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VMI In Terms of Nuclear Softness

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Abstract:

Taking the concept of nuclear softness and the harmonic variable moment of inertia model **VMI**, were used to obtain a new formula ,denoted (VMINS model) which used in calculating the energies of rotational ground bands for some even-even nuclei. The predicted results of the VMINS model are in good agreements with the VMI result and the experimental data.

Keywords: Softness, moment of inertia, angular momentum, rotational band
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1. Introduction

We know that the ground state of rotational bands of deformed nuclei are described by the formula [1,2]

$$E(I) = \frac{\hbar^2 I(I+1)}{2\theta_o} \qquad (1)$$

where θ_0 is the moment of inertia and I is the spin , it takes I^+=2 ,4,6,8, . The predicted energies of Eq. (1) show large deviations from the experimental data [3]. Gupta R.K [4,5] attributed this to the effect of variation of the moment of inertia with the angular momentum I, i.e.

$$E(I) = \frac{\hbar^2}{2\theta(I)} I(I+1) \qquad (2)$$

According to Morinaga's[5]the moment of inertia can be written as

(3) $$\theta(I) = \theta_0(1 + \sigma_1 I + \sigma_2 I^2 + \sigma_3 I^3 + - - + - - -)$$

Where the softness parameter σ_n *is defined as*

$$\sigma_n = \frac{1}{n!}\frac{1}{\theta_o}\frac{\partial^n \theta(I)}{\partial I^n}|I = 0, \quad - - - - (4)$$

$$n = 1,2,3$$

Gupta et al [3,4,5] used the concept of the softness σ_n to state the expression of energy as follow

When take σ_n up to first order "n=1" , which is known NS2 model .; one obtains

$$E(I) = \frac{\hbar^2}{2\theta o(1+\sigma_1 I)} I(I+1) \qquad (5)$$

When take σ_n up to second order "n=2" , which is known NS3 model

$$.E(I) = \frac{\hbar^2}{2\theta_0(1+\sigma_1 I+\sigma_2 I^2)} I(I+1) \qquad (6)$$

Mariscott, et al [3,8.9]proposed the variable mome*nt of inertia model "* VMI" as follow :

$$E(I) = \frac{\hbar^2}{2\theta_0} I(I+1) + \frac{C}{2}(\theta - \theta_0)^2 \quad 7)$$

Where C is a restoring force constant and θ_o is the moment of inertia of the nucleus at spin I=0
In this article we used the concept of softness of nuclear mater in modifying Eq. (7), which is denoted VMINS model. We use the VMINS model to calculate the ground state energies for some even- even nuclei , we compare the above calculations with the results of VMI model Eq.(7) and experimental data. We find that the predicting results of VMINS and VMI [10] are in good agreement with experimental data [10,11,12].

2. Methods and calculations

According to R. K. Gupta (4) the moment of inertia θ(I) can be written as a function of angular momentum I as in Eq.(3)

Taking θ(I) up to first order "n=1" ,then θ(I) becomes

$\theta(I)=\theta_0(1+\sigma_1 I)$, refer to σ_1 by σ . then $\theta(I)=\theta_0(1+\sigma I)$ (8)

Substituting θ(I) from Eq (8) in Eq. (7). One gets:

$$E(I) = \frac{\hbar^2 I(I+1)}{2\theta_o(1+\sigma I)} + \frac{1}{2}C(I\sigma\theta_o)^2 \quad (9)$$

