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A Hybrid Multibody System Algorithm Used in Mathematical Modeling of Delta Robot Mechanisms

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Abstract
Delta robot as a parallel mechanism has been gaining widespread attention. In recent years, researchers have been focused on the construction of serial structured robots. However, few researchers tried to evolve the delta robots in such a system. This study attempted to simplify the delta robot mechanical structure to obtain a kinematically driven Multi- body System Dynamics (MBS) model. The simplified model preserves lower computing costs and faster response than the typical MBS model to be applied in real-time control applications. The simulation results of the simplified MBS model were compared with the results from the typical MBS model of the whole system and the loop closure method, both of which were identical to each other and different from the simplified MBS model. The same motion behavior of the end effector was obtainable using the simplified MBS model and was the same as the realistic behavior. The simplified MBS model created in this study can describe the kinematics of the delta robot, which has prosperous prospects in dynamics, control, and design optimization of the robotic field.

Keywords; Delta robot; MBS modeling; System dynamics.

1. INTRODUCTION

Multibody System Dynamics (MBS) as a modeling algorithm able to instantly monitor positions and their higher derivatives of all bodies which consist of a robotic system. According to masses, external forces, and some driving criteria, the dynamic model iteratively can solve the system. Torque analysis usually used in design stage to select equivalent power of actuators needed to operate the robotic system, however in a real-time control going through this algorithm might be time and power consuming moreover relatively long response time compared to other direct solving algorithms. Alternatives of MBS model usually considers direct transformation from the world frame to local frame of an end effector using the minimum possible coordination variables, the main drawback here is the ambiguity around all other coordinates of the robotic system.

Maneuvering situations may acquire a high responsive control algorithm, besides a specific coordinate state. Hence, the need to develop a model which combines the pros of each method is necessary. Traditional kinematically driven algorithm present in MBS method also uses iterative methods [1, 2], however using this technique hybrid with other direct algebraic method which leads to a set of linear equations to be solved, should be more effective. The iterative methods such as Newton-Raphson are very sensitive to the initial guess passed to the solver, which means that the guess should be close enough to the true state of the system's coordinates otherwise the solution diverges.

The hybrid method which combines kinematically driven procedure and direct method may be a good choice to control a robotic system. Upon known end effector driving constraints, actuators could be controlled by this method directly to achieve the desired output motion. One of the most attractive robotic systems to apply the model to is the delta robot [3, 4]. It is important to note that for the different robotic systems it would be different models using the algorithm stated in this research. In other worlds the notion of generalization is not applicable to our algorithm. Delta robot system (DRS) is a creative system of parallel structure which researchers from different disciplines tried to apply mathematical models and tried to investigate recent advances to validate the efficiency of their hypotheses, as the delta robot provides a mathematical formulation complexity besides the advantages of spatial characteristics, such as high accuracy at higher speeds, high rigidity against relatively heavy payloads, and high repeatability to reach certain positions [5]. DRS consists of three identical parallel chains, oriented to each other by an angle equals $2\pi/3$. Each chain consists of a driven arm by an electric motor, two passive fore- arms, and two connecting rods. These bodies linked to each other by one revolute joint, four spherical joints, and four revolute joints respectively.

Control algorithms developed to drive the end-effector of DRS in a specific motion trajectory, necessitate a computational model of the inverse kinematics to be applied, which governs the spatiotemporal relationships between the desired output coordinates and the required input coordinates [6]. The output of the inverse kinematics model is the angular motion of active arms in each chain, while input of the model is the position vector of end-effector. The different kinematic models could be extended to formulate computational dynamics of DRS when considering system 's inertia and exerted external forces. Some computational models derived from recursive matrices relationships of spatial successive rotations, and from inertia tensors [7]. Another model formulated the Euler- Lagrange equation of DRS based on kinetic and potential energies of bodies in relative motion which caused by a vector of external forces [8, 9]. A model derived to determine in- verse kinematics in form of Jacobian also had been introduced, based on closed-loop vectors equation of a certain position represented within a reference frame which attached to the fixed base of the system [10]. A Multibody model of DRS, based on redundant coordinates formulation, was investigated in another work which handled the validation and simulation of Multibody dynamic model using ADAM software [11, 12].

