustrate the performance of the proposed method. For the conventional power unit 1, For the cogeneration units 2 and 3, For the heat-only unit 4:

$$C_1 = 50\,P1 \quad , \quad 0 \leq P_1 \leq 150\; MW \tag{23}$$

$$C_2 = 0.0345P_2^2 + 14.5\,P_2 + 2650 + 0.03\,H_2^2 + 4.2\,H_2 + 0.031P_2H_2 \tag{24}$$

$$C_3 = 0.0345P_3^2 + 36\,P_3 + 1250 + 0.027\,H_3^2 + 0.6\,H_3 + 0.011P_3H_3 \tag{25}$$

The power and heat demand for the system is 200MW and 115 MWth respectively. The heat-power feasible regions for the cogeneration units are illustrated in **Figs**. 2 & 3.

Fig. 4 shows the characteristics of System 1 which occurs after 500 iterations **Table 1** shows the comparison between the results of the EO along with the other published results. In this system, by comparing the result obtained with the latest results, the EO saved an amount of money amounting to 0.03 $ / h; thus, the total annual saving is 262.8 $ / year.

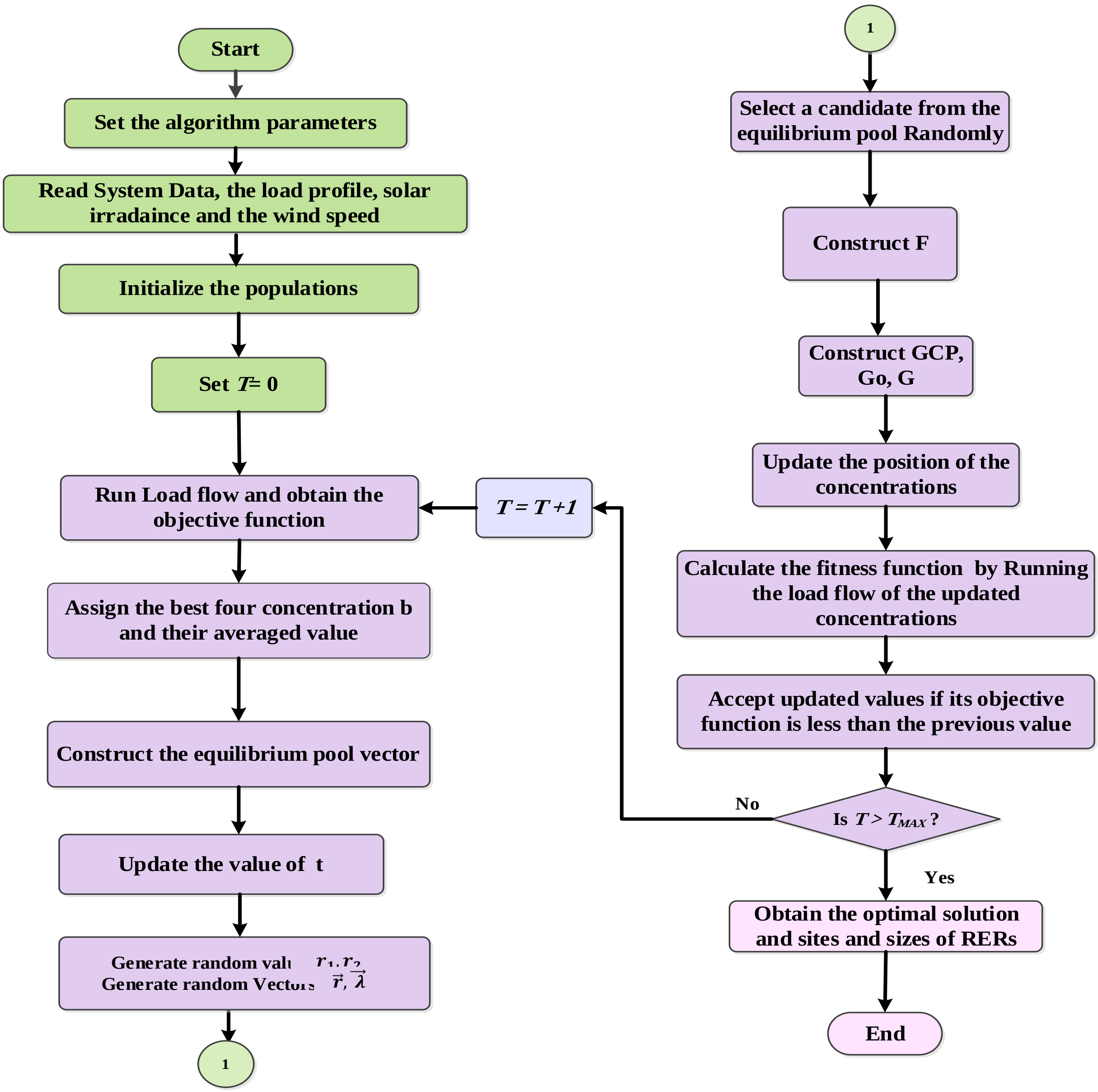

Fig. 1 Flow chart of optimal energy management.

