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BEST PROXIMITY POINT THEOREMS FOR MAIA-TYPE CONTRACTION MAPPINGS

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Abstract. The Maia fixed point theorem is one of the interesting generalizations of the well-known Banach contraction principle. In this manuscript, we introduce two notions called mixed $UC-$property and mixed $P-$property of a pair (A, B) of nonempty subsets of a set X endowed with two metrics. We present two best proximity point theorems which generalize the Maia fixed point theorem.

1. Introduction and preliminaries

Let (X, d) be a metric space and A be a nonempty subset of X. A mapping $T : A \to X$ is a *contraction mapping* if there is a constant $k \in [0, 1)$ such that $d(T\zeta, T\eta) \leq k\, d(\zeta, \eta)$, for all $\zeta, \eta \in A$. A point $\zeta \in A$ is a *fixed point of* T if $T\zeta = \zeta$. The well-known Banach contraction principle states that if $T : A \to A$ is a contraction self-mapping and A is a complete subset of X, then T has a unique fixed point in A and for any $\zeta_0 \in A$, the iterated sequence $\{\zeta_n\}$, where $\zeta_n = T\zeta_{n-1}$, for all $n \in \mathbb{N}$, converges to the unique fixed point. Due to its elementary proof technique and numerous applications in various fields of Mathematics, Banach contraction principle attracts many researchers to obtain various generalizations and extensions of it. For more interesting and important generalizations of Banach contraction principle, one may refer [9, 15].

Maia [10] fixed point theorem is one of the interesting generalizations of Banach contraction principle. It states that

Theorem 1.1 (Maia Fixed Point Theorem)**.** [10] *Let X be a nonempty set together with two metrics d and δ and $T : X \to X$ be a mapping which satisfies the following:*

(1) $d(x, y) \leq \delta(x, y)$*, for all* $x, y \in X$*,*

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(2) *X is complete with respect to d,*
(3) *T is continuous with respect to d,*
(4) *There is a $k \in [0,1)$ such that $\delta(T(x), T(y)) \leq k\,\delta(x,y)$, for all $x, y \in X$.*

Then T has a unique fixed point in X.

It is worth mentioning that the fourth condition in Maia's fixed point theorem can be replaced by Kannan contraction or Meir-Keeler contraction condition. If $\delta = d$ in Theorem 1.1, then Maia's fixed point theorem reduces to Banach contraction principle. More interesting generalizations and applications of Maia's fixed point theorem can be found in [1, 2, 3, 4, 6, 7, 8, 11, 12, 13]

On the other hand, let us consider two nonempty subsets A, B of a metric space X and a mapping $T : A \to B$. If $A \cap B = \emptyset$, then there is no $\zeta \in A$ such that $T\zeta = \zeta$. In such case, we try to find $\zeta \in A$ such that the error $d(\zeta, T\zeta)$ is minimum in some sense. Let $D(A,B) := \inf\{d(a,b) : a \in A,\, b \in B\}$. A point $\zeta \in A$ satisfying $d(\zeta, T\zeta) = D(A,B)$ is known as best proximity point of T in A. If $A = B$, then the best proximity points are nothing but the fixed points of T in A. In this sense, best proximity points are considered as a generalized fixed points of a mapping T.

In this manuscript, we establish two generalizations of Maia fixed point theorem in best proximity point setting. In section 3, we consider two nonempty subsets A and B of a uniformly convex Banach space and a cyclic mapping $T : A \cup B \to A \cup B$. We introduce a notion called mixed UC-Property and establish sufficient conditions for the existence of a best proximity point of T, if T satisfy cyclic contraction conditions with respect to $\|\cdot\|$ and δ. It is worth mentioning that if $A = B$, then Maia's fixed point theorem is obtained as a corollary to our main theorem (Theorem 3.2).