2. MULTIBODY STRUCTURE OF DELTA ROBOT

DRS is a three-transitional degrees of freedom system, sometimes another DOF could be added at the moving plate to allow the end effector to rotate about a vertical axis. The model D3S-800 of delta robots achieves the 4th DOF by means of an actuator fixed within the fixed base, while the rotational motion is transmitted mechanically by universal and prismatic joints to the end effector (Fig. 1). D3S-800 consists of (21) bodies, (17) revolute joints, (12) spherical joints, (2) universal joints, and (1) prismatic joint [10].

This combination of bodies and joints implies to an MBS spatial model consists of (126) coordinates and (122) constraints equations based on Euler's angles representation. Implementation of the simplified model mainly aims to reduce the number of bodies forming the system, in such a method to maintain the same motion of the typical delta system, subsequently it reduces the number of constraints equations, this simplification is expected to improve the computation efficiency and reduce the computational cost.

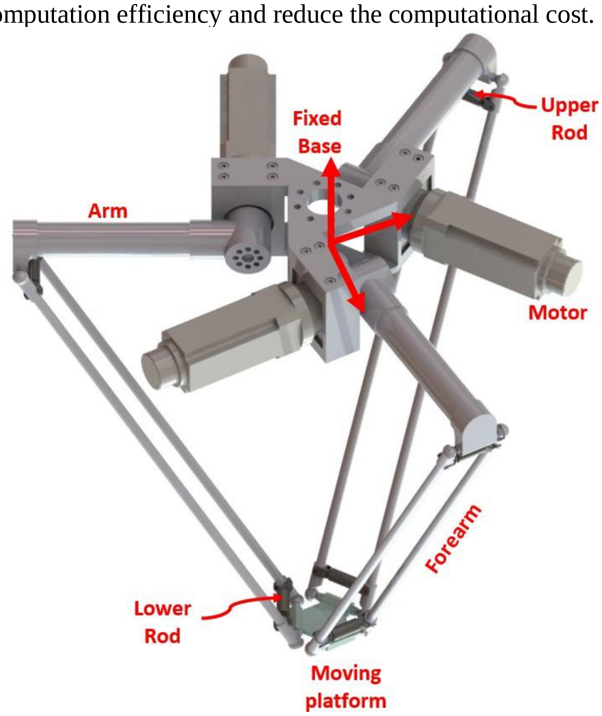

Fig. (1) D3S-800 delta robot system.

3. PARAMETERS ASSIGNMENT OF DELTA SYSTEM

The computational model is established to determine the kinematic relationship between the system 's coordinates. We apply the mathematical calculation process MBS to the incremental robot mechanism. Nodes which represent joints of the system are shown in Fig. 2. Nodes were given small letters except nodes O and P, which are assigned to CGs of the base plate and of the movable end effector respectively.

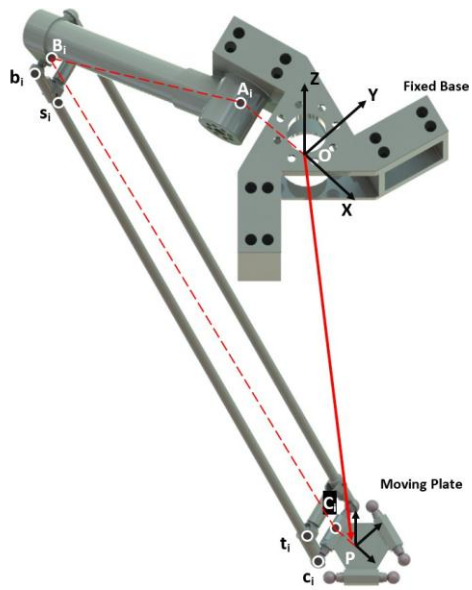

Fig. (2) Parameters assignment of one chain.

Node O is the reference coordinate system located on the same plane which encloses the three ai points. Nodes ai are the positions of revolute joints between the base plate and the active arms and driven by motors to provide needed torques and angular velocities to the system. Node P was attached to CG of the end effector, also in a plane containing six points of spherical joints c_{i1} and c_{i2}. Length of an arm represented as capital letter L, while length of a forearm was given the annotation L_f. The forearms or the passive arms are connected to the active arms at points b_{i1} and b_{i2}, the above two points are connected to the movable platform in points c_{i1} and c_{i2} forming the closed loop (b_{i1} b_{i2} c_{i2} c_{i1}). Another closed loop formed by the connecting rods which functions are to maintain the connectivity of the spherical joints and to prevent the forearms from the undesired rotations about their longitudinal axis. The coordinate system of the end effector body is See and at a distance Lee from the attached coordinate to the movable platform body Sp, the two bodies are connected by a revolute joint. The Z axis of the two coordinate systems is co-linear to each other at any instance, moreover; at the initial home position both are co-linear with the Z axis of the reference coordinate system.