To, determine the parameters $\theta_o, C,$ *and* σ as in Eq.(9) ,We used

the experimental energies at I=2 , E(I)=E(2) we obtain

$$E(2) = \frac{3}{\theta_o(1+2\sigma)} + 2C(\sigma\theta_o)^2 \quad (10)$$

Similarly, At $I = 4$, $E(I) = E(4)$

we obtain $E(4) = \frac{10}{\theta_o(1+4\sigma)} + 8C(\sigma\theta_o)^2$ (11)

and. At $I = 6$, $E(I) = E(6)$

We obtain $E(6) = \frac{21}{\theta_o(1+6\sigma)} + 18C(\sigma\theta_o)^2$ (12)

$$Put \quad R1 = 4E(2) - E(4) = \frac{2+28\sigma}{\theta_o(1+2\sigma)(1+4\sigma)} \quad (13)$$

$$and \quad R2 = 9E(2) - E(6) = \frac{6+120\sigma}{\theta_o(1+2\sigma)(1+6\sigma)} \quad (14)$$

$$put \quad R = \frac{R1}{R2} \quad then \quad R = \frac{1+20\sigma+84\sigma^2}{3+72\sigma+240\sigma^2} \quad (15)$$

Then by simple mainpulation we obtain the follwing quadratic equation

$$(240R - 84)\sigma^2 + (72R - 20)\sigma + (3R - 1) = 0 \quad (16)$$

Take the positive root of σ from Eq.(16) and substitute in Eq.(13) we get

$$\theta_o = \frac{2+28\sigma}{R1(1+2\sigma)(1+4\sigma)} \quad (16)$$

and from .Eq.(10) *we get*

$$C = \frac{\left(E(2) - \frac{3}{\theta_o(1+2\sigma)}\right)}{2(\theta_o\sigma)^2} \quad (17)$$

By using the experimental excitation energies E(2),E(4) &E(6) ref. [11] and Eqs.(15,16, & 17) the parameters θ_o, σ and C for VMINS Eq.(9) are given. These parameters are listed in table (1) for the chosen nuclei.

We calculated the ground state of rotational band for the chosen even –even deformed nuclei. by VMINS Eq..(9), These energies are listed in table (2)

The predicted energies as given by Eq (9) are compared with the experimental data and with the results of VMI model Eq. (7) ref [11,12,13]

Deviation of our results from experimental data are given as

Dev=$\frac{1}{N}\Sigma_{i=1}^{N}(E_{cal}-E_{exp})$

The predicted results for the ground state rotational bands are given systematically in table (3),from this table we noticed that the predicting results of Gd^{152} isotope from $I^{\pi}=2^+$ up to $I^{\pi}=16^+$,Dy^{154} isotope from $I^{\pi}=2^+$ up to $I^{\pi}=16^+$,Yb^{158} isotope from $I^{\pi}=2^+$ up to $I^{\pi}=12^+$,Dy^{162} isotope from $I^{\pi}=2^+$ up to $I^{\pi}=18^+$, Pt^{182} isotope from $I^{\pi}=2^+$ up to.

$I^{\pi}=12^+$,Pt^{184} isotope from $I^{\pi}=2^+$ up to $I^{\pi}=18^+$,Pt^{186} isotope from $I^{\pi}=2^+$ up to $I^{\pi}=12^+$,Pt^{188} isotope from $I^{\pi}=2^+$ up to $I^{\pi}=16^+$,Pt^{196} isotope from $I^{\pi}=2^+$ up to $I^{\pi}=10^+$,Th^{226} isotope from $I^{\pi}=2^+$ up to $I^{\pi}=18^+$,Th^{228} isotope from $I^{\pi}=2^+$ up to $I^{\pi}=18^+$,Th^{230} isotope from $I^{\pi}=2^+$ up to $I^{\pi}=18^+$,Th^{232} isotope from $I^{\pi}=2^+$ up to $I^{\pi}=18^+$, Th^{234} isotope from $I^{\pi}=2^+$ up to $I^{\pi}=12^+$, U^{232} isotope from $I^{\pi}=2^+$ up to $I^{\pi}=18^+$, U^{236} isotope from $I^{\pi}=2^+$ up to $I^{\pi}=18^+$, U^{238} isotope from $I^{\pi}=2^+$ up to $I^{\pi}=18^+$, Pu^{236} isotope from $I^{\pi}=2^+$ up to $I^{\pi}=16^+$, Pu^{240} isotope from $I^{\pi}=2^+$ up to $I^{\pi}=18^+$.and Pu^{242} isotope from $I^{\pi}=2^+$ up to $I^{\pi}=18^+$. It is clear from Table(2) that the predicted results of VMINS model are in close agreement with the results of the VMI model and also, agreement with the experimental data.

