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# Thermal description of hadrons thermodynamics using Tsallis distribution

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**Abstract**

Thermodynamic quantities such as normalized pressure $P/T^4$, normalized energy density $\rho/T^4$ and trace anomaly $(\rho - 3P)/T^4$ are calculated for hadrons in the framework of Tsallis distribution at different values of the baryon chemical potential, $\mu_b = 0, 170, 340, 425$ MeV through temperatures spanning from 120 MeV to 200 MeV. The used values of Tsallis parameter $q$ are 1.15, 1.12, 1.1 and 1.002. The obtained results are compared with the corresponding Lattice Quantum Chromodynamics (LQCD) data. At freezeout temperature of approximately $\simeq .155$ GeV and vanishing chemical potential, the calculated results are shown a good fit with the used lattice results where $q = 1.15$. At non-vanishing chemical potential, the fitting between Tsallis distribution and the lattice results becomes good at $(\mu_b, q) = (.170, 1.12), (.340, 1.1)$, and $(.340, 1.002)$, where the values of $\mu_b$ is represented in GeV. The disagreement between the used model and the corresponding LQCD data alerts for more investigations with other models.

## 1 Introduction

Heavy-ion collision experiments at the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC) are frequently producing the Quark-Gluon Plasma (QGP), the phase of matter that dominated the universe just a few microseconds after the Big Bang [1]. As proven by LQCD simulations [2], the phase transition from hadrons to gluons and quarks is appeared to be crossover especially at lower baryon chemical potential region. The evolution of a high energy heavy-ion collisions after hadronization is defined by two important aspects: the first aspect (chemical freeze-out) represents a vanish of all inelastic collisions between hadrons, and the second aspect (the thermal freeze-out) represents a stop of elastic collisions as well. At equilibrium, two crucial intense state parameters in quantum chromodynamics (QCD), the gauge field approach that explains the strong interactions between both coloured and colourless quarks and gluons including their bound states, are the temperature $T$ and the baryon chemical potential $\mu_b$. The bulk thermodynamic quantities' temperature and density (chemical potential) dependence, generally referred to as the equation of state (EoS), gives the most fundamental characterization of the strongly interacting matter's equilibrium properties. LQCD theory has succeeded very well to describe the EoS through the calculations that performed in $SU(N)$ group theories [3].

At vanishing $\mu_b$, the EoS gives the inputs that helps in visualizing and modelling the evolution of the QCD matter produced in the high energy heavy-ion collisions [4, 5]. In Ref. [6], some of the thermodynamic quantities such as entropy density, pressure density and energy density are calculated at vanishing $\mu_b$ using the hadron resonance gas (HRG) model. In Ref. citebazavov2012chiral, the thermodynamic quantities are noticed to behave smoothly in the region of the phase transition in the frame work of analysing the temperature of the chiral transition $T_c \simeq 154 \pm 9$ MeV. At finite $\mu_b$, LQCD has faced a lot of difficulties arising from the sign problem appears in the formulations and the Eos is not calculated. Many attempts have been made to solve the EoS at finite $\mu_b$ such as the methods based on Taylor expansion in addition to other techniques [7, 8, 9, 10, 11, 12]. These efforts have succeeded to make calculations in a range of $0 \leq \mu_b/T \lesssim 3$ which is covered by the RHIC experiment at energy spanning from $7.7 \leq \sqrt{s_{NN}} \leq 200$ GeV.

In Refs. [13, 14, 15], HRG model is used in LQCD calculations where there are a good agreement especially at lower temperatures region. In HRG model, particles are treated as point-like particles. Previous studies for the themodynamic quantities have been shown in [5, 16, 17, 18]. In this work, the thermodynamic quantities as normalized pressure $P/T^4$, normalized energy density $\rho/T^4$ and trace anomaly $(\rho - 3P)/T^4$ are calculated for hadrons in the framework of Tsallis distribution at different values of the baryon chemical potential, $\mu_b = 0, 170, 340, 425$ MeV through temperatures spanning from 120 MeV to 200 MeV. The used values of Tsallis parameter $q$ are 1.15, 1.12, 1.1 and 1.002. The obtained results are compared with the corresponding Lattice Quantum Chromodynamics (LQCD) data [19, 20]. The Tsallis distribution is based on power law functions distributions which are important in various branches of physics [21]. It includes the Tsallis $q$ parameter in an exponential form that leads to a good statistical fit.