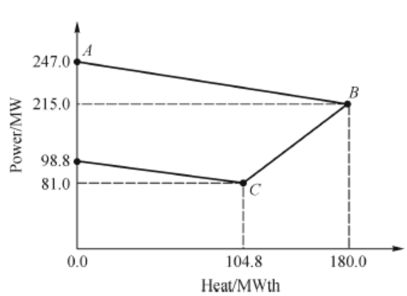

Fig. 2 Feasibility region for cogeneration unit 2 in System1.

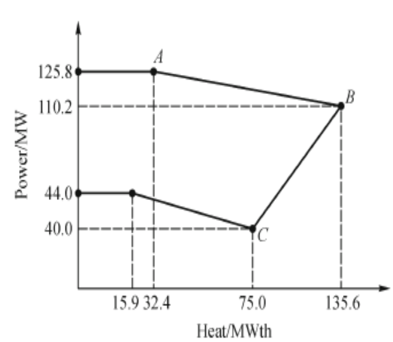

Fig. 3 Feasibility region for cogeneration unit 3 in System1.

Table 1 Comparison between the results of the EO along with the other published results.

Algorithm	P_1	P_2	P_3	H_2	H_3	H_4	Min
ACSA [36]	0.08	150.93	49	48.84	65.79	0.37	9452.2
GA [3]	0	159.23	40.77	39.94	75.06	0	9267.2
RGA [37]	0	158.18	41.82	37	78	0	9263.28
EP [7]	0	160	40	40	75	0	9257.1
FP [38]	0.0014	159.9986	40	40	75	0	9257.1
HS [4]	0	160	40	40	75	0	9257.07
MU-IGA [8]	0	160	40	39.99	75	0	9257.07
SARGA [39]	0	159.99	40.01	40	75	0	9257.07
EMA [11]	0	160	40	40	75	0	9257.07
TVAC – PSO [12]	0	160	40	40	75	0	9257.07
Direct method [13]	0	160	40	40	75	0	9257.07
The proposed algorithm (EO)	0	160	40	39,99	75	0	9257.07

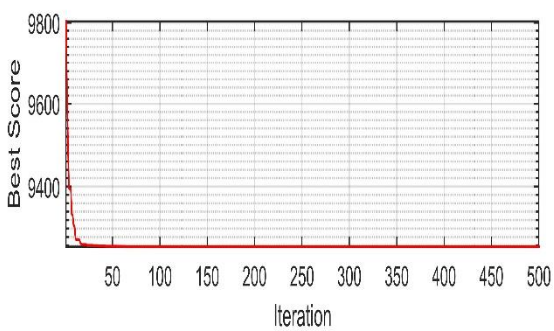

Fig. 4 Characteristics of the Tested System 1 (after 500 iterations).

5.2 Tested System 2

The Tested System involved one conventional power unit, three cogeneration units, and a heat-only unit. For the conventional power unit1:

$$C_1 = 0.00115P_1^3 + 0.00172P_1^2 + 7.6997P_1 + 254.886 \quad 35 \le P_1 \le 135 \quad M_W \tag{26}$$

For the cogeneration units 2 ,3 and 4,

$$C_2 = 0.0345P_2^2 + 36\,P_2 + 1250 + 0.027\,H_2^2 + 0.6\,H_2 + 0.011\,P_2H_2 \tag{27}$$

$$C_3 = 0.0345P_3^2 + 14.5\,P_2 + 2650 + 0.03\,H_3^2 + 4.2\,H_3 + 0.031P_2\,H_2 \tag{28}$$

$$C_4 = 0.072P_4^2 + 20\,P_4 + 1565 + 0.02\,H_4^2 + 2.3\,H_4 + 0.04\,P_4H_4 \tag{29}$$

For the heat-only generated from unit 5,

$$C_5 = 0.038H_5^2 + 2.0109H_5 + 950 \quad 0 \le H_5 \le 60 \quad M_W \tag{30}$$

The heat-power feasible regions for the cogeneration units are illustrated in **Figs**. 5 ,6 and 7