3. Conclusion

The present model Eq. (9) and Eq. (7) predicted the ground state rotational bands for the chosen deformed even-even nuclei and can also be applied to nuclei where the energies of levels are experimentally available. It includes three parameters which are determined straight forward using Eqs.(15,16,17)

Table (1) fitted parameters of VMINS as shown in Eq.(9) for the chosen nuclei.

Nucleus	σ	θ	C
Gd152	0.919971	3.359444	1.56E-03
Yb158	0.713557	3.924487	2.77006E-03
Dy162	0.438181	48.492	5.28E-05
Pt182	0.497481	13.42677	4.72E-04
Pt184	0.487392	12.3735	5.59E-04
Pt186	0.596656	9.127892	7.02E-04
Pt188	0.541221	6.636051	1.89E-03
Pt196	0.596085	4.618888	3.91E-03
Th226	0.172051	48.17122	1.88E-04
Th228	0.243272	67.94872	5.13E-05
Th230	0.331706	75.35878	2.34E-05
Th232	0.364487	81.23367	1.60E-05
Th234	0.451697	77.89022	1.18E-05
U232	0.394995	83.8706	1,26E-05
U234	0.438915	90.12309	8.24E-06
U236	0.503596	84.5	7.61E-06
U238	0,471716	86.64741	8.11E-06
Pu236	0.5276	84.44893	6.89E-06
Pu240	0.55901	86.99158	5.61E-06
Pu242	0.62416	80.27969	5.56E-06

Table (2) the experimental energies (exp) ref.[10, 11] and the predicted energies in Mev of VMINS Eq.(9) and VMI ref.[10] models Eq.(7) for chosen nuclei.

Gd152			
Spin I^+	E(exp)	E(VMINS)	E(VMI)
2^+	0.34428	0.34428	0.34428
4^+	0.7554	0.7554	0.7554
6^+	1.22729	1.2295	1.22729
8^+	1.7467	1.7534	1.753921
10^+	2.3004	2.3191	2.329301

12^+	2.8837	2.9211	2.94748
14^+	3.4991	3.5555	3.602545
16^+	4.1426	4.2194	4.288624
Dev		0.001353	4.62E-03
Yb158			
Spin 1^+	E(exp)	E(VMINS)	E(VMI)
2^+	0.3584	0.3584	0.3584
4^+	0.8349	0.8349	0.8349
6^+	1.4042	1.395	1.4042
8^+	2.048	2.021	2.041748
10^+	2.7454	2.702	2.723713
12^+	3.4285	3.535	3.426956
Dev		0.002326	8.53E-05
Dy162			
Spin 1^+	E(exp)	E(VMINS)	E(VMI)
2^+	0.08066	0.08066	0.08066
4^+	0.26567	0.26567	0.26567
6^+	0.54853	0.5484	0.54853
8^+	0.92128	0.9209	0.923312
10^+	1.37515	1.3749	1.384599
12^+	1.903	1.9031	1.927429
14^+	2.494	2.4988	2.547253
16^+	3.143	3.1564	3.239887
18^+	3.836	3.8711	4.001479
Dev		0.000159	4.48E-03
Pt182			
Spin 1^+	E(exp)	E(VMINS)	E(VMI)
2^+	0.1541	0.1541	0.1541
4^+	0.4175	0.4175	0.4175
6^+	0.7714	0.749	0.7714
8^+	1.2024	1.131	1.198332
10^+	1.695	1.555	1.682042
12^+	2.238	2.015	2.207391
Dev		0.012404	1.87E-04
Pt184			
Spin 1^+	E(exp)	E(VMINS)	E(VMI)
2^+	0.1634	0.1634	0.1634
4^+	0.4365	0.4365	0.4365
6^+	0.7981	0.777	0.7981
8^+	1.23	1.169	1.228558
10^+	1.705	1.603	1.709651
12^+	2.201	2.072	2.224446
14^+	2.723	2.573	2.75719
16^+	3.726	3.293212	3.293212
18^+	3.0869	3.818823	3.818823
Dev		0.084684	0.021285
Pt186			
Spin 1^+	E(exp)	E(VMINS)	E(VMI)
2^+	0.1915	0.1915	0.1915
4^+	0.4901	0.4901	0.4901
6^+	0.8772	0.855	0.8772
8^+	1.3411	1.27	1.335105
10^+	1.8557	1.727	1.846973
12^+	2.407	2.219	2.39676
Dev		0.009494	3.62E-05
Pt188			
Spin 1^+	E(exp)	E(VMINS)	E(VMI)
2^+	0.26589	0.26589	0.26589

