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Investigation of the Nuclear Structure of Even-Even $^{96\text{-}108}$Mo Isotopes

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ABSTRACT

Within the framework of the Interacting Boson Model (IBM-1), we try to examine the ability to characterize the nuclear structure of molybdenum isotopes in the range of $^{96–108}$ Mo. The goal of this study is to use the three basic dynamic symmetries incorporated into the model to investigate the nuclear structure of specific atomic nuclei. There is a significant agreement between the calculated energy levels and the experimental measurements for molybdenum isotopes. We also calculated and compared the probability for reduced electromagnetic transitions, specifically B(E2), with the model's theoretical results. Our research has shown that molybdenum isotopes have a wide range of characteristics. We found that there are different kinds of molybdenum isotopes with atomic numbers 96 to 108. Some of them have U(5) vibrational symmetry, while others have features that are somewhere between U(5) and SU(3) symmetries. These nuclei, commonly referred to as X(5) nuclei, are located in the transition region and exhibit shape coexistence.

1. INTRODUCTION

In recent decades, there has been a notable emphasis on the study of dynamical symmetries due to their remarkable ability to reveal the properties of complex systems. This has resulted in numerous notable findings across several branches of physics, including nuclear, molecular, hadronic, polymer, and nanostructure physics [1-4]. As a generalization and refinement of the exact symmetry concepts, dynamical symmetries provide substantial insight into the nature of complex systems.

The interacting boson model (IBM) is a theoretical framework that uses interacting bosons to explain collective nuclear states. The IBM models nuclei as a system of interacting bosons that depict the behavior of nucleons as a whole. A Hamiltonian that incorporates numerous symmetries is used by the bosons to interact [1]. Given that the Hamiltonian can be defined in terms of a group of generators that satisfy a particular Lie algebra, the IBM has the property of dynamical symmetry. The symmetry of the Lie algebra is where the Hamiltonian's symmetries originate, and because they do, they are referred regarded as dynamical symmetries [3]. The simplest version of this model is called IBM -1 and it does not distinguish between protons and neutrons degree of freedom. In this version of the model, the nucleus is assumed to consist of two types of bosons: s-bosons with $L = 0$ and d-bosons with $L = 2$. The number of bosons is calculated by how many nucleon pairs or hole pairs are required to maintain a closed shell [5,6].

The model may be used for the analysis of medium and heavy nuclei characterized by an even number of protons and an even number of neutrons. IBM has achieved significant progress in elucidating the characteristics of several atomic nuclei, especially those that exhibit collective phenomena like rotational and vibrational motion [7]. The IBM has limitations in its capacity to elucidate the behavior of nuclei at conditions of high excitation energy, whereby the influence of single-particle excitations becomes significant.

The emergence of collective phenomena may be attributed to the presence of amplitudes for many direct product states inside a single nuclear eigenstate, as well as the coherent accumulation of these

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components. Nuclear collectivity is often classified in terms of three benchmark models: the spherical vibrator [8], the axial symmetric rotor [9] and the γ -soft model [10,11]. These are idealized limits codified in the framework of the (IBM) [1,6] in terms of the $U(5)$, $SU(3)$, and $O(6)$ dynamical symmetries, respectively.

The IBM computation of energy levels produces the values of $R_{4/2} = E(4_1^+)/E(2_1^+)$, representing the ratio of excitation energies between the first 4^+ and the first 2^+ excited states. These values are determined to be 2.00, 3.33, and 2.50 for the dynamical symmetries $U(5), SU(3)$, and $O(6)$, respectively.

The purpose of this study is to examine the nuclear structure of molybdenum isotopes ranging from ^{96}M to ^{108}Mo. IBM calculations are conducted to determine the energy of the low-lying levels of these nuclei. It expected to have shape coexistence which occurs when multiple energy levels of a nucleus have comparable energies and, thus, distinct shapes. Consequently, the nucleus is capable of simultaneously existing in multiple states and forms.

