Journal of Basic and Environmental Sciences, 5 (2018) 268-281

ISSN
Online: 2356-6388
Print: 2536-9202

Research paper Open Access

Combination Solution of Additive Type for Two Coupled Weakly Nonlinear Second Order Differential Equations

A. M. Elnaggar, K. M. Khalil and A. M. Omran*

Dept. of Math., Faculty of Science, Benha University, Egypt, B. O. 13518

e.mail: amiramasoud.am@gmail.com*

Abstract

Combinations solution of additive type to a two coupled weakly nonlinear second order differential equations which governed the motion of a two coupled nonlinear oscillator subjected to linear parametric excitation and external excitation. We determined the modulation equations in the amplitude and the phase, steady state solutions, the frequency-response equation and stability analysis of the steady state solutions by MSMS. Numerical study of the frequency-response equations and stability equations are given for different values of the parameters. Results are plotted in group of Figures. Finally discussion and conclusion are given.

Keywords: MSM; combinations solution; external excitation.

Received; 21 Jan 2018, Revised form; 28 Jun 2018, Accepted; 28 Jun 2018, Available online 1 Oct. 2018

1 Introduction

The studies of paper is concerned with combinations solution to a two coupled weakly nonlinear second order differential equations [2, 3, 4, 5, 6, 7, 8, 9, 10, 11].

The present paper devoted to study the combinations solution of additive type to a two coupled weakly nonlinear second order differential equations presented in [1], but in the case $f_1 = f_2 = O(1)$ and $\Omega_1 = \Omega_2$. is devoted to combination solutions of additive type

$\Omega_1 + \Omega_0 \approx \omega_0$, in the case the amplitude of external excitation are equal and of order one, also the frequencies of external excitation are equal [1].

2 Perturbation Analysis

The two weakly coupled nonlinear second order differential equations are the following

$$\begin{aligned} &u_1'' + \omega_1^2 u_1 + \epsilon\beta u_1' - \frac{1}{2}\epsilon\beta u_2' + \epsilon k_1 u_1^2 + \epsilon k_2 u_1^3 + \frac{1}{2}\epsilon h(u_2 - 2u_1)\cos(\Omega_o t) \\ &- \frac{1}{2}\epsilon\mu((u_2 - u_1)^2 u_2' - u_1'(u_2^2 - 2u_1u_2 + 2u_1^2) - u_1^2 \acute{u}_1) + \frac{1}{2}\epsilon\Delta^2 u_2 \\ &= f_1\cos(\Omega_1 t), \end{aligned} \tag{2.1}$$

$$\begin{aligned} &u_2'' + \omega_2^2 u_2 + \epsilon\beta u_2' - \frac{1}{2}\epsilon\beta u_1' + \epsilon k_3 u_2^2 + \epsilon k_4 u_2^3 + \frac{1}{2}\epsilon h(u_1 - 2u_2)\cos(\Omega_o t) \\ &- \frac{1}{2}\epsilon\mu(-(2u_2^2 - 2u_1u_2 + u_1^2)u_2' + (u_2 - u_1)^2 u_1') + \frac{1}{2}\epsilon\Delta^2 u_1 \\ &= f_1\cos(\Omega_1 t), \end{aligned} \tag{2.2}$$

where u_1 and u_2 are the vertical displacements of the micro-cantilevers relative to the origin of the fixed plate, ω_1, ω_2 are the uncoupled natural frequencies and $\beta, \Delta, h, \mu, k_i$ where $i = 1, 2, 3, 4$ are constant.

It is noted that in [1] the excitation are the linear parametric excitation, but in Eqs.(2.1) and (2.2) their are linear parametric and external excitations. Also in [1] they studied only a single degree of freedom for harmonic solution by using secular perturbation theory. In our work here, we studied combination solution of additive type, in the case $f_1 = f_2 = O(1)$ and $\Omega_1 = \Omega_2$.

Using the method of multiple scales, we get a first order uniform solutions of Eqs.(2.1) and (2.2) in the form

$$\begin{aligned} u_1(t;\epsilon) &= u_{10}(T_0, T_1) + \epsilon u_{11}(T_0, T_1) + ..., \\ u_2(t;\epsilon) &= u_{20}(T_0, T_1) + \epsilon u_{21}(T_0, T_1) + ..., \end{aligned} \tag{2.3}$$

where $T_0 = t$ is the first scale associated with changes occurring at the frequencies ω_1, ω_2, Ω_o and Ω_1, and $T_1 = \epsilon t$ is a slow scale associated with modulations in the amplitude. In terms of T_1, the time derivatives become

$$\frac{d}{dt} = D_0 + \epsilon D_1 + ... \qquad \& \qquad \frac{d^2}{dt^2} = D_0^2 + 2\epsilon D_0 D_1 + ..., \tag{2.4}$$

where $D_n = \frac{\partial}{\partial T_n}$. Substituting equations (2.3) and (2.4) into Eqs.(2.1) and (2.2) and equating coefficients of like powers of ϵ one obtains