Next, we consider a nonempty set X with two metrics d, δ and a mapping $T : A \to B$. Note that the iterated sequence of T is not well-defined since T is not a self mapping. In section 4, we introduce a notion called mixed $P-$property and discuss sufficient conditions to ensure the existence of a best proximity point of a contraction nonself mapping T. Again, if we take $A = B$, then Theorem 4.3 reduces to the Maia fixed point theorem.

2. Definitions and Notations

In this section, we state some known definitions and results which we use in subsequent sections.

Let X be a nonempty set together with two metrics d and δ. Consider two nonempty subsets A and B of X. We fix the following notations.

$$\begin{aligned}
D(A,B) &:= \inf\{d(a,b) : a \in A,\, b \in B\}\\
\Delta(A,B) &:= \inf\{\delta(a,b) : a \in A,\, b \in B\}\\
A_0^d &:= \{\zeta \in A : d(\zeta,\eta) = D(A,B) \text{ for some } \eta \in B\}\\
B_0^d &:= \{\eta \in B : d(\zeta,\eta) = D(A,B) \text{ for some } \zeta \in A\}
\end{aligned}$$

The pair (A_0^d, B_0^d) is said to be the proximinal pair associated with the pair (A,B) with respect to the metric d. In [5], Eldred and Veeramani introduced the following notion called cyclic contraction mapping.

Definition 2.1. [5] *Let A, B be nonempty subsets of a metric space X. A mapping $T : A \cup B \to A \cup B$ is said to be cyclic contraction if there is a constant $k \in [0, 1)$ such that*

(1) $T(A) \subseteq B$ *and* $T(B) \subseteq A$*;*
(2) $d(Tx, Ty) \leq kd(x, y) + (1 - k)D(A, B)$, *for all* $x \in A$ *and* $y \in B$.

It is worth mentioning that a cyclic contraction mapping need not be continuous. In [5], Eldred and Veeramani established the following two lemmas and used to obtain best proximity points for cyclic contraction mappings in uniformly convex Banach space setting.

Lemma 2.1. [5] *Let A, B be nonempty closed convex subsets of a uniformly convex Banach space and $D(A, B) = \inf\{\|a - b\| : a \in A, b \in B\}$. Let $\{x_n\}$ and $\{y_n\}$ be sequences in A and $\{z_n\}$ be a sequence in B such that*

(1) $\|x_n - z_n\| \to D(A, B)$,
(2) *For every $\varepsilon > 0$, there is an $N_0 \in \mathbb{N}$ such that, for all $m > n \geq N_0$, $\|y_m - z_n\| \leq D(A, B) + \varepsilon$.*

Then, for every $\varepsilon > 0$, there is an $N_1 \in \mathbb{N}$ such that $\|x_n - y_m\| \leq \varepsilon$, for all $m > n \geq N_1$.

Lemma 2.2. [5] *Let A, B be nonempty closed convex subsets of a uniformly convex Banach space and $D(A, B) = \inf\{\|a - b\| : a \in A, b \in B\}$. Let $\{x_n\}$ and $\{y_n\}$ be sequences in A and $\{z_n\}$ be a sequence in B such that*

(1) $\|x_n - z_n\| \to D(A, B)$,
(2) $\|y_n - z_n\| \to D(A, B)$.

Then, $\|x_n - y_n\| \to 0$.

Later, in [16], the above lemma is coined as a new definition called $UC-$property which is given below.

Definition 2.2. *(UC-Property)*[16] *Let A and B be nonempty subsets of a metric space (X, d). Then (A, B) is said to have $UC-$property if whenever $\{x_n\}$ and $\{y_n\}$ are sequences in A and $\{z_n\}$ is a sequence in B such that $\|x_n - z_n\| \to D(A, B)$ and $\|y_n - z_n\| \to D(A, B)$, then $\|x_n - y_n\| \to 0$*

In this paper, we introduce a notion called mixed $UC-$Property to establish a best proximity point theorem for Maia-Type cyclic contraction mapping.

In recent years, the following geometric notion called $P-$property plays a vital role in proving the existence of best proximity points of non-self mappings.