The present paper is shown as follows: In Section 2, the formalism of the Tsallisi distribution is presented. The obtained results are shown in Section 3. Section 4 is devoted to the summary of the current work.

## 2 Approach Description

One of the main intersting challenge for LQCD theory is estimating all thermodynamic qunatities at non-vanishing $\mu_b$. Several attempts have been made based on Taylor expansion series. In this work, Tsallis distribution [21] is used to calculate the the thermodynamics quantities such as as normalized pressure $P/T^4$, normalized energy density $\rho/T^4$ and trace anomaly $(\rho - 3P)/T^4$.

For a system of high multiplicity of particles, the thermodynamic quantities can be obtained by integrals through a composite function of Tsallis function distribution $f$, modulus of the particle's momentum $p \equiv |p|$ and the energy of the system $E_p \equiv \sqrt{p^2 + m^2}$ [21]. At any Tsallis parameter $q$, the distribution of the particles is given by [21]

$$f \equiv \left[1 + (q-1)\,\frac{E_p(m) - \mu}{T}\right]^{-\frac{1}{q-1}}, \qquad (1)$$

that, represents apart from $q$ and $T$ includes $\mu$ as a used parameter as well. In the limit of $q \to 1$, Eq. (1) reduces to the well-known Boltzmann exponential distribution [21].

The total entropy density $S$, total particle multiplicity number $N$, total energy density $E$, and the pressure density $P$ for the whole system as a function of the Tsallis parameter $q$ are defined as follows [21]

$$\begin{aligned} S &= sV \equiv -gV \int \frac{d^3\boldsymbol{p}}{(2\pi)^3}\left[f^q \ln_q f - f\right], \\ N &= nV \equiv gV \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} f^q, \\ E &= \epsilon V \equiv gV \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} E_p f^q, \\ P &\equiv g \int \frac{d^3\boldsymbol{p}}{(2\pi)^3}\frac{p^2}{3E_p} f^q. \end{aligned} \qquad (2)$$

where $V$ is the volume of the system and $g$ represents the degeneracy. From the above equation, the suggested thermodynamic quantities, i.e., normalized pressure $P/T^4$, normalized energy density $\rho/T^4$ and trace anomaly $(\rho - 3P)/T^4$, using the Tsallis distribution model are obtained by doing a numerical integration over the momentum using the frame work of ROOT data analysis.

# 3 Results and Discussions

In this research, Tsallis distribution model which is based on power exponential factor of the Tsallis parameter $q$ is used to calculate the thermodynamic quantities of hadrons such as normalized pressure $P/T^4$, normalized energy density $\rho/T^4$ and trace anomaly $(\rho - 3P)/T^4$ through temperatures spanning from 120 MeV to 200 MeV. The obtained results are calculated at four different values of $q$ = 1.15, 1.12, 1.1 and 1.002 in case of vanishing and non-vanishing $\mu_b$. The calculated results according to Eqs. (2) are compared to the available LQCD data [22, 23].

Fig. (1) of this work shows the calculated normalized pressure $P/T^4$, normalized energy density $\rho/T^4$, and trace anomaly $(\rho - 3P)/T^4$ (dashed curves) which are normalized by the temperature obtained by using Tsallis distribution based on Eqs. (2). The obtained results are confronted to the corresponding LQCD data shown in ref. [22] (symbols plus error bars) at zero baryon chemical potential ($\mu_b$=0 MeV) for the suggested four values of the Tsallis parameter $q$ = 1.15, 1.12, 1.1 and 1.002. The values of $\chi^2$/dof statistic for the normalized pressure density $P/T^4$, trace anomaly $(\rho - 3P)/T^4$ and normalized energy density $\rho/T^4$ estimated from the used model (Tsallis distribution) in comparison with the corresponding LQCD data shown in [22] for four different values of the baryon chemical potential $\mu_b$ = 0, 170, 340, and 425 MeV.