Order ϵ^0:

$$D_0^2 u_{10} + \omega_1^2 u_{10} - f_1 \cos(\Omega_1 T_0) = 0 \tag{2.5}$$

$$D_0^2 u_{20} + \omega_2^2 u_{20} - f_1 \cos(\Omega_1 T_0) = 0 \tag{2.6}$$

Order ϵ:

$$\begin{aligned} D_0^2 u_{11} + \omega_1^2 u_{11} = -[&\beta D_0 u_{10} - \frac{1}{2}\beta D_0 u_{20} + \mu u_{10}^2 D_0 u_{10} - \frac{1}{2}\mu u_{10}^2 D_0 u_{20} \\ &- \mu u_{20} u_{10} D_0 u_{10} + \mu u_{20} u_{10} D_0 u_{20} + \frac{1}{2}\mu u_{20}^2 D_0 u_{10} \\ &- \frac{1}{2}\mu u_{20}^2 D_0 u_{20} + 2D_0 D_1 u_{10} - h u_{10} \cos(\Omega_0 T_0) \\ &+ \frac{1}{2} h u_{20} \cos(\Omega_0 T_0) + k_2 u_{10}^3 + k_1 u_{10}^2 \\ &+ \frac{1}{2}\Delta^2 u_{20}] \end{aligned} \tag{2.7}$$

$$\begin{aligned} D_0^2 u_{21} + \omega_2^2 u_{21} = -[&-\frac{1}{2}\beta D_0 u_{10} + \beta D_0 u_{20} - \frac{1}{2}\mu u_{20}^2 D_0 u_{10} + \mu u_{20}^2 D_0 u_{20} \\ &+ \mu u_{10} u_{20} D_0 u_{10} - \mu u_{10} u_{20} D_0 u_{20} \\ &+ \frac{1}{2}\mu u_{10}^2 D_0 u_{20} + 2D_0 D_1 u_{20} - h u_{20} \cos(\Omega_0 T_0) \\ &+ \frac{1}{2} h u_{10} \cos(\Omega_0 T_0) + k_4 u_{20}^3 + k_3 u_{20}^2 \\ &- \frac{1}{2}\mu u_{10}^2 D_0 u_{10} + \frac{1}{2}\Delta^2 u_{10}] \end{aligned} \tag{2.8}$$

The solution of Eqs.(2.5) and (2.6) can be expressed in the complex form

$$u_{10} = A e^{i\omega_1 T_0} + \bar{A} e^{-i\omega_1 T_0} + \lambda(e^{i\Omega_1 T_0} + e^{-i\Omega_1 T_0}), \tag{2.9}$$

$$u_{20} = B e^{i\omega_2 T_0} + \bar{B} e^{-i\omega_2 T_0} + \lambda(e^{i\Omega_1 T_0} + e^{-i\Omega_1 T_0}), \tag{2.10}$$

where $\lambda = -\frac{2f_1}{\omega_1 - \Omega_1}$, $\bar{A}$ and $\bar{B}$ are the complex conjugate of A and B respectively. Then

Eqs.(2.7 and 2.8) become,

$$\begin{aligned}
D_0^2 u_{11} + \omega_1^2 u_{11} = & -[e^{i\omega_1 T_0}(i\mu\omega_1 AB\bar{B} + 3k_2 A^2\bar{A} + i\mu\omega_1 A^2\bar{A} \\
& + 2i\omega_1 A' + 6k_2\lambda^2 A + i\beta\omega_1 A + i\lambda^2\mu\omega_1 A) \\
& + e^{i\omega_2 T_0}(-i\mu\omega_2 AB\bar{A} - \frac{1}{2}i\mu\omega_2 B^2\bar{B} - \frac{1}{2}i\beta\omega_2 B + \frac{1}{2}\Delta^2 B) \\
& + e^{i(2\omega_2-\omega_1)T_0}(i\mu\omega_2 B^2\bar{A} - \frac{1}{2}i\mu\omega_1 B^2\bar{A}) \\
& + e^{i(\omega_2-2\omega_1)T_0}(i\mu\omega_1 B\bar{A}^2 - \frac{1}{2}i\mu\omega_2 B\bar{A}^2) \\
& - \frac{1}{2}he^{i(\Omega_0-\omega_1)T_0}\bar{A} + e^{i\Omega_1 T_0}(6k_2\lambda A\bar{A} + i\lambda\mu\Omega_1 A\bar{A} + 3k_2\lambda^3 \\
& + \frac{1}{2}i\beta\lambda\Omega_1 + \frac{\Delta^2\lambda}{2} + \frac{1}{2}i\lambda^3\mu\Omega_1) + 2k_1\lambda\bar{A}e^{i(\Omega_1-\omega_1)T_0} \\
& + 2k_1 A\bar{A} + \frac{1}{4}he^{i(\Omega_0-\omega_2)T_0}\bar{B} - \frac{1}{4}h\lambda e^{i(\Omega_0+\Omega_1)T_0} \\
& + e^{3i\Omega_1 T_0}(k_2\lambda^3 + \frac{1}{2}i\lambda^3\mu\Omega_1) + k_1\lambda^2 e^{2i\Omega_1 T_0} + 2k_1\lambda^2] \\
& + NST. + c.c,
\end{aligned} \tag{2.11}$$