Definition 2.3. [14] *A pair (A, B) of nonempty subsets of a metric space (X, d) is said to have $P-$property if and only if $d(x_1, x_2) = d(y_1, y_2)$ for any $x_1, x_2 \in A$ and $y_1, y_2 \in B$ with $d(x_1, y_1) = D(A, B)$ and $d(x_2, y_2) = D(A, B)$.*

In this article, we introduce a new notion called mixed $P-$property (Definition 4.6), which generalizes the idea of $P-$property. We use mixed $P-$property to obtain unique best proximity point for a nonself contraction mapping involving two metrics.

3. Maia-Type Cyclic Contraction

Now, we define a new class of mapping called generalized cyclic contraction with respect to two metrics.

Definition 3.4. *Let X be a nonempty set together with two metrics d, δ such that $d(\zeta,\eta) \le \delta(\zeta,\eta)$, for all $\zeta,\eta \in X$. Let A, B be nonempty subsets of X and let $D(A,B) := \inf\{d(\zeta,\eta) : \zeta \in A, \eta \in B\}$. A mapping $T : A\cup B \to A\cup B$ is a generalized cyclic contraction mapping with respect to d and δ if there is a constant $k \in [0,1)$ such that*

1. *$T(A) \subseteq B, T(B) \subseteq A$*
2. *$\delta(T\zeta, T\eta) \le k\delta(\zeta,\eta) + (1-k)D(A,B)$, for all $\zeta \in A, \eta \in B$.*

When $d = \delta$, then the above definition reduces to Eldred's cyclic contraction mapping given in Definition 2.1. When $A = B$, then the above definition reduces to the usual $\delta-$contraction. Now, we introduce a notion called mixed UC-Property as follows:

Definition 3.5. *Let X be a nonempty set together with two metrics d, δ such that $d(\zeta,\eta) \le \delta(\zeta,\eta)$, for all $\zeta,\eta \in X$. Let A, B be nonempty subsets of X and let $D(A,B) := \inf\{d(\zeta,\eta) : \zeta \in A, \eta \in B\}$. We say that the pair (A,B) has mixed UC-Property with respect to d and δ if for every sequence $\{\zeta_n\}, \{\eta_n\}$ in A and $\{z_n\}$ in B with $\delta(\zeta_n, z_n) \to D(A,B)$ and $\delta(\eta_n, z_n) \to D(A,B)$, then $\delta(\zeta_n,\eta_n) \to 0$.*

If $d = \delta$, then the above definition reduces to the usual $UC-$Property given in Definition 2.2. Now, let us prove the main result of this section.

Theorem 3.2. *Let A and B be nonempty closed convex subsets of a uniformly convex Banach space X and $T : A\cup B \to A\cup B$ be a cyclic mapping. Suppose there is a metric function $\delta : X\times X \to \mathbb{R}$ satisfying the following conditions :*

1. *$\|\zeta - \eta\| \le \delta(\zeta,\eta)$, for all $\zeta,\eta \in X$,*
2. *T is a generalized cyclic contraction with respect to $\|\cdot\|$ and δ,*
3. *T is continuous with respect to $\|\cdot\|$,*
4. *the pair (A,B) satisfies mixed UC-property with respect to $\|\cdot\|$ and δ.*

Then T has a best proximity point in A with respect to $\|\cdot\|$.

