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Treatment of Medical Cyclotrons Shielding Cost with Interval Programming

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ABSTRACT

The majority of radiation fields are synthesis of various radiation types. The most significant radiations include primary gamma rays, secondary gamma rays, thermal neutrons, and fast neutrons. These kinds of radiation are intended to be attenuated by thermo composite shielding materials. Modern medical cyclotrons that generate significant quantities of short-lived radioisotopes must be operated safely which requires effective radiation shielding. For the shielding design, a precise cost-benefit analysis is needed. For functions involving several variables, like the medical cyclotron shielding design's cost-benefit analysis, the traditional optimization method is complicated. Hence the cost-benefit analysis of a medical cyclotron has been formulated and solved using interval programming. The main objective of this paper is to find the thickness of the concrete shielding wall to provide the maximum radiation safety at the lowest operational and material costs. We formulate an interval shielding cost optimization problem of medical cyclotron. In this optimization problem the interval parameters (parameter is a constant may be vary) will be in both objective functions and constraints. We demonstrate that an interval optimization approach is a suitable method for the radiological shielding design of concrete vaults holding cyclotron targets to generate medicinal radioisotopes.

1. INTRODUCTION

A cyclotron is an instrument that causes charged particles to travel in a circular motion at an accelerated fluence, accelerating them to high speeds. It is the most component in the production of radioactive substances that are used in medicine as cancer treatments and diagnostic tracers. Shielding is the process of using materials to block radiation [1]. The safe operation of medical cyclotrons that generate significant quantities of short-lived radioisotopes depends on effective radiation shielding [2]. The best shielding design for medical cyclotrons must carefully balance radiological, economic, and frequently social issues, similar to the containment shielding of medical linear accelerators [3]. It is necessary to balance the costs of radiological health and radiation protection. The cost of radiation protection is affected by several factors as the type of ionizing radiation field generated by the cyclotron and its operational state. Also it depends on the shielding material cost, the dose reduction degree, and the estimated net profit from selling the radioactive materials. Numerous researchers have published on the mathematical techniques for accelerator shielding optimization [4,5] within the guiding principle of radiation safety ALARA (As Low As Reasonably Achievable) guidelines [6]. A metaheuristic optimization method based on genetic algorithm for radiation shielding design was presented by Zhenping Chen et al. [7]. Also Yao Cai et al. [8] presented optimization model of shielding design using metaheuristic for reducing neutrons and gamma rays dose equivalent. Zhenping Chen et al. [9] introduced radiation shielding design problem using genetic algorithm with multi-objective optimization strategies. A comparison of two multi-objective optimization methods for composite radiation shielding materials was introduced by Yao Cai et al.

For functions involving several variables, like the medical cyclotron shielding design's cost-benefit analysis, the traditional optimization method is overly complicated and prone to major errors. This research focuses on the target vault of a medical cyclotron's optimized shielding thickness calculation.

The coefficients of the problems are always handled as predictable numbers in conventional mathematical programming. The coefficient values, however, are frequently simply approximations in practice. Interval programming methods [11] have emerged as a popular

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tool for resolving choice problems with interval parameter values in order to get around these challenges. The boundaries of the uncertain coefficients are all that are needed in interval programming. Olivera et al. [12] have reviewed the methodological facets of interval programming research that has been done in the past.

In this study, we provide an optimization model with interval parameter for the radiation treatment problem of medical cyclotrons, since the cyclotron shielding thickness is described by an interval parameter. In reality, the interval decision, that enables various levels of control is excellent for these real-world problems.

2. NONLINEAR INTERVAL NUMBER PROGRAMMING CONCEPTS

We can consider general nonlinear interval number programming (NINP) issue with uncertain interval parameter in the objective function and restrictions is as follows:

$$\min f\left(X\right)$$

Subject to:

$$g_i\left(X, M\right) \geq (=, \leq)\left[v_i^L, v_i^R\right] \qquad i = 1, 2, \ldots, l$$

$$X \in \Psi^n, M_i = \left[M_i^L, M_i^R\right], \qquad i = 1, 2, \ldots, n$$

Where Ψ^n is the range of the n-dimensional decision vector X. The *i*th constraint and the objective function, respectively, are denoted by g_i and *f*. They are all continuous and nonlinear functions of *X*. $\left[M_i^L, M_i^R\right]$ is the interval number represents a bounded collection of real numbers between the boundaries. The superscripts *L* and *R* stand for an interval number's lower and upper bounds, respectively. *V* represents the *i*th constraint's allowed interval number. The objective function's or a constraint's potential values for each *X* will take the form of an interval rather than an actual number.