$$\begin{aligned}
D_0^2 u_{21} + \omega_2^2 u_{21} = & -[e^{i\omega_2 T_0}(i\mu\omega_2 AB\bar{A} + 3k_4 B^2\bar{B} + i\mu\omega_2 B^2\bar{B} + 2i\omega_2 B' \\
& + 6k_4\lambda^2 B + i\beta\omega_2 B + i\lambda^2\mu\omega_2 B) \\
& + e^{i\Omega_1 T_0}(i\lambda\mu\Omega_1 A\bar{A} - \lambda\mu A\bar{A} + 6k_4\lambda B\bar{B} \\
& + 2i\lambda\mu\Omega_1 B\bar{B} - \lambda\mu B\bar{B} + 3k_4\lambda^3 + i\beta\lambda\Omega_1 - \frac{\beta\lambda}{2} + \frac{\Delta^2\lambda}{2} + \frac{1}{2}i\lambda^3\mu\Omega_1) \\
& + e^{i(\Omega_1-\omega_2)T_0}(2k_3\lambda\bar{B} \\
& + e^{i\omega_1 T_0}(\lambda\mu A\bar{B} - i\lambda\mu\Omega_1 A\bar{B})) \\
& + e^{i\omega_1 T_0}(-i\mu\omega_1 AB\bar{B} - \frac{1}{2}i\mu\omega_1 A^2\bar{A} - \frac{1}{2}i\beta\omega_1 A + \frac{1}{2}\Delta^2 A) \\
& + e^{i(\omega_1-2\omega_2)T_0}(i\mu\omega_2 A\bar{B}^2 - \frac{1}{2}i\mu\omega_1 A\bar{B}^2) \\
& + \frac{1}{4}he^{i(\Omega_0-\omega_1)T_0}\bar{A} \\
& - \frac{1}{2}he^{i(\Omega_0-\omega_2)T_0}\bar{B} + 2k_3 B\bar{B} \\
& - \frac{1}{4}h\lambda e^{i(\Omega_0+\Omega_1)T_0} \\
& + e^{3i\Omega_1 T_0}(k_4\lambda^3 + \frac{1}{2}i\lambda^3\mu\Omega_1) + k_3\lambda^2 e^{2i\Omega_1 T_0} + 2k_3\lambda^2] \\
& + NST. + c.c.
\end{aligned} \tag{2.12}$$

From Eqs.(2.11) and (2.12) there exist combination solution of additive type.

3 Combination Solution of Additive Type $\Omega_0 + \Omega_1 \approx \omega_1$

In this case, we have

$$\Omega_0 + \Omega_1 \approx \omega_1 \qquad \text{i.e. } \Omega_0 + \Omega_1 = \omega_1 + \epsilon\sigma_1,$$

where σ_1 is detuning parameter, by eliminating the secular terms from Eqs.(2.11) and (2.12) yields

$$\begin{aligned} &- i\mu\omega_1 AB\bar{B} - 3k_2 A^2\bar{A} - i\mu\omega_1 A^2\bar{A} - 2i\omega_1 A' \\ &- 6k_2\lambda^2 A - i\beta\omega_1 A - i\lambda^2\mu\omega_1 A + \frac{1}{4}h\lambda e^{i\sigma_1 T_1} = 0 \end{aligned} \tag{3.1}$$

$$\begin{aligned} &- i\mu\omega_2 AB\bar{A} - 3k_4 B^2\bar{B} - i\mu\omega_2 B^2\bar{B} - 2i\omega_2 B' \\ &- 6k_4\lambda^2 B - i\beta\omega_2 B - i\lambda^2\mu\omega_2 B = 0 \end{aligned} \tag{3.2}$$

where the prime indicates the derivative with respect to T_1. Writing A and B in the polar form as $A = \frac{1}{2}a_1(T_1)e^{i\delta(T_1)}$ and $B = \frac{1}{2}a_2(T_1)e^{i\gamma(T_1)}$ into Eqs.(3.1) and (3.2) where $a_1(T_1)$, $a_2(T_1)$, $\delta(T_1)$ and $\gamma(T_1)$ are real-valued functions, representing, the amplitudes and phases of the response, by separating real and imaginary parts, we obtain the following modulation equations:

$$8\omega_1 a_1' = a_1^3(-\mu)\omega_1 - a_1 a_2^2\mu\omega_1 - 4a_1\beta\omega_1 - 4a_1\lambda^2\mu\omega_1 + 2h\lambda\sin(\phi_1), \tag{3.3}$$

$$8a_1\omega_1\phi_1' = 3a_1^3 k_2 + 24a_1 k_2\lambda^2 + 8a_1\sigma_1\omega_1 - 2h\lambda\cos(\phi_1), \tag{3.4}$$

$$8\omega_2 a_2' = -a_1^2 a_2\mu\omega_2 + a_2^3(-\mu)\omega_2 - 4a_2\beta\omega_2 - 4a_2\lambda^2\mu\omega_2, \tag{3.5}$$

$$8\omega_2\gamma' = 3k_4 a_2^2 + 24k_4\lambda^2, \tag{3.6}$$

where, $\phi_1 = T_1\sigma_1 - \delta$. For steady state solution, $a_1' = a_2' = \phi_1' = \gamma' = 0$, in Eqs.(3.3), (3.4), (3.5) and (3.6) we obtain