2. A testation of $^{96\text{-}108}$Mo isotopes

In the course of time, several nuclei have been selected by different authors as vibrators [12-15], deformed rotor [6], and γ -unstable [10]. The candidate nucleus to be selected as one of the three types of the model if it satisfies all the following conditions:

i) It has at least nine levels with known spin and parity [17,18].

ii) All levels predicted by IBM-1 are observed within the sensitivity of the experiment performed.

iii) Levels outside the model space need to have reasonable explanations.

iv) satisfying the IBM-1 predictions for electromagnetic transitions.

v) The energy levels may be accurately represented by one of the IBM-1 formulas, namely equation (1) for vibrators, equation (2) for a deformed rotor, and equation (3) for γ -unstable nuclei.

$$E(U(5)) = \varepsilon n_d + \alpha n_d(n_d + 4) + 2\beta\tau(\tau + 3) + 2\gamma L(L + 1), \quad \textbf{(1)}$$

where n_d , τ and L are the quantum numbers for the number of d -bosons, the d -boson seniority, and the level spin, respectively. The factors ε , α , β and γ are adjustable parameters.

$$E(SU(3)) = E_0 - k[\lambda(\lambda + 3) + \mu(\mu + 3) + \lambda\mu - 2N(2N + 3)] + k'L(L + 1), \quad \textbf{(2)}$$

where λ, μ are the quantum numbers classifying the rotational states, N is the total number of bosons and L is the level spin, respectively, the factors k and k' are adjustable parameters. The formula obtained by IBM-1 for the so-called γ -unstable nuclei is

$$E(O(6)) = E_0 + \frac{A}{4}(N - \sigma)(N + \sigma + 4) + B\tau(\tau + 3) + CL(L + 1), \quad \textbf{(3)}$$

where σ , τ, N and L are the quantum numbers for the number of d-bosons, the d-boson seniority, the total number of bosons and the level spin, respectively. The factors A , B and C are adjustable parameters.

Table (1:) The IBM parameters for even-even molybdenum $^{96\text{-}108}$Mo isotopes.

		$U(5)$				$O(6)$			$SU(3)$	
	$R_{4/2}$	ε	α	β	γ	A	B	C	k	k'
^{96}Mo	2.092	897.458	-44.094	-3.930	8.238	220.967	68.660	20.864	16.841	46.827
^{98}Mo	1.918	1068.255	-77.197	5.721	5.894	201.630	68.845	19.810	14.507	45.603
^{100}Mo	2.121	560.970	-21.438	5.715	5.365	162.770	59.316	19.810	12.155	39.851
^{102}Mo	2.507	333.000	6.035	-0.157	3.665	112.593	47.769	12.099	7.376	29.849
^{104}Mo	2.917	71.16	61.980	-12.677	0.887		54.839	2.823	6.971	24.325
^{106}Mo	3.045	88.250	51.185	-12.404	2.324		45.793	5.527	5.937	22.721
^{108}Mo	2.924	55.999	49.133	-13.732	3.953		38.672	9.296	3.385	25.012

In Table 1, the adjustable model parameters $^{96-108}$Mo isotopes are presented. The determination of the degree of agreement between observed (experimental) and estimated (fitted) energy levels in each nucleus is achieved by using two relevant values.

One of the primary factors to be taken into consideration is the mean absolute deviation,

$$\Delta = \frac{1}{N_L}\sum_i^{N_L}\left|E_i^{ex} - E_i^{fit}\right|, \qquad \textbf{(4)}$$

where E_i^{exp} and E_i^{fit} are the experimental and best-fit energies in keV of the *i* th level while N_L is the number of levels. The second is the quality factor defined by

$$Q = \frac{W_i}{N_L - b}\sum_i\left(E_i^{exp} - E_i^{fit}\right)^2, \qquad \textbf{(5)}$$

Here, b represents the number of adjustable parameters, while $W_i = 0.01$ denotes the weighting factor selected to align with a uniform uncertainty of 10keV on the level energies [6, 12, 13]. Table 2. Present the quality factor Q and the absolute average deviation Δ for three sets of fitted energy levels for even-even molybdenum $^{96-108}$Mo isotopes.