At zero chemical potential, the calculated results of the suggested thermodynamic quantities have a good agreement with the corresponding LQCD data. In case of trace anomaly, there is a slight deviation between Tsallis distribution and the corresponding LQCD data. It is clear from the results that as the value of $q$ is increased, a discrepancy between the obtained results and the LQCD results appeared.

Fig. 2 shows the calculated normalized pressure $P/T^4$, normalized energy density $\rho/T^4$, and trace anomaly $(\rho - 3P)/T^4$ (dashed curves) which are normalized by the temperature obtained by using Tsallis distribution based on Eqs. (2). The obtained results are confronted to the corresponding LQCD data shown in ref. [22] (symbols plus error bars) at zero baryon chemical potential ($\mu_b$=170 MeV) for the suggested four values of the Tsallis parameter $q$ = 1.15, 1.12, 1.1 and 1.002. At $q$ = 1.1, the statistical fit between the obtained data and the corresponding LQCD data becomes good.

Fig. 3 shows the calculated normalized pressure $P/T^4$, normalized energy density $\rho/T^4$, and trace anomaly $(\rho - 3P)/T^4$ (dashed curves) which are normalized by the temperature obtained by using Tsallis distribution based on Eqs. (2). The obtained results are confronted to the corresponding LQCD data shown in ref. [22] (symbols plus error bars) at zero baryon chemical potential ($\mu_b$=340 MeV) for the suggested four values of the Tsallis parameter $q$ = 1.15, 1.12, 1.1 and 1.002. At $q$ = 1.1, the statistical fit between the obtained data and the corresponding LQCD data becomes good.

Fig. 4 shows the calculated normalized pressure $P/T^4$, normalized energy density $\rho/T^4$, and trace anomaly $(\rho - 3P)/T^4$ (dashed curves) which are normalized by the temperature obtained by using Tsallis distribution based on Eqs. (2). The obtained results are confronted to the corresponding LQCD data shown in ref. [22] (symbols plus error bars) at zero baryon chemical potential ($\mu_b$=425 MeV) for the suggested four values of the Tsallis parameter $q$ = 1.15, 1.12, 1.1 and 1.002.

Tsallis distribution model is used to compute the thermodynamic quantities of hadrons such as normalized pressure $P/T^4$, normalized energy density $\rho/T^4$ and trace anomaly $(\rho - 3P)/T^4$ through temperatures spanning from 120 MeV to 200 MeV. The obtained results are calculated at four different values of $q$ =