$$a_1^3(-\mu)\omega_1 - a_1 a_2^2\mu\omega_1 - 4a_1\beta\omega_1 - 4a_1\lambda^2\mu\omega_1 + 2h\lambda\sin(\phi_1) = 0 \tag{3.7}$$

$$3a_1^3 k_2 + 24a_1 k_2\lambda^2 + 8a_1\sigma_1\omega_1 - 2h\lambda\cos(\phi_1) = 0 \tag{3.8}$$

$$a_1^2 a_2 \mu\omega_2 + a_2^3 \mu\omega_2 + 4a_2\beta\omega_2 + 4a_2\lambda^2\mu\omega_2 = 0 \tag{3.9}$$

$$a_2^2 + 8\lambda^2 = 0 \tag{3.10}$$

Squaring and adding the equations (3.7), (3.8), (3.9) and (3.10), we get the following frequency-response equations:

$$M_1 a_1^6 + M_2 a_1^4 + M_3 a_1^2 - 4h^2\lambda^2 = 0, \tag{3.11}$$

and

$$a_2^6(9k_4^2 + \mu^2\omega_2^2) + M_4 a_2^2 + M_5 a_2^4 = 0, \tag{3.12}$$

where

$M_1 = 9k_2^2 + \mu^2\omega_1^2$

$M_2 = 144k_2^2\lambda^2 + 48k_2\sigma_1\omega_1 + 2a_2^2\mu^2\omega_1^2 + 8\beta\mu\omega_1^2 + 8\lambda^2\mu^2\omega_1^2$

$M_3 = 576k_2^2\lambda^4 + 384k_2\lambda^2\sigma_1\omega_1 + a_2^4\mu^2\omega_1^2 + 8a_2^2\beta\mu\omega_1^2 + 8a_2^2\lambda^2\mu^2\omega_1^2 + 16\beta^2\omega_1^2 + 32\beta\lambda^2\mu\omega_1^2 + 16\lambda^4\mu^2\omega_1^2 + 64\sigma_1^2\omega_1^2$

$M_4 = a_1^4\mu^2\omega_2^2 + 8a_1^2\beta\mu\omega_2^2 + 8a_1^2\lambda^2\mu^2\omega_2^2 + 576k_4^2\lambda^4 + 16\beta^2\omega_2^2 + 32\beta\lambda^2\mu\omega_2^2 + 16\lambda^4\mu^2\omega_2^2$

$M_5 = 2a_1^2\mu^2\omega_2^2 + 144k_4^2\lambda^2 + 8\beta\mu\omega_2^2 + 8\lambda^2\mu^2\omega_2^2$

For stability analysis from equations (3.3), (3.4), (3.5) and (3.6) by putting:

$$\begin{aligned} a_1 &= a_{10} + a_{11}(T_1) \qquad & \phi_1 &= \phi_{10} + \phi_{11}(T_1), \\ a_2 &= a_{20} + a_{21}(T_1) \qquad & \gamma &= \gamma_{10} + \gamma_{11}(T_1), \end{aligned} \tag{3.13}$$

where a_{10}, a_{20}, ϕ_{20} and ϕ_{30} are solutions of Eqs.(3.7), (3.8), (3.9) and (3.10) and a_{11}, a_{21}, ϕ_{21} and ϕ_{31} are perturbations which are assumed to be small. Substituting Eq.(3.13) into Eqs.(3.3), (3.4), (3.5) and (3.6), linearizing the resulting equations and noticing that the steady-state values satisfy Eqs.(3.7), (3.8), (3.9) and (3.10), then we obtain

$$\begin{aligned} a'_{11} = & \frac{(3a_{10}^3 k_2 + 24a_{10}k_2\lambda^2 + 8a_{10}\sigma_1\omega_1)}{8\omega_1}\phi_{11} \\ & + \frac{(-3a_{10}^2\mu\omega_1 - a_{20}^2\mu\omega_1 - 4\beta\omega_1 - 4\lambda^2\mu\omega_1)}{8\omega_1}a_{11} \\ & + \frac{(-2a_{10}a_{20}\mu\omega_1)}{8\omega_1}a_{21} \end{aligned} \tag{3.14}$$

$$a'_{21} = -\frac{(a_{10}a_{20}\mu)}{4}a_{11} + \frac{(a_{10}^2(-\mu) - 3a_{20}^2\mu - 4\beta - 4\lambda^2\mu)}{8}a_{21} \tag{3.15}$$

$$\phi'_{11} = \frac{(9a_{10}^2k_2 + 24k_2\lambda^2 + 8\sigma_1\omega_1)}{8a_{10}\omega_1}a_{11} + \frac{(a_{10}^3\mu\omega_1 + a_{10}a_{20}^2\mu\omega_1 + 4a_{10}\beta\omega_1 4a_{10}\lambda^2\mu\omega_1)}{8a_{10}\omega_1}\phi_{11} \tag{3.16}$$

$$\gamma'_{11} = \frac{3k_4(3a_{20}^2 + 8\lambda^2)}{8a_{20}\omega_2}a_{21} \tag{3.17}$$

The previous system of Eqs.(3.14)-(3.17) can be written in the simple form

$$[\dot{H}] = [A][H] \qquad ; \qquad [H]^T = [a_{11}, a_{21}, \phi_{11}, \gamma_{11}] \tag{3.18}$$