3. P-Factor

Deformation and collectivity in nuclei are a result of configuration mixing. The primary driving force for configuration mixing is the interaction between valence protons and neutrons (p-n). Subsequently, a struggle between p and n tends to propel the structural development of nuclei. The determination of the collectivity in nuclei may be achieved by the utilization of valence p-n interactions. The P-factor, as stated in equation (6), provides a concise depiction of the p-n interaction [16].

$$P = \frac{N_n N_p}{N_n + N_p}, \qquad \textbf{(6)}$$

The numbers of valence protons and neutrons, denoted as N_p and N_n respectively, are expected to have a substantial impact on collectivity and the initiation of deformation as the P -factor increases [19-22].

4. Transition ratio

The B(E2) ratio for transition between low laying levels is a very sensitive tool in investigating the collective behavior of the nuclei [6,12,13]. we can qualify the collective nature of the selected nuclei by considered and comparing the ratio

$$R = \frac{B(E2;4_g^+ \to 2_g^+)}{B(E2;2_g^+ \to 0_g^+)}, \qquad \textbf{(7)}$$

the theoretical values for the three dynamical symmetry limits are:

1 – For the $U(5)$ limit :

$$R = \frac{2(N-1)}{N}, \qquad \textbf{(8)}$$

2-For the $SU(3)$ limit :

$$R = \frac{10}{7}\frac{(2N^2+3N-5)}{(2N^2+3N}, \qquad \textbf{(9)}$$

3-For the $O(6)$ limit :

$$R = \frac{10}{7}\frac{(N^2+4N-5)}{(N^2+4N}, \qquad \textbf{(10)}$$

The calculation of the $B(E2)$ transition ratio between low-laying levels is performed using the available experimental data for $^{96-108}$Mo isotopes. We compare the outcomes with the predictions of the three dynamical symmetry thresholds. The comparison results are presented in Table 3. In most cases, the agreement between the calculation and the experimental data is not satisfactory for the model. However, two nuclei are slightly closer to the data predictions of the models. Specifically, ^{100}Mo is close to the $U(5)$ limit, while ^{106}Mo is close to the $SU(3)$ limit with error 0.3, (see Table 3).

Table (2):The quality factor Q and the absolute average deviation Δ for three sets of fitted energy levels for even-even molybdenum $^{96-108}$Mo isotopes.

Nucleus			$U(5)$		$O(6)$		$SU(3)$	
	N_b	P	Δ	Q	Δ	Q	Δ	Q
^{96}Mo	6	2.67	130.7	314.5	286.0	1477.0	287.1	1370.8
^{98}Mo	7	3.43	140.4	473.8	340.3	1940.4	287.2	1356.8
^{100}Mo	8	4.00	**70.1**	**109.8**	203.3	714.5	245.4	916.4
^{102}Mo	9	4.44	109.8	273.3	163.8	465.0	**105.6**	**191.0**
^{104}Mo	10	4.8	**79.4**	**140.1**	**80.1**	**146.0**	**90.5**	**163.1**
^{106}Mo	11	5.09	95.2	156.3	84.2	139.6	137.2	283.9
^{108}Mo	12	5.33	**59.6**	**76.3**	**51.1**	**69.8**	**30.7**	**16.3**

Table (3): The comparison of the experimental data with calculations of B(E2) values for U(5), SU(3) and O(6) for even-even molybdenum $^{96\text{-}108}$Mo isotopes.