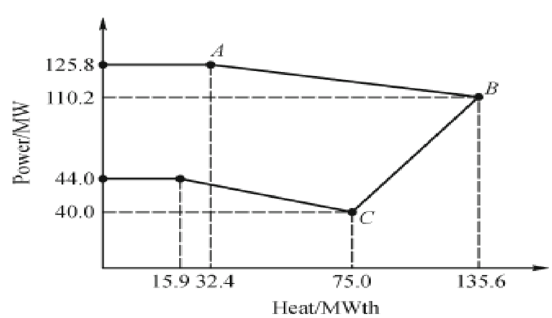

Fig.5 Feasibility region for cogeneration unit 2 in System2.

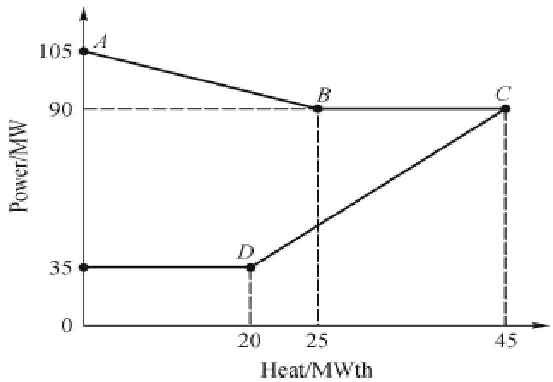

Fig. 6 Feasibility region for cogeneration unit 3in System 2.

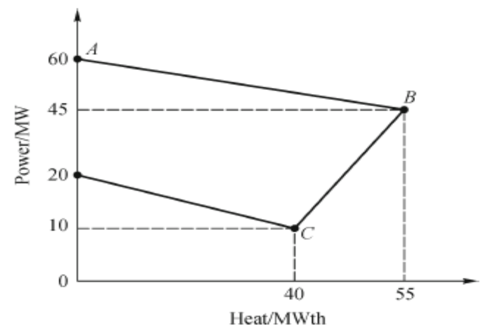

Fig.7 Feasibility region for cogeneration unit 4in System 2.

Table 2 shows the comparison between the results of the EO along with the other published results, The power and heat demand for the system are 300MW and 150 MWth respectively Profile 1. by comparing the result obtained with the latest results, the EO saved an amount of money amounting to 6.119 $ / h; thus, the total annual saving is 53602.44 $/year.

Fig. 8 shows the characteristics of System 2 which occurs after 500 iterations profile 1.

Table 2 Comparison between the EO results & the other published results for the system of 300MW and 150 MWth.

Algorithm	**P_1 (MW)**	**P_2 (MW)**	**P_3 (MW)**	**P_4 (MW)**	**H_2 (MWth)**	**H_3 (MWth)**	**H_4 (MWth)**	**H_5 (MWth)**	**Min.**
GA [3]	135.00	70.81	10.84	83.28	80.54	39.81	0.00	29.64	13.779.50
RCGA [15]	134.9904	49.9525	25.0827	89.9744	73.5089	35.8519	1.2916	39.3476	13,776.14
HS [4]	134.74	48.20	81.09	16.23	23.92	100.85	6.29	38.70	13,723.20
CPSO [40]	135	40.7309	19.2728	105	64.4003	26.4119	0	59.1955	13,692.5212
IWO [27]	134.73	40.00	75.00	20.86	37.60	104.41	0	37.40	13,683.65
FA [9]	134.74	40	20.25	105	75	27.87	0	47.12	13,683.22
GSA [41]	135	41.7806	18.1736	105	74.089	37.3336	0	38.5713	13,671.149
Proposed algorithm (EO)	135.00	40.00	20. 54	104. 45	77.25	27.23	0.00	45.53	13,665.03

Table 3 Comparison between the EO results & the other published results for the system of 250MW and 175 MWth.