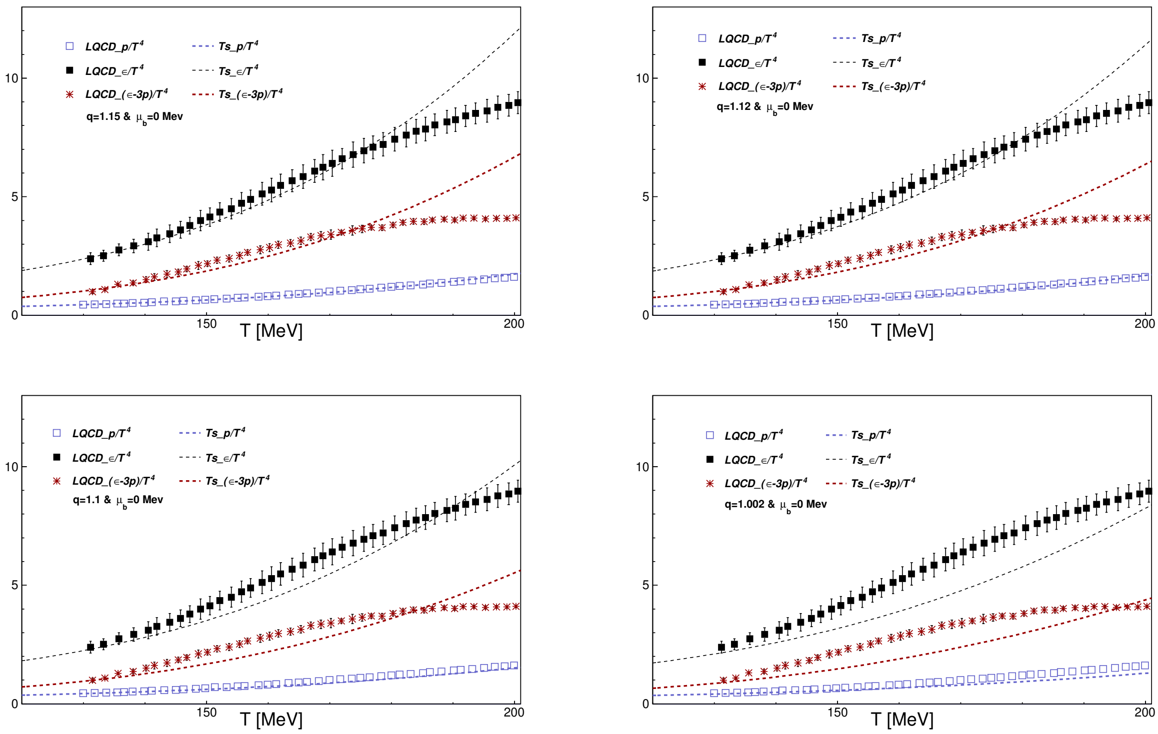


**Fig.** 1: Normalized pressure density $P/T^4$, normalized energy density $\rho/T^4$ and trace anomaly $(\rho - 3P)/T^4$ (dashed curves) computed using Tsallis distribution in comparison with the corresponding LQCD data given in ref. [22] (symbols plus error bars), at $\mu_b = 0$ MeV taking into account for four different values of the Tsallis parameter $q$.