Nucleus	N_b	R_E	R $U(5)$	R $O(6)$	R $SU(3)$
^{96}Mo	6	2.0 ± 0.3	1.667	1.310	1.349
^{98}Mo	7	0.756 ± 0.015	1.714	1.336	1.369
^{100}Mo	8	1.84 ± 0.17	1.750	1.354	1.382
^{102}Mo	9	1.2 ± 0.3	1.778	1.368	1.391
^{104}Mo	10	1.20 ± 0.09	1.800	1.378	1.398
^{106}Mo	11	1.4 ± 0.3	1.818	1.385	1.403
^{108}Mo	12		1.833	1.391	1.407

5. DISCUSSION

In this study the isotopes of molybdenum ranging from ^{96}Mo to ^{108}Mo were investigated. The observed that a substantial variation in the $R_{4/2}$ value, which ranged from 1.92 to 3.05. This suggests that the nuclei undergo a transition from an oscillating state specified by the $U(5)$ symmetry to a state that closely resembles the $SU(3)$ symmetry. Figure 1 illustrates the correlation between the mass number and the $R_{4/2}$ ratio for molybdenum $^{96\text{-}108}$Mo isotopes. Consequently, there exist several isotopes that exhibit transitional properties between $U(5)$ and $SU(3)$, in accordance with the theoretical framework known as $X(5)$ [23, 24, 27]. This observation is supported by the data shown in Tables 2 to 8, as well as Figure 1.

The isotope ^{104}Mo has been verified to exhibit the properties of the transitional nucleus referred to as $X(5)$ nucleus [28]. This confirmation is based on the presence of both experimental and theoretical energy levels, see Table 7. Notably, the values of $R_{4/2} = 2.92$ and $P = 4.8$ associated with this isotope are deemed significant and are indicative of the $X(5)$ classification for this nucleus. Furthermore, both Δ and Q values are within the accepted limits for the three sets of basic theoretical models of the IBM-1, demonstrating that the nuclei can be described by the basic three dynamical symmetries in the model, which is a distinctive feature of transitional nuclei. Despite the reduced probability of quadrupole electric transition being 1.20 ± 0.09, which suggests the nuclei that belong to $SU(3)$, we agree with previous research studies that this is $X(5)$ nucleus.

For instance, the value of $\frac{E(6_1^+)}{E(2_1^+)} = 5.62$ from Table 4, as compared to $X(5)$ value of 5.43. Similarly, the value of $\frac{E(8_1^+)}{E(2_1^+)} = 8.956$, as compared to $X(5)$ value of 8.48. Finally, the value of $\frac{E(0_2^+)}{E(2_1^+)} = 4.61$, as compared to 5.65 of $X(5)$ model [27, 29].

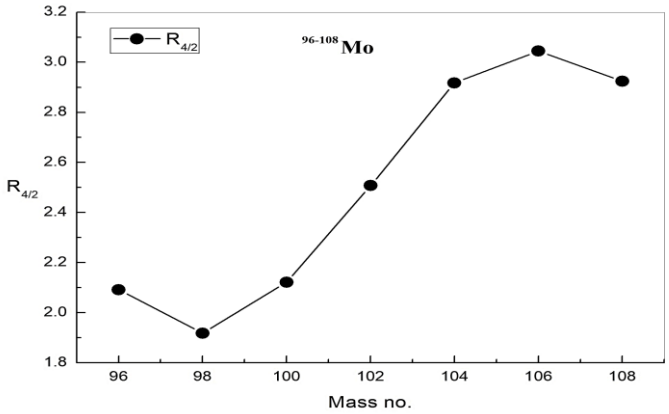

Fig. (1): The relation between the ratio $R_{4/2}$ of the molybdenum $^{96\text{-}108}$Mo isotopes and its mass number.