4. SIMPLIFICATION OF DELTA ROBOT MBS MODEL

For the system simplification, first, we assumed that the motor of the fourth DOF is attached directly to the movable platform, and the rotational motion of the end effector about the vertical Z-axis is the direct output of the motor instead of transmitting the motion from the fixed base by means of mechanical linkages. Application of this modification will have no effect on the desired control task; however, it contributes to simplify the MBS model significantly by eliminating 3 bodies, 2 universal joints and 1 prismatic joint which means the number of systems coordinate reduced by 12 coordinates and the number of constraints reduced by 13 equations. The second simplification is at the end effector, back to the first modification which is the rotation motion about the vertical axis is directly delivered from an actuator fixed at the movable platform. Subsequently, in the inverse kinematics or forward kinematics, this variable will be inserted in the equations without any mathematical manipulations, hence, we can consider that the end effector is the movable platform in the mathematical model and this motion will be considered directly in the final equations of the system. Since the last rotational motion is no longer present in the modified model, the DOFs of the system are converted to 3 after simplification. The last simplification is the elimination of the two connecting rods at each chain eliminates 12 revolute joints at the points s_{ij} and t_{ij} which necessitates 60 constraints equations to be modeled, alternatively, we use only 6 constraints equations which are the axial rotational motion of each forearm is equal to zero. The newly added 6 DOFs preserve the same functionality of these eliminated rods. Also, this simplification reduces the number of coordinates by 36 coordinates. The numeric values which are shown in Table.1 were provided based on the Euler Angles representation. In sum, a Delta robot machine which has 3 DOFs was developed in this part. The simplified Delta robot machine consists of a fixed base and a movable base connected to each other by means of three chains, each chain only consists of three bodies, one arm and two forearms. The simplified model consists of 3 revolute joints and 12 spherical joints. A comparison between the typical model and the equivalent simplified model is exhibited in Table 1.

Table (1) MBS Model simplification effect

Property	Typical system model	Simplified model
No. of bodies (nb)	17	11
No. of coordinates (6 nb)	102	66
No. of revolute joints	15	3
No. of spherical joints	12	12
Assumed constraints	0	6
Constraints of fixed joint with ground	6	6
No. of constraints equ.	111	63

5. Parameters of The Introduced MBS Model

Parameters of D3S-800 delta system is provided in Table 2. These values were fed to the MBS approach to generate the constraints equations and the Jacobian matrix. The fixed base radius is the radius of a circle that passes through the three points of the revolute joints of arms, while the radius of the movable platform is the radius of a circle passes through the 6 points of the lower spherical joints between the platform and the forearms. The length of the arm in this study is shown in Fig. 3. Based on assumptions afore mentioned and Euler Angles representation, coordinate of a body n written as:

$$q^n = [R^n_x \;\; R^n_y \;\; R^n_z \;\; \varphi^n \;\; \theta^n \;\; \psi^n]^T \qquad (1)$$

Three revolute joints (the joints between the fixed base and the three arms) exist in this model. The body i in this case is the fixed base and the body j is the arm of concern, the fixed base position and rotation vectors are $R^0 = [0\ 0\ 0]^T$ and $\theta^0 = [0\ 0\ 0]^T$ respectively, on the other hand the arms position vectors are variables and time dependents which take the form $R^i = [R^i_x \;\; R^i_y \;\; R^i_z]^T$, while the rotation vectors take the form $\theta^i = [\alpha^i \;\; \pi^i/2 \;\; \psi^i]^T$. Position and rotation vectors of a forearm is in the following forms respectively;
$R^{ij} = [R^{ij}_x\ R^{ij}_y\ R^{ij}_z]^T$ and $\theta^{ij} = [\varphi^{ij}\ \pi^{ij}/2\ \psi^{ij}]^T$. Figure 2 explains the rotation vector from first coordinate system to the n^{th} coordinate system. which equals to 30◦, 50◦, and 270◦ for n arm equals to 1, 2, and 3 respectively. The rotation vectors of each arm consist of 3 parts first, a constant angle α about the Z axis, a constant rotation angle of π/2 about the current X axis, and an unknown, time dependent angle ψ which equals to the motor rotation angle fed to each arm. The simplified delta model consists of 12 spherical joints, 6 of them are between arms and forearms which would be referred in this study as Upper Spherical Joint (USJ), and the other six joints are between the forearms and the movable platform and would be referred as Lower Spherical Joint (LSJ). Each chain in the simplified model consists of an arm i and 2 forearms i j.