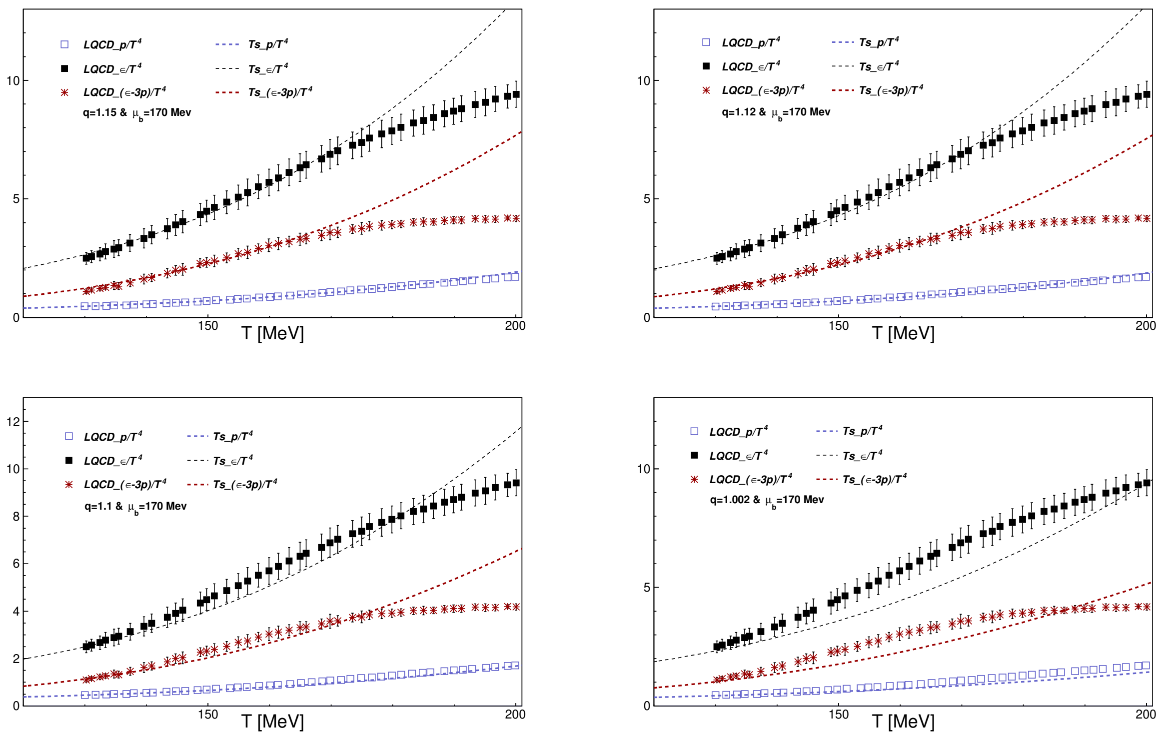


**Fig.** 2: The same shown in Fig. 1 but at $\mu_b = 170$ MeV.

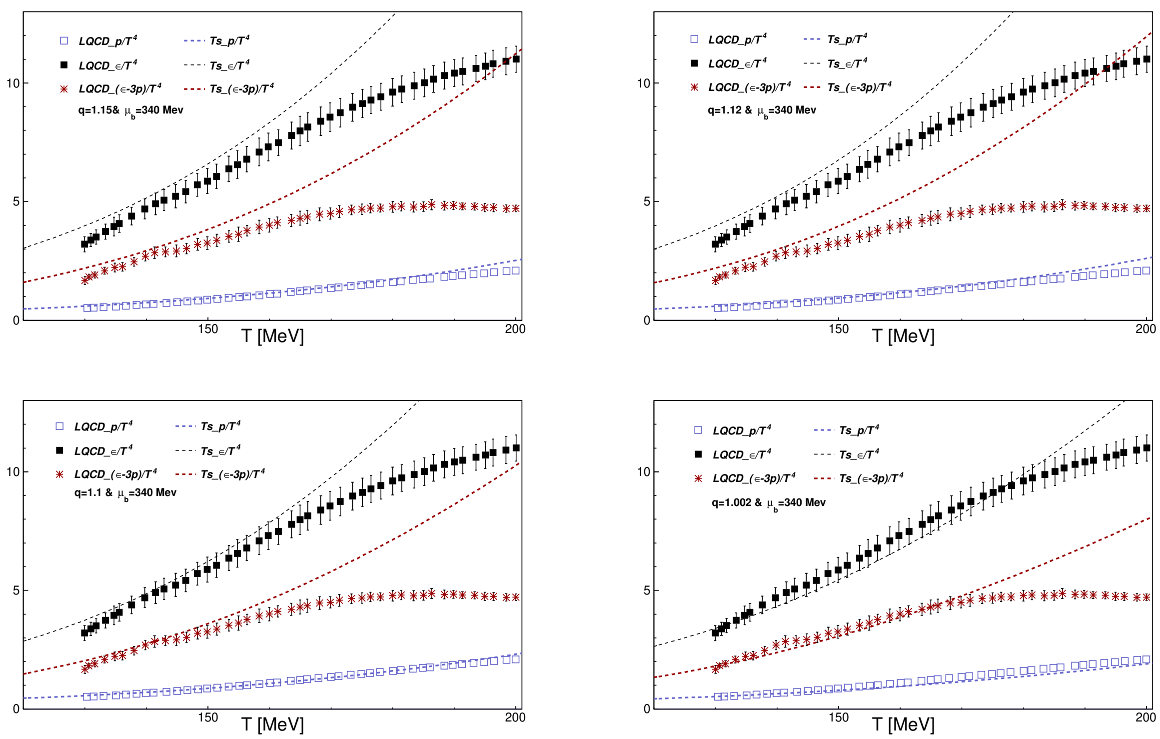


**Fig.** 3: The same shown in Fig. 1 but at $\mu_b = 340$ MeV.

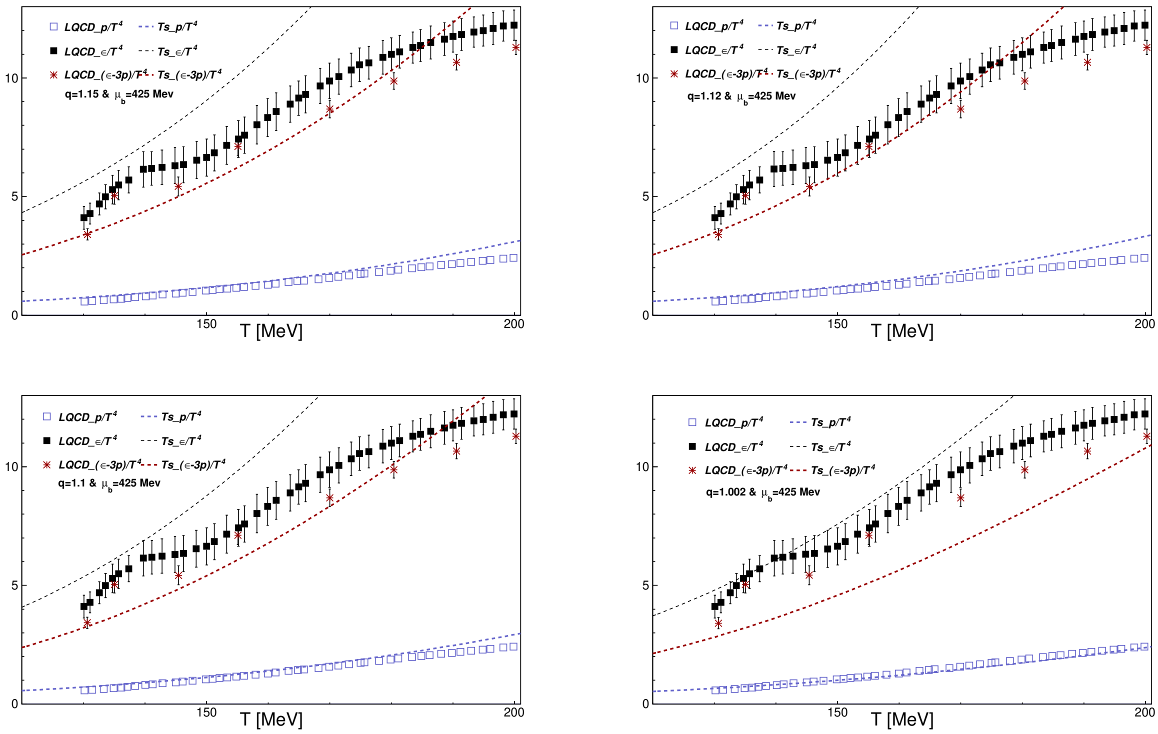


**Fig.** 4: The same shown in Fig. 1 but at $\mu_b = 425$ MeV.

1.15, 1.12, 1.1 and 1.002 in case of vanishing and non-vanishing $\mu_{\rm b}$. The calculated results according to Eqs. (2) are compared to the available LQCD data [22, 23]. The best statistical fit occurs at $q = 1.002$ which is very close to the case of Boltzmann distribution. Although not all the obtained results can describe the LQCD data well especially at large value of the Tsallis parameter $q$ but at least succeeded to explain it qualitatively. The deviation of the used model form LQCD data encourage to search for another model to have a good agreement with LQCD data.

## 4 Conclusions

Tsallis distribution model is used to compute the thermodynamic quantities of hadrons such as normalized pressure $P/T^4$, normalized energy density $\rho/T^4$ and trace anomaly $(\rho - 3P)/T^4$ through temperatures spanning from 120 MeV to 200 MeV. The obtained results are calculated at four different values of $q$ = 1.15, 1.12, 1.1 and 1.002 in case of vanishing and non-vanishing $\mu_{\rm b}$. The calculated results are compared to the available LQCD data. The best statistical fit occurs at $q = 1.002$ which is very close to the case of Boltzmann distribution. The deviation of the used model form LQCD data encourage to search for another model to have a good agreement with LQCD data. A future work using Non-extensive thermodynamic and an interacting HRG models shall be considered.

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