On the other hand, the nucleus ^{100}Mo can be described using $U(5)$ dynamical symmetry, as reflected by the values of Δ and Q within the acceptable limits indicating its dynamic symmetry, see Table 6. Furthermore, the value of $R_{4/2} = 2.12$ and the reduced transition probability is 1.84 ± 0.17, which is very close to the computed value of 1.75 using IBM-1, and the low value of the paring strength $P = 2.667$ as shown in Tables 2, and 4 which indicated low deformation and nearly spherical shape of this nucleus.

However, we cannot accurately describe both nuclei $^{96\text{-}98}$Mo using the first version of the interacting boson model, as shown clearly in Table 2 and Table 5 due to the high value of Δ and Q. For the nucleus ^{102}Mo, although, the moderately high values of Δ and Q, we are unable to describe this nucleus using the IBM-1. Although, Berun et al. [30] considered this nucleus to be deformed with O(6) dynamical symmetry, I didn't agree with this conclusion.

As for the ^{106}Mo nucleus, the value of $R_{4/2} = 3.045$, the coefficient $P = 5.09$, the values of $\frac{E(6_1^+)}{E(2_1^+)} = 6.023$,

$\frac{E(8_1^+)}{E(2_1^+)} = 9.841$, $\frac{E(0_2^+)}{E(2_1^+)} = 5.576$ and, the value of the reduced transition probability of pointing towards the possibility of following the critical point symmetry X(5), as shown in Table 4, and the low values of Δ and Q for U(5) and O(6) dynamic symmetries energy level mixing shown in Table 7.

Table (4): A comparison between the X(5) theoretical prediction and the experimental energy levels for $^{104\text{-}108}$Mo isotopes.

Levels ratio	$X(5)$	^{104}Mo	^{106}Mo	^{108}Mo
$\frac{E(4_1^+)}{E(2_1^+)}$	2.91	2.917	3.045	2.924
$\frac{E(6_1^+)}{E(2_1^+)}$	5.43	5.62	6.023	5.65
$\frac{E(8_1^+)}{E(2_1^+)}$	8.48	8.956	9.841	9.09
$\frac{E(0_2^+)}{E(2_1^+)}$	5.65	4.61	5.576	
$B_{4/2}$	1.58	1.20 ± 0.09	1.4 ± 0.3	
P	5	4.8	5.09	5.33

In the case of ^{108}Mo, the value of $R_{4/2} = 2.923$ and the coefficient $P = 5.33$ suggest the probability of following the the critical point symmetry $X(5)$ as shown in Table 4. However, the lack of practical data on the reduced transition probability makes confirming this difficult, in addition to the incomplete energy levels structure for the nucleus, thus rendering it challenging to reach a fair decision regarding its nature. Nonetheless, some ratios of energy levels in this nucleus agree with the expected values from the critical point symmetry $X(5)$, as in Table 4. Moreover, the Δ and Q values, see Table 8, fall within the recognized thresholds for the three sets of fundamental theoretical models of the IBM-1. This indicates that the nuclei may be accurately characterized by the three basic dynamical symmetries in the model, which is a unique characteristic of transitional nuclei. All this makes the nucleus favored for this transitional symmetry. Nevertheless, further practical studies on this nucleus are needed in the current situation to obtain more precise energy levels that enable us to resolve the matter.

Table (5): The experimental, the fitted energy levels, the quality factor Q, and the absolute average deviation for the fitted energy levels for $^{96\text{–}98}$Mo nuclei.