Proof. Let $\zeta_0 \in A$ and put $\zeta_n = T\zeta_{n-1}$, for all $n \in \mathbb{N}$. Since T is a generalized cyclic contraction with respect to $\|\cdot\|$ and δ, there is a $k \in [0,1)$ satisfying

$$\delta(\zeta_n, \zeta_{n+1}) \le k^n\delta(\zeta_0,\zeta_1) + (1-k^n)D(A,B), \text{ for all } n \in \mathbb{N}.$$

From the inequality $D(A,B) \le \|\zeta_n - \zeta_{n+1}\| \le \delta(\zeta_n,\zeta_{n+1})$, we have $\delta(\zeta_n,\zeta_{n+1}) \to D(A,B)$ and hence $\|\zeta_n - \zeta_{n+1}\| \to D(A,B)$. Particularly, $\delta(\zeta_{2n}, T\zeta_{2n}) \to D(A,B)$ and $\delta(T^2\zeta_{2n}, T\zeta_{2n}) \to D(A,B)$. By mixed UC-Property, we conclude $\delta(\zeta_{2n}, T^2\zeta_{2n}) \to 0$. In similar manner, we have $\delta(\zeta_{2n+1}, T^2\zeta_{2n+1}) \to 0$.

Now, we claim that $\{\zeta_{2n}\}$ is a Cauchy sequence in A. In view of Lemma 2.1, it is sufficient to show that for each $\varepsilon > 0$, there is an $N \in \mathbb{N}$ such that $\|\zeta_{2m} - T\zeta_{2n}\| < D(A,B) + \varepsilon$, for all $n, m \ge N$.

If not, then there is an $\varepsilon > 0$ and for all $k \in \mathbb{N}$, there are $m_k > n_k \ge k$ satisfying

$$\delta(\zeta_{2m_k}, T\zeta_{2n_k}) \ge \|\zeta_{2m_k} - T\zeta_{2n_k}\| \ge D(A,B) + \varepsilon. \quad (1)$$

The m_k can be chosen such that it is the least integer greater than n_k satisfying (1). Then,

$$\begin{aligned} D(A,B) + \varepsilon &\le \|\zeta_{2m_k} - T\zeta_{2n_k}\| \le \delta(\zeta_{2m_k}, T\zeta_{2n_k}) \\ &\le \delta(\zeta_{2m_k}, \zeta_{2(m_k-1)}) + \delta(\zeta_{2(m_k-1)}, T\zeta_{2n_k}) \\ &\le \delta(\zeta_{2m_k}, \zeta_{2(m_k-1)}) + D(A,B) + \varepsilon. \end{aligned}$$

Since $\delta(\zeta_{2m_k}, \zeta_{2(m_k-1)}) \to 0$, we have $\delta(\zeta_{2m_k}, T\zeta_{2n_k}) \to D(A,B) + \varepsilon$. Then,

$$\begin{aligned}\delta(\zeta_{2m_k}, T\zeta_{2n_k}) &\leq \delta(\zeta_{2m_k}, \zeta_{2(m_k+1)}) + \delta(\zeta_{2(m_k+1)}, T\zeta_{2(n_k+1)}) \\ &\qquad + \delta(T\zeta_{2(n_k+1)}, T\zeta_{2n_k}) \\ &\leq \delta(\zeta_{2m_k}, T^2\zeta_{2m_k}) + k^2\delta(\zeta_{2m_k}, T\zeta_{2n_k}) + (1-k^2)D(A,B) \\ &\qquad + \delta(T^2\zeta_{2n_k+1}, \zeta_{2n_k+1})\end{aligned}$$

By taking limits on both sides, we arrived the inequality

$$D(A,B) + \varepsilon \leq k^2\varepsilon + D(A,B),$$

which leads to a contradiction to the fact $k < 1$.

Hence, for each $\varepsilon > 0$, there exist $N \in \mathbb{N}$ such that $\|\zeta_{2m} - T\zeta_{2n}\| < D(A,B) + \varepsilon$, for all $n, m \geq N$. By Lemma 2.1, $\{\zeta_{2n}\}$ is a Cauchy sequence with respect to $\|\cdot\|$. Since A is complete, there exists $\zeta^* \in A$ such that $\zeta_{2n} \to \zeta^*$. Also, the continuity of T with respect to $\|\cdot\|$, we have $T\zeta_{2n} \to T\zeta^*$. Now,

$$\begin{aligned}D(A,B) &\leq \|\zeta^* - T\zeta^*\| \\ &\leq \|\zeta^* - \zeta_{2n}\| + \|\zeta_{2n} - T\zeta_{2n}\| + \|T\zeta_{2n} - T\zeta^*\| \to D(A,B).\end{aligned}$$

Hence, $\|\zeta^* - T\zeta^*\| = D(A,B)$. That is, T has a best proximity point in A with respect to the norm. □

We could not show the uniqueness of best proximity point in above result. Note that if $A = B$, then the above theorem reduces to Maia fixed point theorem in uniformly convex Banach space setting.