$[A]$ is a 4x4 matrix and the elements of which are functions of $a_{11}, a_{21}, \phi_{11}$ and γ_{11}. Stability of the steady state solution is now decided by the nature of the eigenvalues of the matrix $[A]$. Equations (3.14), (3.15), (3.16) and (3.17) admit of solutions on the form $(a_{11}, a_{21}, \phi_{11}, \gamma_{11}) = (c_1, c_2, c_3, c_4)e^{mT_1}$. Then the eigenvalues are given by the following equation

$$m^4 + R_1m^3 + R_2m^2 + R_3m + R_4 = 0, \tag{3.19}$$

where $R_1 = 4096a_1a_2\omega_1^2\omega_2$,
$R_2 = 512a_1a_2\omega_1^2\omega_2(3a_1^2\mu + 3a_2^2\mu + 4(\beta + \lambda^2\mu))$,
$R_3 = -64a_1a_2\omega_2(96k_2\sigma_1\omega_1(a_1^2 + 4\lambda^2) + 9k_2^2(3a_1^4$
$+ 32a_1^2\lambda^2 + 64\lambda^4) + \omega_1^2(a^4\mu^2 + 2a_1^2\mu(a_2^2\mu + 4(\beta + \lambda^2\mu))$
$+ a_2^4\mu^2 + 8a_2^2\mu(\beta + \lambda^2\mu) + 16((\beta + \lambda^2\mu)^2 + 4\sigma_1^2)))$,
$R_4 = -8a_1a_2\omega_2(96k_2\sigma_1\omega_1(a_1^2$
$+ 4\lambda^2)(a_1^2\mu + 3a_2^2\mu + 4(\beta + \lambda^2\mu)) + 9k_2^2(3a_1^4$
$+ 32a_1^2\lambda^2 + 64\lambda^4)(a_1^2\mu + 3a_2^2\mu + 4(\beta + \lambda^2\mu))$
$+ \omega_1^2(3a_1^6\mu^3 + a_1^4\mu^2(9a_2^2\mu + 28(\beta + \lambda^2\mu))$
$+ a_1^2\mu(9a_2^4\mu^2 + 56a_2^2\mu(\beta + \lambda^2\mu) + 16(5(\beta + \lambda^2\mu)^2$
$+ 4\sigma_1^2)) + (3a_2^2\mu + 4(\beta + \lambda^2\mu))(a_2^4\mu^2$
$+ 8a_2^2\mu(\beta + \lambda^2\mu) + 16((\beta + \lambda^2\mu)^2 + 4\sigma_1^2))))$.

According to the Routh-Hurwitz criterion [11], the fixed points are stable if and only if

$$\begin{aligned} &R_1 > 0, \qquad R_4 > 0, \qquad R_1R_2 - R_3 > 0 \\ &and \qquad R_3(R_1R_2 - R_3) - R_1^2R_4 > 0. \end{aligned} \tag{3.20}$$

4 Numerical results and discussion

By solving numerically the frequency response equations (3.11), (3.12) and stability condition (3.20). The numerical results are plotted in groups of figures, which represent the variation of the amplitudes (a_1 and a_2) with the detuning parameter (σ_1) for given values of the other parameters. In all figures, the solid (hollow) symbols represent stable (unstable) solutions.

Figure(1)-(16) represent the frequency response curves for combination solution additive type. In Fig.(1), the magnitude of the parameter are ($h = 7, \mu = .08, \beta = -.1, k_2 = 2, k_4 = .07, \omega_1 = .4, \omega_2 = .03, \lambda = -.05$), we have two systematic branches about $\sigma_1 = 0$ stable (unstable) solutions. In Fig.(2) for a_2 we have also, two curves with multivalued solutions, where the values of σ_1 are unstable region.

- By increasing (decreasing) (μ and (ω_2), the magnitude of (a_1) have no change (i.e. we have a saturation phenomenon) Fig.(11) and Fig.(17).

- By increasing (decreasing) (h), we observe that for a_1, the range of the definition and the zone of multivalued are an increased (decreased) and the stable solutions are decreasing (increasing) where $\sigma_1 = -3.8$ and $\sigma_1 = -6.5$ Fig.(3), for (a_2) the curves are increased (decreased) for the zones of the definition $\sigma_1 = -3.9$ and $\sigma_1 = -6.5$ Fig.(4).

- By decreasing (β), we note that the magnitude of (a_1) are decreased and the region of stable are vanish Fig.(5). But for (a_2) the magnitude are increased and curves turn out to be a straight line and the region of stable are decreasing at the decreasing for (a_2) Fig.(6).

- By increasing (decreasing) (k_2), for (a_1 the semi-oval expand and move to right (left) which is given by an increase (decrees) in the zone of multivaled and the region of the stable solutions vanish at decreasing k_2 Fig.(7). We observe that for (a_2), the curves are increased (decreased) of the range of definitions and the curves move to left (right) Fig.(8).

- By decreasing (increasing) (λ), for $(a_1$ the zone of multivaled and the region of the stable solutions are increased (decreased) and the part of stable solution are increasing (decreasing) Fig.(9). We observe that for (a_2), the curves are increased (decreased) of the region of definitions and the curves bent to upward (downward) by decreasing (increasing) for $(\lambda$ and $k_4)$ Fig.(10) and Fig.(12).