	^{96}Mo				^{98}Mo			
Levels	**Exp**	**U(5)**	**O(6)**	**SU(3)**	**Exp**	**U(5)**	**O(6)**	**SU(3)**
$E(2^+_1)$	778.2	744.4	399.8	281.0	787.4	798.8	394.2	273.6
$E(0^+_2)$	1148.1	1265.8	1235.9	1111.5	734.8	1210.2	1239.2	1131.5
$E(2^+_2)$	1497.8	1286.0	811.8	1392.5	1432.2	1395.3	807.3	1405.1
$E(4^+_1)$	1628.2	1516.7	1103.9	936.5	1510.0	1560.3	1084.7	912.1
$E(0^+_3)$	1330.0	1624.9	1546.8	1818.8	1963.1	1789.6	1613.0	1914.9
$E(2^+_3)$	1625.9	1833.8	2047.7	1392.5	1758.5	1700.1	2046.5	1405.1
$E(3^+_1)$	1978.5	1822.6	1486.3	1673.4	2104.7	1931.0	1476.9	1678.8
$E(4^+_2)$	1869.6	1954.4	1653.2	2048.0	2223.9	2025.3	1635.4	2043.6
$E(6^+_1)$	2440.8	2316.9	2112.2	1966.7	2343.6	2284.7	2071.2	1915.3
$E(2^+_4)$	2095.8	2057.6	1946.6	2099.8	1871.0	2193.8	2007.3	2188.5
$E(2^+_5)$	2426.1	2199.1	2358.6	2099.8	2206.6	1987.9	2420.4	2188.5
$E(4^+_3)$	2219.4	2288.2	2339.8	2048.0	2240.0	2358.8	2323.9	2043.6
$E(5^+_1)$	2438.5	2453.0	2548.4	2516.3	2506.6	2476.7	2522.0	2499.6
$E(6^+_2)$	2755.1	2650.7	2798.8	3078.2	2678.7	2618.2	2759.7	3046.9
$E(8^+_1)$	2978.4	3145.0	3424.7	3371.5	2854.2	2971.8	3354.0	3283.4
Δ		**130.7**	**286**	**287.1**		**140.4**	**340.3**	**287.2**
Q		**314.5**	**1477**	**1370.8**		**473.8**	**1940.4**	**1356.8**

Table (6): The same as table 5 for $^{100\text{-}102}$Mo nuclei

	^{100}Mo				^{102}Mo			
Levels	**Exp**	**U(5)**	**O(6)**	**SU(3)**	**Exp**	**U(5)**	**O(6)**	**SU(3)**
$E(2^+_1)$	535.6	563.9	356.1	239.1	296.6	379.3	276.3	186.2
$E(0^+_2)$	695.1	864.7	1067.7	1094.0	698.3	682.5	982.0	797.6
$E(2^+_2)$	1063.8	1043.4	712.0	1333.1	847.9	780.1	603.7	983.8
$E(4^+_1)$	1136.0	1193.6	989.4	797.0	743.7	852.7	739.3	620.5
$E(0^+_3)$	1504.7	1438.5	1464.9	1896.2	1334.0	1202.4	1103.9	1407.6
$E(2^+_3)$	1463.9	1342.8	1779.7	1333.1	1144.5	1140.5	1585.6	983.8
$E(3^+_1)$	1607.4	1567.2	1305.4	1572.2	1245.5	1264.6	1098.2	1169.9
$E(4^+_2)$	1771.4	1653.1	1463.9	1891.0	1398.4	1306.1	1175.7	1418.1
$E(6^+_1)$	1847.2	1889.1	1899.7	1673.8	1327.9	1420.3	1388.9	1303.1
$E(2^+_4)$	1766.5	1942.3	1821.1	2135.4	1249.7	1739.5	1380.3	1593.8
$E(2^+_5)$					1608.0	1619.9	1707.6	1593.8
$E(4^+_3)$	2103.1	2092.5	2057.1	1891.0	2239.0	1812.2	1721.3	1418.1
$E(5^+_1)$	2289.4	2199.8	2255.2	2289.5				
$E(6^+_2)$	2310.0	2328.6	2492.9	2767.7	2010.4	1926.3	1934.4	2100.7
$E(8^+_1)$	2627.2	2650.5	3087.2	2869.3	2018.8	2082.0	2225.1	2233.8
Δ		**70.1**	**203.3**	**254.4**		**109.8**	**163.8**	**105.6**
Q		**109.8**	**714.5**	**916.4**		**273.3**	**465**	**191**