Algorithm	**P_1 (MW)**	**P_2 (MW)**	**P_3 (MW)**	**P_4 (MW)**	**H_2 (MWth)**	**H_3 (MWth)**	**H_4 (MWth)**	**H_5 (MWth)**	**Min.**
GA [3]	119.22	45.12	15.82	69.89	78.94	22.63	18.4	54.99	12,327.37
HS [4]	134.67	52.99	10.11	52.23	85.69	39.73	4.18	45.4	12,284.45
CPSO [40]	135	40.3446	10.0506	64.606	70.9318	39.9918	4.0773	60	12,132.858
IWO [27]	134.59	40	10.94	64.47	75	38.98	8.81	52.21	12,134.33
FA [9]	134.81	40	10	65.18	75	40	16.97	43.02	12,119.86
GSA [41]	135	39.9998	10	64.9807	74.9844	40	17.8939	42.1095	12,117.37
Proposed algorithm (EO)	135.00	40.05	10.06	64.87	78.48	39.74	0.00	56.76	12,111.47

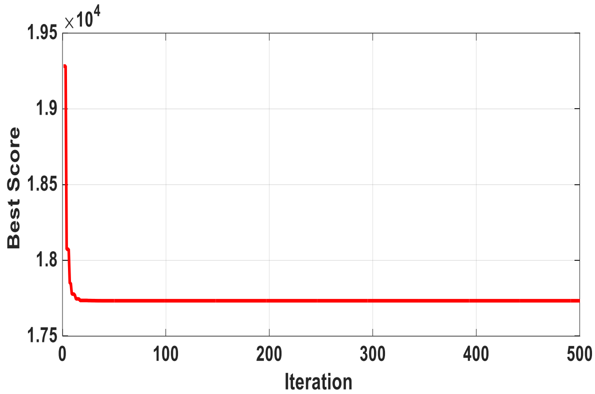

Fig. 8 Characteristics of the Tested System 2 (after 500 iterations profile 1).

Table 3 shows the comparison between the EO results & the other published results, the power & heat demand for the system are 250MW & 175 MWth respectively Profile 2. by comparing the result obtained with the latest results, the EO saved an amount of money amounting to **5.9** $ / h; thus, the total annual saving is 53602.44 $ /year.

Fig. 9 shows the characteristics of System 2 which occurs after 500 iterations profile 2.

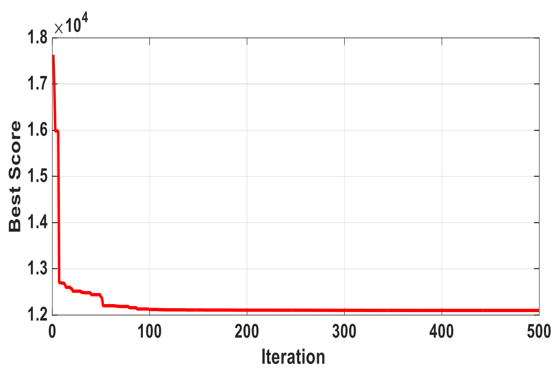

Fig. 9 Characteristics of the Tested System 2 (after 500 iterations profile2).

5.3 Tested System 3

Data of this system are adopted from [5] Four conventional thermal power units, two cogeneration units, and a heat-only unit make up the tested system. In relation to the standard thermal power units 1, 2, 3, and 4,

$$C_1 = 0.008P_1^2 + 2P_1 + 25 + \left|100\, sin\left(0.042(10 - P_1)\right)\right| \quad (31)$$

$$C_2 = 0.003P_2^2 + 1.8\, P_2 + 10 + \left|140\, sin\left(0.04(20 - P_2)\right)\right| \quad (32)$$

$$C_3 = 0.0012P_3^2 + 2.1P_3 + 100 + \left|160\, sin\left(0.038(30 - P_3)\right)\right| \quad (33)$$

$$C_4 = 0.001P_4^2 + 2P_4 + 120 + \left|180\, sin\left(0.037\, (40 - P_4)\right)\right| \quad (34)$$

For the cogeneration units 5 and 6 ,

$$C_5 = 0.0345\, P_5^2 + 14.5\, P_5 + 2650 + 0.03\, H_5^2 + 4.2H_5 + 0.031P_5H_5 \quad (35)$$

$$C_6 = 0.0345\, P_6^2 + 36\, P_6 + 1250 + 0.027\, H_6^2 + 0.6\, H_6 + 0.011\, P_6H_6 \quad (36)$$