- By increasing (decreasing) (μ), we note that for (a_1) are decreased (increased) of the region of definition Fig.(13). But for (a_2) the curves turn out to be two curves right and left and the curves convergent at the zero Fig.(14).

- By increasing (decreasing) (ω_1), for $(a_1$ the semi-oval expand and move to right (left) which is given by an increase (decrees) in the zone of multivaled Fig.(15). We observe that for (a_2), the curves are increased (decreased) of the region of definitions and the curves move to left (right) Fig.(16).

- By increasing (decreasing) (ω_2), we note that the magnitude for (a_2) are increased (decreased) and the region of definitions are increase (decrease) and the curves bent to upward (downward) Fig.(18).

5 Summery and Conclusion

This paper is devoted to study analytically the combination solutions od additive type to a two coupled weakly nonlinear second order differential equations which represent the dynamical behavior of MEMS, in the case $f_1 = f_2 = f$ and $\Omega_1 = \Omega_2$.
From the figures we note that:

- The region of stability affect for decreasing the parameters ω_1, k_2 for a_1 and for increasing (decreasing) the parameters k_4 and ω_2 have no change i.e. we have a saturation phenomenon for a_1.

- The range of definitions bent to upward (downward) by decreasing (increasing) for a_2 for the parameters λ and k_4 at the inverse the parameter ω_2.
- For increasing, μ we observe that for a_2 the branches bent to the right branch and the left branch and convergent to zero for the parameter μ.