Table (2) Parameters of D3S-800 delta system

Parameter	Symbol	Value (mm)
Radius of Fixed base	R	125
Radius of movable platform	R	62
Effective length of arm	L	370
Length of forearm	L_f	960
Length of connecting rod	L_r	95
Distance $b_{ij}\ s_{ij}$	d_1	45
Distance $b_{ij}\ t_{ij}$	d_2	915

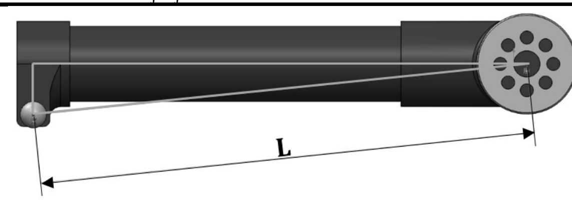

Figure 3. Effective arm length L.

6. FLARM and Driving Constraints

The simplified delta robot system presented in this study will not meet the exact motion of the typical system unless another 6 constraints are added to the system. These 6 constraints are concerned with the forearms to prevent the rotational motion about the longitudinal axes, these constraints lead to the same functionality of the six eliminated connecting rods. The Forearms Longitudinal Axis Rotational Motion (FLARM) constraints would be added to the constraints. A system is considered as kinematically driven when the number of constraints equations including the driving constraints is equal to the number of the coordinate's variables "q. This fact has a great advantage in real-time control routines, because the dynamics of the system are no longer necessary to determine the velocities and accelerations of the system coordinates, resulting in a faster computational procedure with a reduced computational cost. This is the case in the simplified MBS model of delta system presented in this study. The number of constraints equations without the driving constraints is 63, and this number changed to 66 when 3 driving constraints, which represent the velocity vector of the end effector V along the global Cartesian axes XYZ, were added. This number equals the number of coordinates present in the system.

7. Constraints Equations and Jacobian Matrix

The resulting constraints vector contains 66 constraints equations including the driving constraints. Equation 2 illustrates how the different system's constraints are structured in a constraints vector C(q,t), the constraints are classified into five main categories as indicated.

$$C(q^i,t) = [C_f \ \ C_{rev} \ \ C_{sph} \ \ C_{FALRM} \ \ C_{in}]^T = 0 \qquad (2)$$

The partial time derivative of equation 2 with respect to all coordinates of the system, results in equation 1 which is the Jacobian matrix "C, square matrix of 66x66 dimensions, multiplied by the vector of infinitesimal change of the system's coordinate "dq" equals to 0. Both independent and dependent coordinates are indexed inside the vector dq which means a separation of coordinates is needed for mapping the time derivatives of the dependent coordinates "dq" with the time derivatives of the independent coordinates "dq", this procedure of separation and portioning the Jacobian matrix follows. Partitioning is also advantageous for the efficiency of computations and reducing the processing time. where the subscript "n" is the number of the system's coordinates, "nd" is the number of dependent coordinates, and "n-n" is the number of independent coordinates. The core study of this work aims to track an object moving in space by a set of linear velocities. The end effector is assumed to move toward the target object, and this means that the independent coordinates of the system are the end effector linear velocities in the Cartesian space and all other "63" coordinates are the dependent coordinates to be con- trolled. The independent coordinates vector q is the partitioning procedure presented in this section was programed directly in the Jacobian subroutine, which resulting into two Jacobian matrices. This system of linear equations is in the form "AX=B" which is solved using MATLAB programing code by using the linear solver "lin-solve", matrix "A" represents the dependent Jacobian matrix "C", while vector "B" represents the multiplication Cqi.

8. Exact Instantaneous Dependent Coordinates Calculations

In this section the presented simplified MBS model of delta robot will be compared with another method [7]. This method, as mentioned above in the literature calculates the Jacobian matrix by using the minimum coordinates set, which only relates the components of the linear velocity of the end effector to the velocity of actuated revolute joints. This work aims to calculate the inverse kinematics, the inverse kinematics of delta robot. The angular velocities ω1, ω2 and ω3 calculated, these 3 velocities in the MBS method are extracted from the resulted vector dq using loop closure method, see Fig. 4.