For the heat-only unit 7,

$$C_7 = 0.038\, H_7^2 + 2.0109\, H_7 + 950, \quad 0 \le H_7 \le 60 \text{ MWth} \quad (37)$$

$$C_1 = 0.0028\, P_1^2 + 8.1\, P_1 + 550 + \left|300\, \sin\left(0.035(0 - P_1)\right)\right| \quad (\$/h) \quad 0 < P_1 < 680 \quad (38)$$

$$C_2 = 0.00056\, P_2^2 + 8.1\, P_2 + 309 + \left|200\, \sin\left(0.042\, (0 - P_2)\right)\right| \quad (\$/h) \quad 0 < P_2 < 360 \quad (39)$$

$$C_3 = 0.00056\, P_3^2 + 8.1\, P_3 + 309 + \left|200\, \sin\left(0.042\, (0 - P_3)\right)\right| \quad (\$/h) \quad 0 < P_3 < 360 \quad (40)$$

$$C_4 = 0.00324\, P_4^2 + 7.74\, P_4 + 240 + \left|200\, sin\left(0.063\, (60 - P_4)\right)\right| \quad (\$/h) \quad 60 < P_4 < 180 \quad (41)$$

$$C_5 = 0.00324\, P_5^2 + 7.74\, P_5 + 240 + \left|200\, sin\left(0.063\, (60 - P_5)\right)\right| \quad (\$/h) \quad 0 < P_5 < 180 \quad (42)$$

The heat-power feasible regions for the cogeneration nits are illustrated in **Figs.** 10 & 11.

Table 4 Comparison between the EO results & the other published results, for the system are 600MW and 150 MWth respectively.

Algorithm	P_1 (MW)	P_2 (MW)	P_3 (MW)	P_4 (MW)	P_5 (MW)	P_6 (MW)	H_5 (MWth)	H_6 (MWth)	H_7 (MWth)	Min
RCGA [15]	74.5357	99.3518	174.7196	211.0170	100.9363	44.1036	24.3678	72.52 70	53.1052	10,712.86
PSO [42]	18.4626	124.2602	112.7794	209.8158	98.8140	44.0107	57.9236	32.7603	59.3161	10,613
EP [7]	61.3610	95.1205	99.9427	208.7319	98.8	44	18.0713	77.5548	54.3739	10,390
AIS [43]	50.1325	95.5552	110.7515	208.7688	98.8	44	19.4242	77.0777	53.4981	10,355
DE [44]	44.2118	98.5383	112.6913	209.7741	98.8217	44	12.5379	78.3481	59.1139	10,317
RCO [45]	43.9457	98.5888	112.932	209.7719	98.8 44	12.0974	78.0236	59.8	79	10,317
ECSA [46]	53.7610	98.5039	112.5996	209.7993	93.0872	40.2022	33.6571	72.6890	43.6539	10,121.9466
KHA [47]	46.3835	104.1223	64.3729	246.1853	98.9736	40.7401	0	66.71	83.29	10,111.1501
EMA [48]	52.6847	98.5398	112.6734	208.8158	93.8341	40. 29242	75	45.75	79	10,111.0732
TVAC – PSO [12]	47.3383	98.5398	112.6735	209.81582	92.3718	40	37.8467	74.9999	37.1532	10,100.3164
TLBO [49]	45.266	98.5479	112.6786	209.8284	94.4121	40.0062	25.8365	74.9970	49.1666	10,094.8384
OTLBO [50]	45.886	98.5398	112.6741	209.8141	93.8249	40.0002	29.2914	75.0002	45.7084	10,094.3529
CSO [51]	45.4909	98.5398	112.6734	209.8158	94.1838	40	27.1786	75	47.8214	10,094.1267
Proposed algorithm (EO)	45.4909	98.5398	112.6734	209.8158	94.1838	40	27.1786	75	47.8214	10,044.126

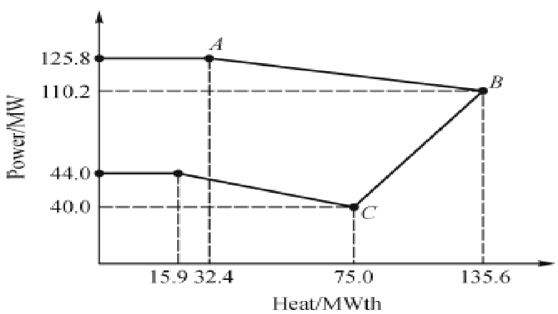

Fig.10 Feasibility region for cogeneration unit 5 in System3.

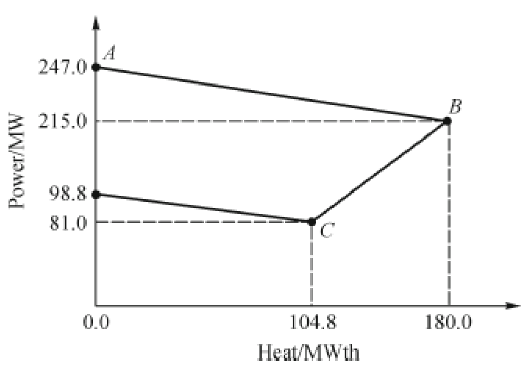

Fig. 11 Feasibility region for cogeneration unit 6 in System 3.