4^+	0.67134	0.67134	0.67134
6^+	1.18427	1.1672	1.18427
8^+	1.78225	1.7312	1.774532
10^+	2.43714	2.3508	2.413765
12^+	2.81007	3.075268	3.075268
14^+	3.1391	3.733876	3.733876
16^+	3.6273	4.365853	4.365853
Dev		0.122125	0.121267
Pt196			
Spin 1^+	E(exp)	E(VMINS)	E(VMI)
2^+	0.35568	0.355568	0.355568
4^+	0.87685	0.87685	0.87685
6^+	1.527	1.5035	1.527
8^+	2.255	2.2108	2.27097
10^+	2.995	2.9847	3.075401
Dev		0.000412	1.34E-03
Th226			
Spin 1^+	E(exp)	E(VMINS)	E(VMI)
2^+	0.0722	0.0722	0.0722
4^+	0.22643	0.22643	0.22643
6^+	0.4503	0.4446	0.4473
8^+	0.7219	0.7139	0.723415
10^+	1.0403	1.0255	1.046157
12^+	1.3952	1.3735	1.408889
14^+	1.7815	1.7533	1.806416
16^+	2.1958	2.1617	2.234613
18^+	2.6351	2.596	2.690163
Dev		0.00044	1.08E-03
Th228			
Spin 1^+	E(exp)	E(VMINS)	E(VMI)
2^+	0.05776	0.057759	0.057759
4^+	0.1869	0.186823	0.186823
6^+	0.3782	0.378	0.378179
8^+	0.6225	0.622	0.624501
10^+	0.9118	0.911	0.919771
12^+	1.2394	1.239	1.259002
14^+	1.5995	1.603	1.638025
16^+	1.9881	1.999	2.053329
18^+	2.4079	2.423	2.501933
Dev		3.99144E-05	1.67E-03
Th230			
Spin 1^+	E(exp)	E(VMINS)	E(VMI)
2^+	0.0532	0.0532	0.0532
4^+	0.1741	0.1741	0.1741
6^+	0.357	0.3563	0.3566
8^+	0.592	0.5931	0.595399
10^+	0.8797	0.8797	0.885866
12^+	1.2078	1.2063	1.223941
14^+	1.5729	1.5729	1.606048
16^+	1.9715	1.9746	2.029024
18^+	2.3978	2.4084	2.490061
Dev		1.38E-05	1.16E-02
Th232			
Spin 1^+	E(exp)	E(VMINS)	E(VMI)
2^+	0.04937	0.049369	0.049369
4^+	0.16212	0.16212	0.16212
6^+	0.3331	0.3334	0.3332
8^+	0.5569	0.5576	0.55814