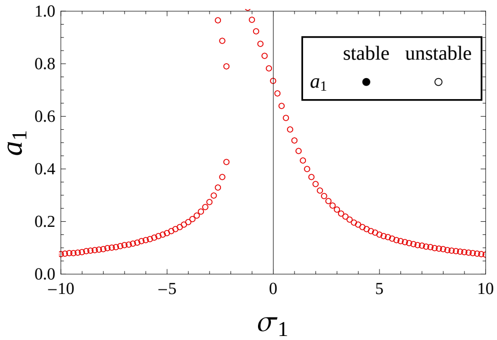

Figure 1: ***Variation of the amplitude of the steady-state solution*** a_1

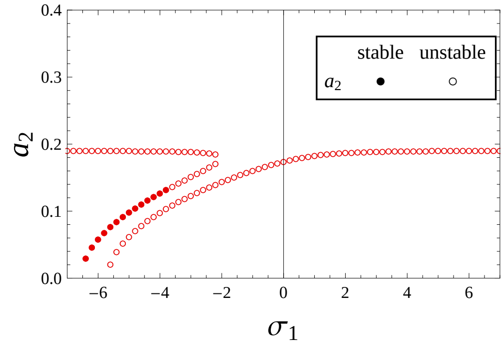

Figure 2: ***Variation of the amplitude of the steady-state solution*** a_2

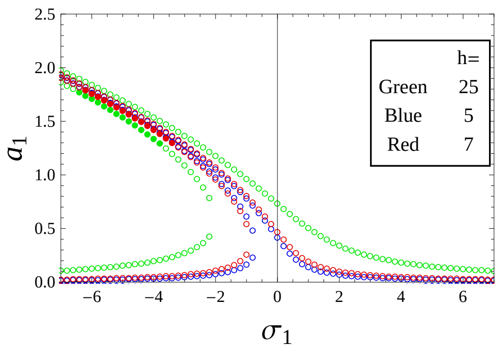

Figure 3: ***Variation of the parameter*** h

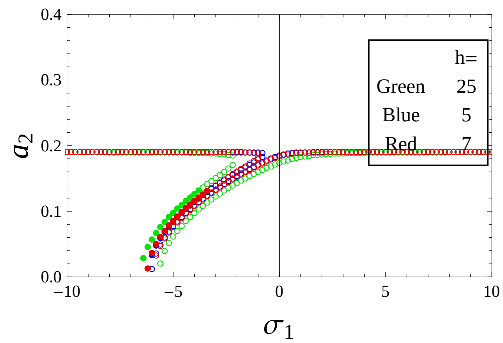

Figure 4: ***Variation of the parameter*** h

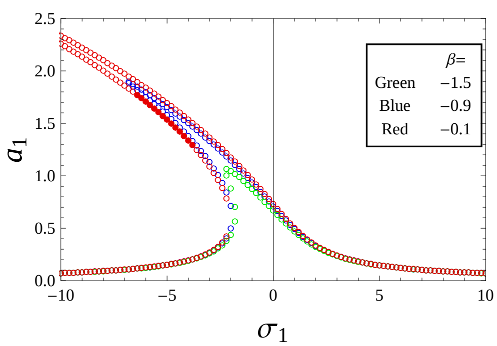

Figure 5: ***Variation of the parameter*** β

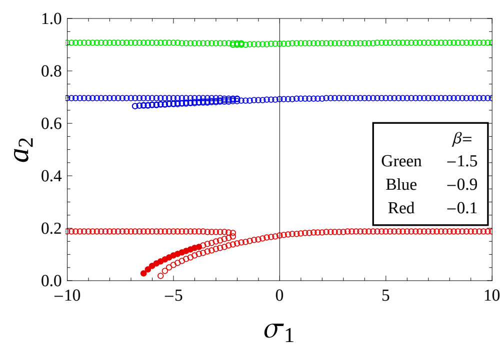

Figure 6: ***Variation of the parameter*** β

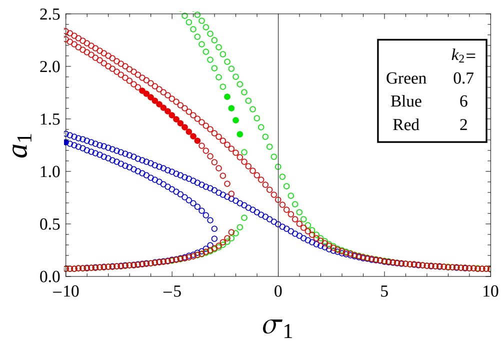

Figure 7: ***Variation of the parameter*** k_2

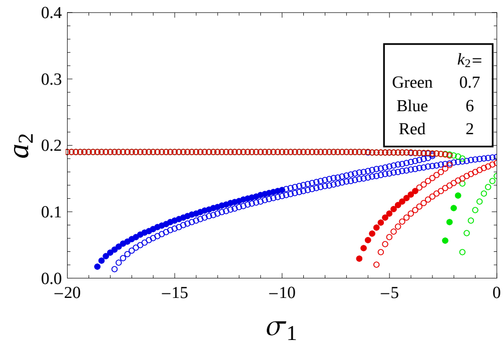

Figure 8: ***Variation of the parameter*** k_2

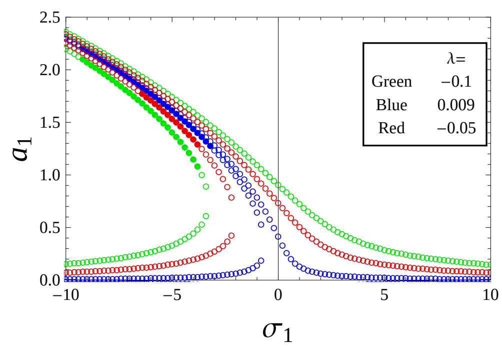

Figure 9: ***Variation of the parameter*** λ

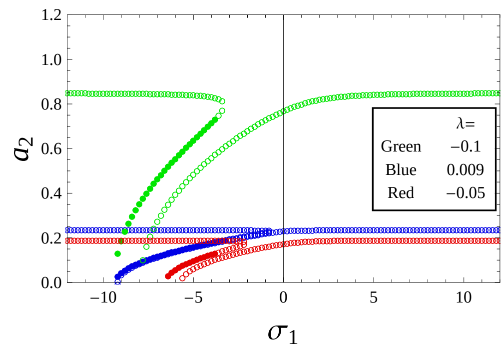

Figure 10: ***Variation of the parameter*** λ

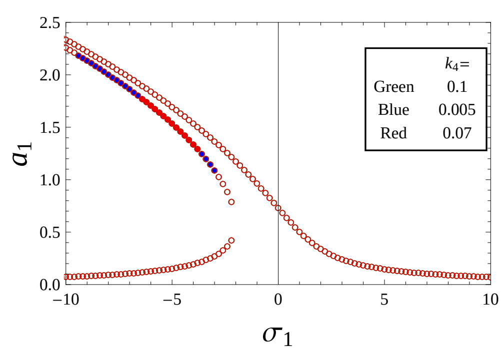

Figure 11: ***Variation of the parameter*** k_4

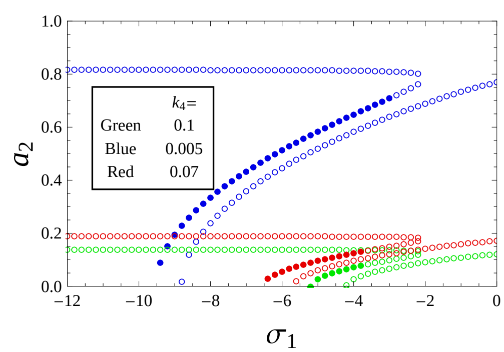

Figure 12: ***Variation of the parameter*** k_4

References

[1] M. C. Cross and Ron Lifshitz , Response of parametrically driven nonlinear coupled oscillators with application to micromechanical and nanomechanical resonator arrays, Physical Review B 67: 134302, (2003).