In this section, the tested system considered both the valve-point effects and transmission losses to examine the effectiveness of the EO. **Table 4** shows the comparison between the results of the EO along with some other published results, the power and heat demand were 600MW and 150 MWth respectively. By comparing the result obtained with the latest results, the EO saved an amount of money amounting to 50 $ / h; thus, the total annual saving is 438000 $/year.

Fig. 12 shows the characteristics of System 3 which occurs after 500 iterations.

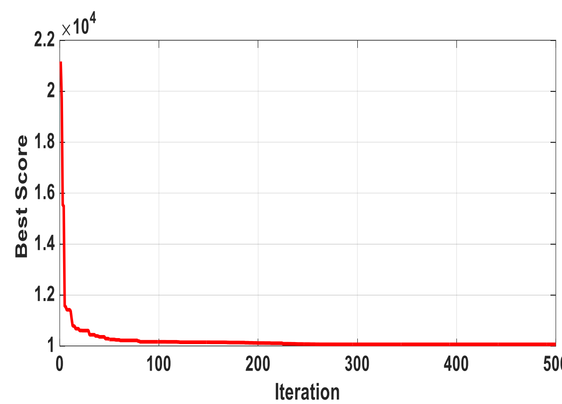

Fig. 12 Characteristics of the Tested System 3 (after 500 iterations).

5.4 Tested System 4

A large-scale test system consists of 24 units is considered. Among twenty-four units, units 1–13 are power-only units, 14 – 19 are cogeneration units and 20–24 are heat-only units. The fuel cost function includes valve point effects, The power and heat demand for the system is 2350MW and 1250 MWth respectively. In relation to the standard thermal power units 1-13, For the conventional thermal power units.

$$C_6 = 0.00324\,P_6^2 + 7.74\,P_6 + 240 + \left|200\,\sin\left(0.063\,(60 - P_6\,)\right)\right| \quad (\$/h) \tag{43}$$

$$0 < P_5 < 180$$

$$C_7 = 0.00324\,P_7^2 + 7.74\,P_7 + 240 + \left|200\,sin\left(0.063\,(60 - P_7\,)\right)\right| \quad (\$/h) \tag{44}$$

$$0 < P_7 < 180$$

$$C_8 = 0.00324\, P_8^2 + 7.74\ P_8 + 240 + \left|200\ sin\left(0.063\,(60 - P_8\,)\right)\right| \quad (\$/h) \tag{45}$$

$$0 < P_8 < 180$$

$$C_9 = 0.00324\, P_9^2 + 7.74\ P_9 + 240 + \left|200\ sin\left(0.063\,(60 - P_9\,)\right)\right| \quad (\$/h) \tag{46}$$

$$0 < P_5 < 180$$

$$C_{10} = 0.00284\, P_{10}^2 + 8.6\ P_{10} + 126 + \left|100\ sin\left(0.084\,(40 - P_{10}\,)\right)\right| (\$/h) \tag{47}$$

$$40 < P_{10} < 120$$

$$C_{11} = 0.00284\, P_{11}^2 + 8.6\ P_{11} + 126 + \left|100\ sin\left(0.084\,(40 - P_{11}\,)\right)\right| \ (\$/h) \tag{48}$$

$$40 < P_{10} < 120$$

$$C_{12} = 0.00284\, P_{12}^2 + 8.6\ P_{12} + 126 + \left|100\ sin\left(0.084\,(55 - P_{12}\,)\right)\right| \ (\$/h) \tag{49}$$

$$40 < P_{10} < 120$$

$$C_{13} = 0.00284\, P_{13}^2 + 8.6\ P_{13} + 126 + \left|100\ sin\left(0.084\,(55 - P_{13}\,)\right)\right| \ (\$/h) \tag{50}$$

$$40 < P_{10} < 120$$

For the cogeneration units,

$$C_{14} = 0.0345\, P_{14}^2 \ + 14.5\ P_{14} + 2650 + 0.03\ H_{14}^2 + 4.2\ H_{14} + 0.031\ P_{14}H_{14}\ \ (\$/h) \tag{51}$$

$$C_{15} = 0.0435\, \mathrm{P}_{15}^2\ + 36\ \ \mathrm{P}_{15}\ + 1250 + 0.027\ \mathrm{H}_{15}^2 + 0.6\ \mathrm{H}_{15} + 0.011\, \mathrm{P}_{15}\mathrm{H}_{15}\quad (\$/h) \tag{52}$$