Table (7): The same as table 5 for $^{104\text{-}106}$Mo nuclei

	^{104}Mo				^{106}Mo			
Levels	**Exp**	**U(5)**	**O(6)**	**SU(3)**	**Exp**	**U(5)**	**O(6)**	**SU(3)**
$E(2^+_1)$	192.19	290.295	236.292	145.951	171.549	272.83	216.333	136.328
$E(0^+_2)$	886.08	886.08	987.098	794.698	956.55	790.72	824.265	748.062
$E(2^+_2)$	812.36	643.198	565.325	940.65	710.48	570.531	491.088	884.39
$E(4^+_1)$	560.68	668.04	604.845	486.505	522.32	635.596	568.468	454.427
$E(0^+_3)$								
$E(2^+_3)$					1149.84	1268.29	1315.35	884.39
$E(3^+_1)$	1028.35	1080	1020.97	1086.6	885.17	948.872	890.591	1020.72
$E(4^+_2)$	1214.82	1094.2	1043.56	1281.2	1067.77	986.052	934.808	1202.49
$E(6^+_1)$	1079.97	1133.24	1105.66	1021.66	1033.34	1088.3	1056.41	954.297
$E(2^+_4)$								
$E(2^+_5)$								
$E(4^+_3)$	1583.5	1593.61	1591.94	1281.2	1434.73	1389.26	1392.73	1202.49
$E(5^+_1)$	1475.67	1611.36	1620.17	1524.46	1306.81	1435.74	1448	1429.7
$E(6^+_2)$	1724.39	1632.65	1654.05	1816.36	1563.25	1491.51	1514.33	1702.36
$E(8^+_1)$	1721.79	1685.88	1738.73	1751.42	1688.26	1630.93	1680.15	1635.94
Δ		**79.4**	**80.1**	**90.5**		**95.2**	**84.2**	**137.2**
Q		**140.1**	**146**	**163.1**		**156.3**	**139.6**	**283.9**

Table (8): The same as table 5 for ^{108}Mo nucleus

	^{108}Mo			
Levels	**Exp**	**U(5)**	**O(6)**	**SU(3)**
$E(2^+_1)$	192.791	250.991	210.463	150.07
$E(0^+_2)$				
$E(2^+_2)$	586.01	486.132	442.497	617.254
$E(4^+_1)$	563.69	624.235	572.634	500.234
$E(0^+_3)$				
$E(2^+_3)$				
$E(3^+_1)$	782.97	823.796	807.65	767.324
$E(4^+_2)$	978.29	902.712	882.014	967.418
$E(6^+_1)$	1090.7	1119.73	1086.51	1050.49
$E(2^+_4)$				
$E(2^+_5)$				
$E(4^+_3)$				
$E(5^+_1)$	1232.1	1323.17	1361.69	1217.53
$E(6^+_2)$	1507.8	1441.54	1473.24	1517.67
$E(8^+_1)$	1752.8	1737.48	1752.1	1800.84
Δ		**59.6**	**51.1**	**30.7**
Q		**76.3**	**69.8**	**16.3**

6. CONCLUSION

This study investigates the structural changes in the Molybdenum isotope chain $^{96-108}$Mo using the IBM-1 framework, which is commonly employed to analyze the structure of low-lying excited states in even-even nuclei. The analysis has concluded that the isotopes $^{104-108}$Mo fall between the vibrational $U(5)$ and axial deformed rotor SU(3) limits, indicating that they are located in the transition region $U(5)-SU(3)$. These isotopes may have characteristics of $X(5)$ symmetry while ^{100}Mo could be considered as $U(5)$ nucleus.

The IBM tool offers a valuable foundation for understanding the nuclear structure of molybdenum isotopes in low-lying excited states. It demonstrates the progressive transformation from spherical to deformed shapes as the excitation energy increases. Unfortunately, we have not been able to find enough experimental data to make fair conclusion for all isotopes under investigation.

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