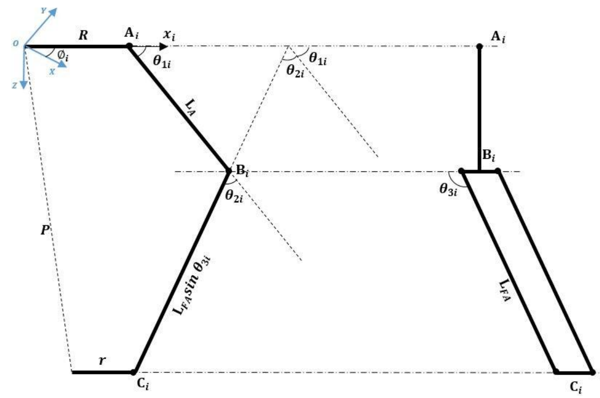

Fig. (4) Loop closure method.

9. The analysis and verification of the simplified MBS model

As mentioned above, the objective of this simplification is to provide a simplified kinematically driven MBS model to be applied in a real time control application in such that the desired motion of the end effector is maintained unchanged. In general, the mechanical topology of delta robots allows the end effector to move linearly along the 3 Cartesian coordinates while restricting the rotational motion about any of them. An arbitrary path motion was selected to verify the results, the path parameters was a linear path motion with a velocity of "0.8 m/s" and simulation time of "250 ms" al- lowed the end effector to travel "20 cm" starting from the home position and moving upward with an elevation view inclination angle of "45∘" and with a plan view inclination angle of "45∘". As shown in Fig. 5, the presented simplified MBS model succeeded to achieve this objective, however, results exhibit a great deviation in the angular velocities of the actuated revolute joints for the same desired linear motion behavior of the end effector. Figure 5 shows a correspondence between the angular velocities of the end effector "df" and dψ, which was represented by black and blue lines, respectively. The negative sign means that the angular velocity in an opposite direction, while the angular velocity dθ is maintained equals to θ. Referring to the convention of Euler angles followed in this work, and according to the graph, the end effector rotates about the current Z-axis by an angular velocity equals "df" followed by an angular rotation dψ of the same magnitude in an opposite direction about the same Z-axis, which is remained untransformed due the zero angular velocity θ . These 3 consecutive rotations result in a zero rotational transformation of the end effector in space which is equivalent to the typical motion of the end effector of the typical delta robot system.

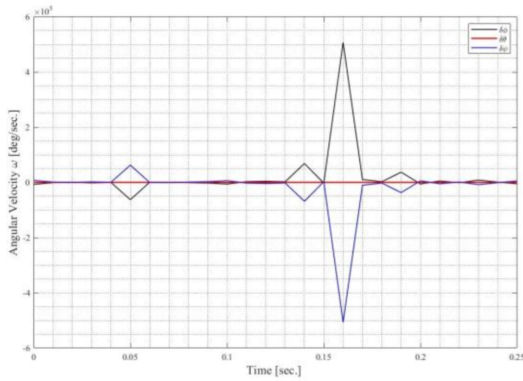

Fig. (5) Angular velocities of the end effector.

On the other hand, the simplified model failed in driving the actuated arms with the same velocities obtained by using the other models. Both typical MBS model results obtained from Adams simulation and the loop closure method results obtained from a MATLAB subroutine coincided with each other, but the results did not match with the presented simplified MBS model. It was found that the closer angle between the path of motion and the inclination axes of the kinematic chains, the closer results of driving velocities could be obtained. This means that by using the simplified model, there will be at least one kinematic chain which have a driving velocity to be close to the real system and there will be also at least one kinematic chain which exhibits a great deviation from the desired velocity. The test motion path had an angle of 45° in space. Hence, the results of the first chain drive which has an inclination angle of 30° are very close to the exact real results which is shown in Fig. 6, and for the other two chains which have inclination angles of 150° showed differences, see Fig. 7. The figures below illustrate the deviation between the angular velocities using the simplified MBS model compared to the exact velocities obtained by using the loop closure method [3], and the deviation between the angular velocities using the simplified MBS model compared with the velocities of the typical MBS model for the same path parameters afore mentioned. The results of the simplified model are represented by a black color, the results were compared to both the angular velocities obtained from the Adams simulation (solid red line) and the angular velocities obtained by using the loop closure method (dashed green line) which coincides with the angular velocities.