$$C_{16} = 0.0345\, \mathrm{P}_{16}^2\ \ + 14.5\ \mathrm{P}_{16} + 2650 + 0.03\ \mathrm{H}_{16}^2 + 4.2\ \mathrm{H}_{16} + 0.031\ \mathrm{P}_{16}\mathrm{H}_{16}\ \ (\$/h) \tag{53}$$

$$C_{17} = 0.0435\, \mathrm{P}_{17}^2\ + 36\ \ \mathrm{P}_{17}\ + 1250 + 0.027\ \mathrm{H}_{17}^2 + 0.6\ \mathrm{H}_{17} + 0.011\, \mathrm{P}_{17}\mathrm{H}_{17}\quad (\$/h) \tag{54}$$

$$C_{18} = 0.1035\, \mathrm{P}_{18}^2\ \ + 34.5\ \mathrm{P}_{18} + 2650 + 0.025\ \mathrm{H}_{18}^2 + 2.203\ \mathrm{H}_{18} + 0.051\ \mathrm{P}_{18}\mathrm{H}_{18}\ \ (\$/h) \tag{55}$$

$$C_{19} = 0.072\, \mathrm{P}_{19}^2\ + 20\ \ \mathrm{P}_{19}\ + 1565 + 0.02\ \ \mathrm{H}_{19}^2 + 2.34\ \mathrm{H}_{19} + 0.040\, \mathrm{P}_{19}\mathrm{H}_{19}\ \ (\$/h) \tag{56}$$

For the heat-only units,

$$C_{20} = 0.038\ h_{20}^2\ \ + 2.0109\ h_{20} + 950\ (\$/h) \qquad 0 < P_{20} < 2695.20 \tag{57}$$

$$C_{21} = 0.038\ h_{21}^2\ \ + 2.0109\ h_{21} + 950 \qquad 0 < P_{21} < 60 \tag{58}$$

$$C_{22} = 0.038\ h_{22}^2\ \ + 2.0109\ h_{22} + 950 \qquad 0 < P_{22} < 60 \tag{59}$$

$$C_{23} = 0.052\ h_{22}^2\ \ + 3.0651\ h_{23} + 480 \qquad 0 < P_{23} < 120 \tag{60}$$

$$C_{24} = 0.052\ h_{24}^2\ \ + 3.0651\ h_{24} + 480 \qquad 0 < P_{24} < 120 \tag{61}$$

Comparing the EO with others; it can be noted that the EO is the lowest in cost, as the cost of generating energy and heat reaches 57920.26$. Data of this system are adopted from [2], published results. **Table 5** shows the comparison between the results of the EO along with the other published results, The power and heat demand for the system is 2350MW and 1250 MWth respectively.

The heat-power feasible regions for the cogeneration units are illustrated in **Figs**. 13,14,15 & Fig. 16 shows the characteristics of System 4 which occurs after 500 iterations. by comparing the result obtained with the latest results, the EO saved an amount of money amounting to 86.73 $ / h; thus, the total annual saving is 759754.8 $ /year.

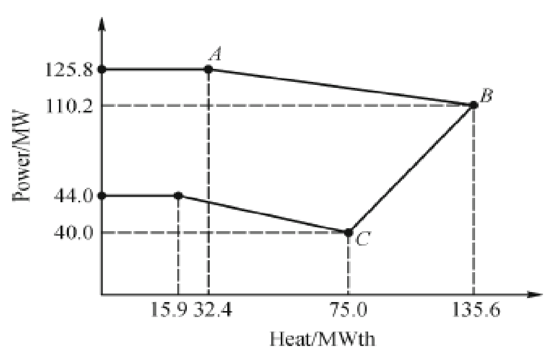

Fig. 13. Feasibility region for cogeneration unit 14,16 in System4.

Table 5 Comparison between the EO results & the other published results, for the system are 235MW and 1250 MWth respectively.