10^+	0.827	0.8292	0.832973
12^+	1.1374	1.4963	1.154165
14^+	1.4833	1.4963	1.923324
16^+	1.8595	1.8843	2.365916
18^+	2.2634	2.3046	2.844039
Dev		0.014588	5.16E-02
Th234			
Spin 1^+	E(exp)	E(VMINS)	E(VMI)
4^+	0.16	0.163	0.163
6^+	0.331	0.32	0.3365
8^+	0.555	0.53	0.566523
10^+	0.843	0.78	0.849829
12^+	1.1602	1.05	1.183443
Dev		0.00279	9.83E-05
14^+	1.454	1.4573	1.483885
16^+	1.8285	1.8377	1.883202
18^+	2.2317	2.2502	2.320977
Dev		4.87E-05	3.05E-03
U232			
Spin 1^+	E(exp)	E(VMINS)	E(VMI)
2^+	0.04762	0.047572	0.047572
4^+	0.15659	0.15657	0.15657
6^+	0.3228	0.3226	0.3226
8^+	0.5412	0.5404	0.541721
10^+	0.806	0.8051	0.81039
12^+	1.1117	1.112	1.125408
14^+	1.454	1.4573	1.483885
16^+	1.8285	1.8377	1.883202
18^+	2.2317	2.2502	2.320977
Dev		4.87E-05	3.05E-03
U236			
Spin 1^+	E(exp)	E(VMINS)	E(VMI)
2^+	0.045242	0.045242	0.045242
4^+	0.149476	0.149476	0.149476
6^+	0.309784	0.3097	0.309784
8^+	0.52224	0.5224	0.52345
10^+	0.7823	0.7834	0.787943
12^+	1.0853	1.089	1.100901
14^+	1.4263	1.4357	1.460119
16^+	1.7991	1.8203	1.863532
18^+	2.2021	2.24	2.30921
Dev		0.000221	3.12E-02
U238			
Spin 1^+	E(exp)	E(VMINS)	E(VMI)
2^+	0.044916	0.044916	0.044916
4^+	0.14838	0.14838	0.14838
6^+	0.30718	0.3076	0.30718
8^+	0.5181	0.5189	0.518354
10^+	0.7759	0.7783	0.779166
12^+	1.0767	1.0821	1.087086
14^+	1.4155	1.4268	1.439769
16^+	1.7884	1.8094	1.83504
18^+	2.1911	2.2269	2.270877
Dev		0.000209	0.03552
Pu236			
Spin 1^+	E(exp)	E(VMINS)	E(VMI)
2^+	0.0446	0.04463	0.04463
4^+	0.145	0.14745	0.14745

6^+	0.305	0.29	0.3058
8^+	0.5157	0.49	0.517189
10^+	0.7735	0.72	0.779279
12^+	1.0743	0.98	1.08988
14^+	1.4136	1.27	1.44693
16^+	1.786	1.59	1.848493
Dev		0.402771	6.62E-04
Dev		6.4E-06	9.71E-03
Pu240			
Spin 1^+	E(exp)	E(VMINS)	E(VMI)
2^+	0.04283	0.042824	0.042824
4^+	0.14169	0.14169	0.14169
6^+	0.29431	0.2941	0.294319
8^+	0.4976	0.497	0.49856
10^+	0.7414	0.747	0.752383
12^+	1.0418	1.0406	1.053866
14^+	1.3756	1.3746	1.401194
16^+	1.7456	1.7461	1.792647
18^+	2.152	2.1526	2.226595
Dev		3.39E-07	1.24E-02
Pu242			
Spin 1^+	E(exp)	E(VMINS)	E(VMI)
2^+	0.04454	0.04454	0.04454
4^+	0.1472	0.1473	0.1473
6^+	0.3059	0.305	0.3064
8^+	0.5176	0.515	0.520042
10^+	0.7787	0.772	0.786506
12^+	1.0867	1.074	1.104144
14^+	1.4317	1.417	1.471379
16^+	1.8167	1.797	1.8867
18^+	2.236	2.212	2.348658
Dev		0.000149	3.37E-02

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