4. Maia-Type Nonself Contraction Mapping

In this section, we introduce a notion called mixed $P-$property and provide sufficient conditions for the existence of best proximity points for a Maia-Type nonself contraction mapping.

Definition 4.6 (mixed $P-$property)**.** *Let X be a nonempty set together with two metrics d and δ. Let A, B be two nonempty subsets of X and $D(A,B)$ is the distance between A and B with respect to d. Then the pair (A,B) is said to have mixed $P-$property with respect to d and δ if and only if $\delta(\zeta_1, \zeta_2) = \delta(\eta_1, \eta_2)$, for any $\zeta_1, \zeta_2 \in A$ and $\eta_1, \eta_2 \in B$ satisfying $d(\zeta_1, \eta_1) = D(A,B)$ and $d(\zeta_2, \eta_2) = D(A,B)$.*

If we take $d = \delta$ in Definition 4.6, then it reduces to the usual $P-$property given in Definition 2.3.

Definition 4.7. [17] *Consider $\mathbb{R}^2$ with a function $\delta : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$ defined as follows:*

$$\delta((x,y),(u,v)) := \begin{cases} |y-v|, & \text{if } x = u \\ |x-u| + |y| + |v|, & \text{otherwise.} \end{cases}$$

Then δ is a metric on $\mathbb{R}^2$, known as river metric.

Now, we give an example of two metrics d and δ in $\mathbb{R}^2$ and a pair (A,B) of subsets of $\mathbb{R}^2$ which satisfies $P-$property with d but fails to have mixed $P-$property.

Example 4.1. *Let $A = \{(x,2) : 1 \le x \le 3\}$ and $B = \{(x,1) : 1 \le x \le 3\}$ be two nonempty subsets of $\mathbb{R}^2$. Let $d((x,y),(u,v)) = |x-u| + |y-v|$, and δ is the river metric on $\mathbb{R}^2$. Then $d((x,y),(u,v)) \le \delta((x,y),(u,v))$, for all $(x,y),(u,v) \in \mathbb{R}^2$. Then, it is easy to see that the pair (A,B) has $P-$property with respect to d but fails to have mixed $P-$property with respect to d and δ.*

Now, we use the mixed $P-$property to establish the following best proximity point theorem.

Theorem 4.3. *Let X be a nonempty set together with two metrics d and δ such that $d(\zeta,\eta) \le \delta(\zeta,\eta)$, for all $\zeta,\eta \in X$. Let A, B be two nonempty subsets of X such that A_0^d is nonempty and the pair (A,B) satisfies mixed $P-$property. Let $T : A \to B$ be a mapping satisfying $T(A_0^d) \subseteq B_0^d$. Suppose that*

1. *A is complete with respect to d,*
2. *T is continuous with respect to d,*
3. *there exist $k \in [0,1)$ such that $\delta(T\zeta,T\eta) \le k\,\delta(\zeta,\eta)$, for all $\zeta,\eta \in A$.*

Then T has a unique best proximity point in A with respect to d.