Algorithm	CPSO [40]	TVAC – PSO [12]	PSO [42]	GSO[52]	IGSO[52]	Proposed Algorithm (EO)
$P_1(MW)$	680	538.5587	627.7455	628.152	628.324	536.786
P_2	0	224.4608	76.2285	299.4778	227.3588	299.065
P_3	0	224.4608	299.5794	154.5535	225.9347	148.3117
P_4	180	109.8666	159.4386	60.846	110.3721	159.4924
P_5	180	109.8666	61.2378	103.8538	110.2461	60.0022
P_6	180	109.8666	60	110.0552	160.1761	60.00073
P_7	180	109.8666	157.1503	159.0773	108.3552	158.6485
P_8	180	109.8666	107.2654	109.8258	110.5379	60.7442
P_9	180	109.8666	110.1816	159.992	110.5672	158.1718
P_{10}	50.5304	77.521	113.9894	41.103	75.7562	112.9635
P_{11}	50.5304	77.521	79.7755	77.7055	41.8698	77.3716
P_{12}	55	120	91.1668	94.9768	92.4789	55.0041
P_{13}	55	120	115.6511	55.7143	57.514	55.0013
P_{14}	117.4854	88.3514	84.3133	83.9536	82.5628	90.2864
P_{15}	45.9281	40.5611	40	40	41.4891	40.0045
P_{16}	117.4854	88.3514	81.1796	85.7133	84.771	101.5109
P_{17}	45.9281	40.5611	40	40	40.5874	40.04673
P_{18}	10.0013	10.0245	10	10	10.001	10.0117
P_{19}	42.1109	40.4288	35.097	35	31.0978	35.0427
$H_{14}(MWth)$	125.2754	108.9256	106.6588	106.4569	105.6717	110.0114
H_{15}	80.1175	75.4844	74.998	74.998	76.2843	75.0038
H_{16}	125.2754	108.9256	104.9002	107.4073	106.9125	116.3104
H_{17}	80.1174	75.484	74.998	74.998	75.5061	75.0385
H_{18}	40.0005	40.0104	40	40	39.9986	40.9999
H_{19}	23.2322	22.4676	19.7385	20	18.2205	23.19965
H_{20}	415.9815	458.702	469.3368	466.2575	468.2278	449.4373
H_{21}	60	60	60	60	59.9867	59.9999
H_{22}	60	60	60	60	59.9814	59.99999
H_{23}	120	120	119.6511	120	119.6074	119.99999
H_{24}	120	120	119.7176	119.8823	119.603	119.9989
Min.	59736.26	58122.74	58225.74	58049.01	58006.99	57920.26

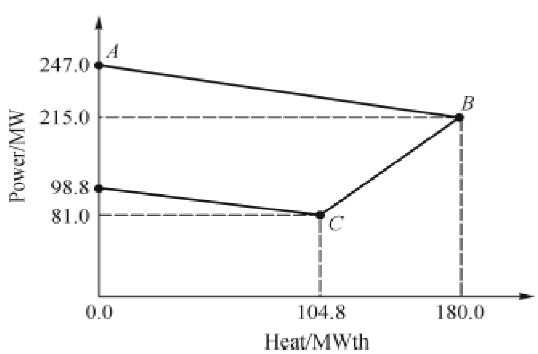

Fig. 14. Feasibility region for cogeneration unit 15,17, in System 4.

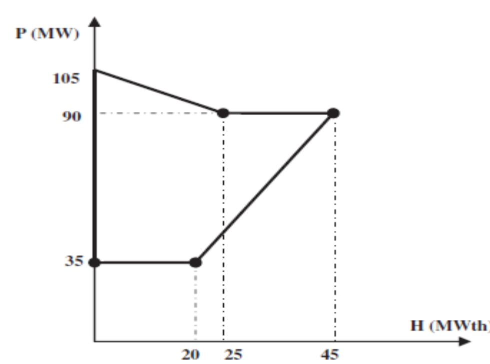

Fig.15. Feasibility region for cogeneration unit 19, in System 4.

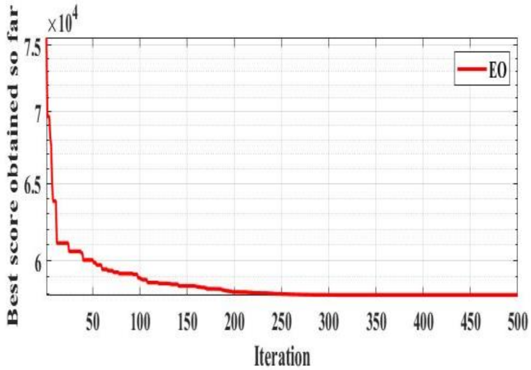

Fig.16 Characteristics of the Tested System 4 (after 500 iterations).

6 Conclusions

CHP systems generate both electricity and thermal energy with high efficiency. The economic dispatch problem aims to meet power and heat demands at the lowest possible cost. This work studied different tested systems. The EO was used to solve the economic dispatch problem in these systems, and its effectiveness was compared with that of other algorithms. The EO provided the best solution among the algorithms compared.

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