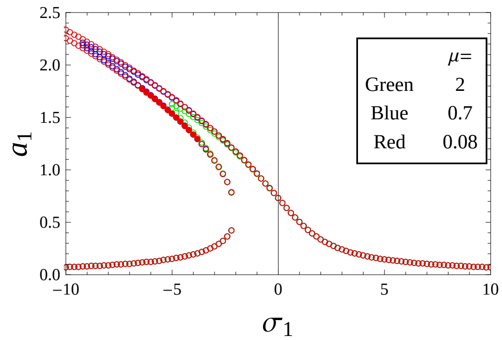

Figure 13: ***Variation of the parameter*** μ

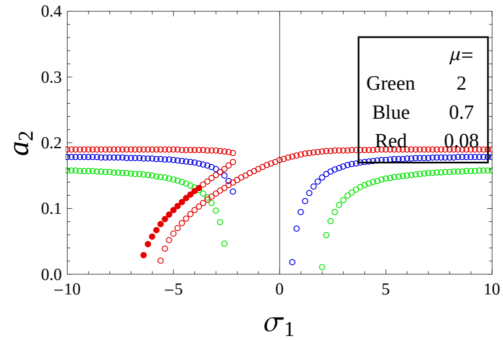

Figure 14: ***Variation of the parameter*** μ

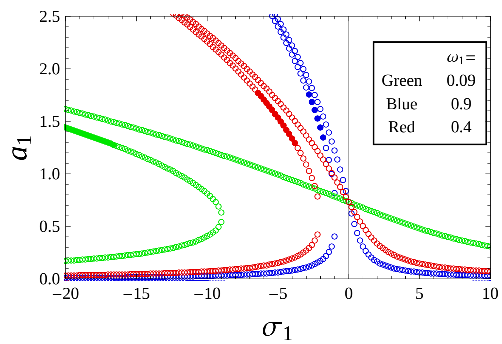

Figure 15: ***Variation of the parameter*** ω_1

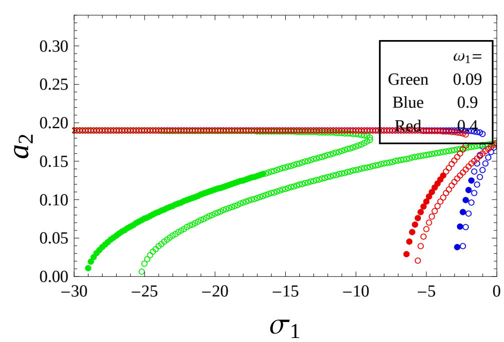

Figure 16: ***Variation of the parameter*** ω_1

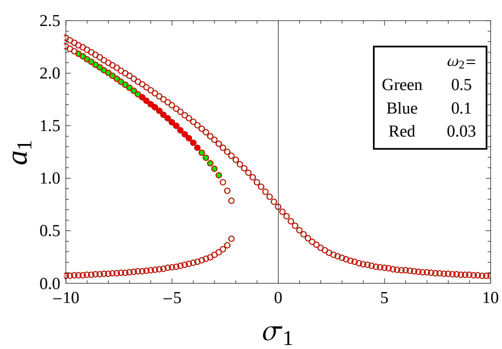

Figure 17: ***Variation of the parameter*** ω_2

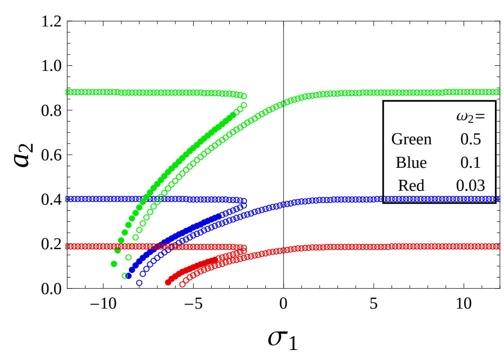

Figure 18: ***Variation of the parameter*** ω_2

[2] F. M. Alsaleem, M. I. Younis and H. M. Ouakad, On the nonlinear resonances and dynamic pull-in of electrostatically acuated resonators, J. Micromeshanics and Micro-Engineering 19(4): 1-14, (2009).

[3] A. F. El-Bassiouny and M. Eissa, Resonance of nonlinear systems subjected to multi-parametrically excited structures: Comparison between two methods, response and

stability, Physica Scripta 70(2-3): 101, (2004).

[4] B. E. DeMartini, Development of Nonlinear and Coupled Microelectromechanical Oscillators for Sensing Applications Ph.D. dissertation, University of California 2008.

[5] E. Buks and M. L. Roukes, Electrically tunable collective response in a coupled micromechanical Array, Journal of Microelectromech Syst. 11(6): 802-807, (2002).

[6] E. M. Abd El-Rahman and A. H. Nayfeh, Secondary resonances of electrically actuated resonant microsensors, Journal of Micromech and Microeng 13(3): 491-901, (2003).

[7] N. Kacem, S. Baguet, S. Hentz and R. Dufour, Computational and quasi-analytical models for nonlinear vibrations of resonant MEMS and NEMS sensors, International Journal of Non-Linear Mechanics 46(3): 532-542, (2011).

[8] N. Kacem, S. Hentz, H. Fontaine, V. Nguyen, P. Robert, B. Legrand and L. Buchaillot, From MEMS to NEMS: Modelling and characterization of the non linear dynamics of resonators, a way to enhance the dynamic range, NST Nanotech 3(4): 619-622, (2008).

[9] R. Lifshitz and M. C. Cross, Response of parametrically driven nonlinear coupled oscillators with application to micromechanical and nanomechanical resonator arrays, Physical Review B 67(13): 134302, (2003).

[10] J. Zhu, C. Q. Ru and A. Mioduchowski, High order subharmonic parametric resonance of nonlinearly coupled micromechanical oscillators, European Physica B 58(4): 411-421, (2007). bibitembbb H. Askari, Z. Saadatnia, E. Esmailzadeh, and D. Younesian, Multi-frequency excitation of stiffened triangular plates for large amplitude oscillations, J. Sound and Vib. 333(22): 5817-5835, (2014).

[11] M. R. M. Rao, Ordinary Differential Equations, Theorey and Applications, Edward Arnold, London 1981.