Proof. Let $\zeta_0 \in A_0^d$. Since $T\zeta_0 \in B_0^d$, there is $\zeta_1 \in A_0^d$ such that $d(\zeta_1,T\zeta_0) = D(A,B)$. Since $T\zeta_1 \in B_0^d$, there is $\zeta_2 \in A_0^d$ such that $d(\zeta_2,T\zeta_1) = D(A,B)$. By repeating the process, we obtain a sequence $\{\zeta_n\}$ in A_0^d satisfying the equality $d(\zeta_n,T\zeta_{n-1}) = D(A,B)$, for all $n \in \mathbb{N}$. Seeing that $d(\zeta_n,T\zeta_{n-1}) = D(A,B)$ and $d(\zeta_{n+1},T\zeta_n) = D(A,B)$, by mixed $P-$property, we have $\delta(\zeta_n,\zeta_{n+1}) = \delta(T\zeta_{n-1},T\zeta_n)$.

Thus, $\delta(\zeta_n,\zeta_{n+1}) = \delta(T\zeta_{n-1},T\zeta_n) \le k\,\delta(\zeta_{n-1},\zeta_n)$, for all $n \in \mathbb{N}$. Now, it is easy to see that $\delta(\zeta_n,\zeta_{n+1}) \le k^n\delta(\zeta_0,\zeta_1)$ and hence, for any $n \le m$,

$$d(\zeta_n,\zeta_m) \le \delta(\zeta_n,\zeta_m) \le \sum_{j=n}^{m-1} \delta(\zeta_j,\zeta_{j+1}) \le \frac{k^n}{1-k}\delta(\zeta_0,\zeta_1) \to 0, \text{ as } n \to \infty$$

Hence $\{\zeta_n\}$ is Cauchy with respect to d. Since A is complete, there is $\zeta^* \in A$ such that $\zeta_n \to \zeta^*$. Since T is continuous with respect to d, $d(T\zeta_n,T\zeta^*) \to 0$. Then

$$D(A,B) \le d(\zeta^*,T\zeta^*) \le d(\zeta^*,\zeta_n) + d(\zeta_n,T\zeta_{n-1}) + d(T\zeta_{n-1},T\zeta^*) \to D(A,B).$$

Hence $d(\zeta^*,T\zeta^*) = D(A,B)$.

Suppose that there is $\eta^* \in A$ such that $\zeta^* \ne \eta^*$ and $d(\eta^*,T\eta^*) = D(A,B)$. Then by mixed $P-$property, $\delta(\zeta^*,\eta^*) = \delta(T\zeta^*,T\eta^*) < \delta(\zeta^*,\eta^*)$, a contradiction. Hence the best proximity point is unique. □

When $A = B$, the above theorem reduces to the Maia's fixed point theorem (Theorem 1.1). The following example shows the condition that X is complete with respect to d can not be relaxed.

Example 4.2. *Let $X = \mathcal{C}[0,1]$ be the set of all continuous real-valued functions defined on $[0,1]$. Consider the following subsets of X.*

$$A := \left\{ f \in \mathcal{C}[0,1] : f(t) = t, \text{if } t \in \left[0,\frac{1}{2}\right] \text{ and } \frac{1}{2} \le f(t) \le t, \text{if } t \in \left[\frac{1}{2},1\right] \right\}$$

$$B := \left\{ g \in \mathcal{C}[0,1] : g(t) = \frac{1}{2}, \text{if } t \in \left[0,\frac{1}{2}\right] \text{ and } \frac{1}{2} \le g(t) \le t, \text{if } t \in \left[\frac{1}{2},1\right] \right\}$$

Consider the two metrics,

$$d(f,g) = \int_0^1 |f(t) - g(t)|dt,$$
$$\delta(f,g) = \sup\{|f(t) - g(t)| : t \in [0,1]\}.$$

Clearly, $d(f,g) \leq \delta(f,g)$*, for all* $f, g \in X$ *and* X *is not complete with respect to* d*. It is easy to see that* A_0^d*,* B_0^d *are nonempty subsets of* A, B *respectively and* $D(A,B) = \frac{1}{8}$*, the area of triangle whose vertices are* $(0,0)$*,* $(0, \frac{1}{2})$ *and* $(\frac{1}{2}, \frac{1}{2})$*.*