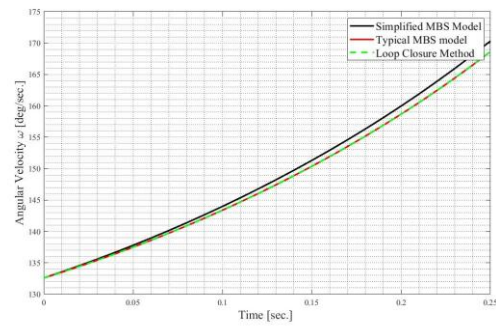

Fig. (6) Angular velocities of arm 1 (Arm angle: 30◦).

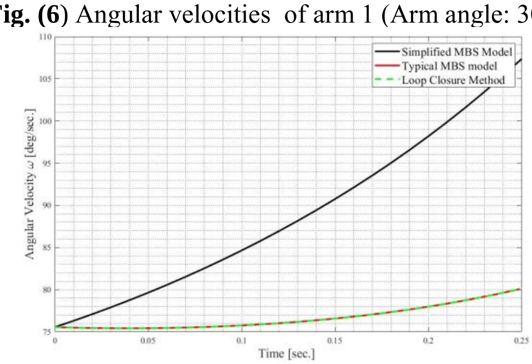

Fig. (7) Angular velocities of arm 2 (Arm angle:150◦).

In general, if the path motion had a plan view inclination angle equals to the inclination angle of one kinematic chain (e.g., 30°), then the driving velocity of that chain obtained by the simplified MBS model will completely agree with the other two methods, and in this case the 3 lines in graphs will coincide to each other and the end effector preserved a horizontal orientation within all the simulation time. The figures shows that the velocities of the second actuated arm (i.e. arm inclination angle is 150°) and at the same time the path motion had a plan inclination angle of the same magnitude, these 3 methods are completely identical and coincide to each other regardless the magnitude of the inclination angle of the elevation view. MBS model shows that the simplified model does not match the ex- act results. The first reason accused of causing this error is the simplification of the MBS model. During the simplification process, the connecting rods between the forearms were removed and new constraints were substituted. One possibility of not having the same desired velocities from the actuated arms is the new added FLARM constraints. Another possible source of errors is the sparse pattern of the Jacobian matrix C; this sparse pattern is very sensitive to the slight changes of the matrix values and causes errors in the results, see Fig.8. To overcome the deviation error and proceed using the simplified model, two necessary steps are important. First, reformulation of the constraints by an alternative way based on another perception instead of those considered in the presented model. Second, to obtain accurate and exact results, a precondition of the Jacobian matrix and iterative methods could be used rather than directly solving the linear system “Ax=B”. The simplified model needs more modifications to obtain better results, and this is what we will do in our going and future work

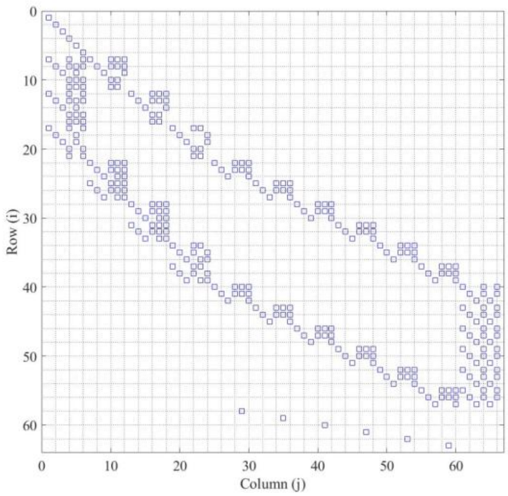

Fig. (8) Sparse pattern of the Jacobian matrix C.

CONCLUSIONS

This research built a simplified model of the Delta robot mechanical structure to obtain a kinematically driven MBS model in which the number of constraint equations is equal to the number of absolute coordinates. The advantage of the kinematically driven systems is the reduced computational cost and the faster response than the typical MBS model to be applied in a real time control application. The results of the simplified MBS model were compared with the results from another two methods which are the typical MBS model of the full system and the loop closure method, and both was identical to each other, but the simplified MBS model did not agree with them. The same motion behavior of the end effector was obtainable by using the simplified MBS model and was the same as the realistic behavior. Multibody system dynamics modeling was intended to be used because of its ability to describe the kinematics of all the bodies of a system, which is an important objective frequently demanded in the robotic dynamics field. This type of simplification is very important in the case of complex systems that require many coordinates, which causes a long solution period, which affects the quality of the results.

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