Let $T : A \to B$ *be a mapping defined by*

$$T(f)(t) = \begin{cases} \frac{1}{2}, & \text{if } t \in \left[0, \frac{1}{2}\right] \\ \frac{tf(t)}{2} + \frac{3}{8}, & \text{if } t \in \left[\frac{1}{2}, 1\right]. \end{cases}$$

It is easy to see that T *is continuous with respect to* d *and it satisfies* $\delta(Tf_1, Tf_2) \leq \frac{1}{2}\delta(f_1, f_2)$*, for all* $f_1, f_2 \in A$*. But there is no* $f \in A$ *such that* $d(f, Tf) = \frac{1}{8} = D(A,B)$*. That is,* T *has no best proximity points in* A *with respect to* d*.*

References

[1] Ansari, Arslan Hojat; Khan, Mohammad Saeed; Rakočević, Vladimir (2020) Maia type fixed point results via C-class function.Acta Univ. Sapientiae Math.12, no.2, 227–244.

[2] Balazs, Margareta-Eliza (2018), Maia type fixed point theorems for Ćirić-Prešić operators. Acta Univ. Sapientiae Math. 10, 18–31.

[3] Bera, Ashis; Mondal, Pratikshan; Garai, Hiranmoy; Dey, Lakshmi Kanta (2023) On Maia type fixed point results via implicit relation .AIMS Math.8, no.9, 22067–22080.

[4] Berinde, Vasile (2022), Maia type fixed point theorems for some classes of enriched contractive mappings in Banach spaces. Carpathian J. Math. 38, 35–46.

[5] Eldred, A. Anthony; Veeramani, P. (2006), Existence and convergence of best proximity points. J. Math. Anal. Appl. 323, 1001–1006.

[6] Filip, Alexandru-Darius(2022), Some applications of Maia's fixed point theorem in the case of operators with Volterra property with respect to a subinterval. Miskolc Math. Notes 23, 183–192.

[7] Filip, Alexandru-Darius (2022), Some applications of Maia's fixed point theorem for Fredholm integral equation systems. Stud. Univ. Babeş-Bolyai Math. 67, 189–202.

[8] Kedim, Imed; Berzig, Maher (2021), Fixed point theorems for Maia α-ψ contractive type mappings with applications. J. Math. Anal. Appl. 504, Paper No. 125381, 12 pp.

[9] Kirk W. A. and B. Sims (Editors) (2001), Handbook of metric fixed point theory. Kluwer Academic Publishers, Dordrecht.

[10] Maia, Maria Grazia (1968), Un'osservazione sulle contrazioni metriche (Italian). Rend. Sem. Mat. Univ. Padova 40 , 139–143.

[11] Mureşan, Anton S. (2007), From Maia fixed point theorem to the fixed point theory in a set with two metrics. Carpathian J. Math. 23 , 133–140.

[12] Olgun, Murat; Alyıldız, Tuğçe; Biçer, Özge; Altun, Ishak (2021) Maia type fixed point results for multivalued F-contractions.Miskolc Math. Notes, Vol.22, no.2, 819–829.

[13] Petruşel, Adrian; Petruşel, Gabriela; Horvath, Leonard (2024) Maia type fixed point theorems for multi-valued Feng-Liu operators.J. Anal. Vol. 32, no.1, 73–83.

[14] Raj, V. Sankar (2011), A best proximity point theorem for weakly contractive non-self mappings. Nonlinear Anal. 74, 4804–4808.

[15] Rhoades, B.E. (1977), A comparison of various definitions of contractive mappings. Trans. Amer. Math. Soc. 226, 257–290.

[16] Suzuki, Tomonari; Kikkawa, Misako; Vetro, Calogero (2009), The existence of best proximity points in metric spaces with the property UC. Nonlinear Anal. 71, 2918–2926.

[17] Victor Bryant (1985), Metric Spaces : Iteration and Applications, Cambridge University Press.

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