2.1 Treatment of the interval objective function

The uncertain objective function with interval numbers in interval mathematics can be split into two deterministic objective functions by doing the following [13]:

$$m\left(f\left(\bar{x}\right)\right) = \frac{1}{2}\left(f^L\left(\bar{x}\right) + f^R\left(\bar{x}\right)\right),$$

$$w\left(f\left(\bar{x}\right)\right) = \frac{1}{2}\left(f^L\left(\bar{x}\right) - f^R\left(\bar{x}\right)\right),$$

where *m* is called the midpoint value and *w* is called the radius of the interval number.

where $f^L\left(\bar{x}\right)$ and $f^R\left(\bar{x}\right)$ are given as follows:

$$f^L\left(\bar{x}\right) = \min f\left(\bar{x}\right) \text{ and } f^R\left(\bar{x}\right) = \max f\left(\bar{x}\right)$$

The multi objective optimization is handled using the linear combination approach [14]. This approach ensures that a limited number of Pareto solutions are obtained. Therefore, it is appropriate for these kinds of problems, where:

$$f_1\left(\bar{x}\right) = \alpha_1 m\left(f_1\left(\bar{x}\right)\right) + \alpha_2 w\left(f_1\left(\bar{x}\right)\right),$$

$$\alpha_1, \alpha_2 \geq 0,$$

$$\alpha_1 + \alpha_2 = 1 \cdot$$

2.2 Treatment of the interval constraints

The possibility degree of interval number indicates how much one interval number differs from another.

Let $g_i\left(\bar{x}\right) = \left[a_{ij}^L, a_{ij}^R\right] X_j \leq \left[v_i^L, v_i^R\right]$

then

$$\left[a_{ij}^L, a_{ij}^R\right] X_j - \left[v_i^L, v_i^R\right] \leq 0$$

The possibility degree can be described according to [13] as follows:

$$P_{(b \geq E)} = \begin{cases} 0 & b < E^L \\ \dfrac{b - E^L}{E^R - E^L} & E^L \leq b < E^R \\ 1 & b \geq E^R \end{cases}$$

where $E = \left[g_i^L\left(\bar{x}\right), g_i^R\left(\bar{x}\right)\right]$ is the *i*th constraint function's interval.

$P_{(b \geq E)} \geq \theta_i$ represents the *i*th constraint of the possibility degree since a predetermined possibility degree level is $0 \leq \theta_i \leq 1$, $i = 1, 2, \ldots, l$. We can convert the uncertain form into the certain form as follows by using the possibility degree concept.**:**

$$P_{\left(\left[a_{ij}^L, a_{ij}^R\right] X_j - \left[v_i^L, v_i^R\right] \leq 0\right)} \geq \theta_i \quad , \quad i = 1, 2, \ldots, l$$

then

$$\frac{0 - a_{ij}^L x_j + v_i^L}{\left(a_{ij}^R x_j - v_i^R\right) - \left(a_{ij}^L x_j - v_i^L\right)} \geq \theta_i \quad , \qquad i = 1, 2, \ldots, l$$

$$0 \leq \theta_i \leq 1$$

3. SHIELDING CALCULATIONS FOR THE RADIATION FIELD NEAR THE CYCLOTRON

In this study, the source terms were the experimentally [15] determined neutron (H_N) and gamma (H_G) dose equivalent rates at a distance of 1 meter from a thick copper plate attacked with 30 MeV protons:

$$H_N\left(Svh^{-1}\mu A^{-1}m^2\right) = 1.4 \pm 12\%$$

$$H_G\left(Svh^{-1}\mu A^{-1}m^2\right) = 0.11 \pm 11\%$$

These values will serve our mathematical model.

The total dose equivalent H_t (neutron & gamma) at the exterior of the concrete shielding is determined using the deterministic shielding calculation method [2] where:

$$H_t = 2.4I \exp\left(\frac{\left(-x/\lambda\right)}{(c+x)^2} \right) \quad \textbf{(1)}$$

where

I [μA] is the proton current beam on the target,

x [m] is the thickness of concrete shielding,

c [m] is the distance between the internal surface of vault wall and the target,

and λ [m] is concrete neutron attenuation length which equal to 0.126 m [16].

The fast evaporation neutrons slow down to thermal energy level after multiple scattering with the hydrogen atoms of the water molecules found in the concrete shielding. The neutrons are released from the thick copper target plate. The target vault's neutron fluence ϕ_n given by neutron.cm^{-2}. s^{-1} is stated as [17]:

$$\phi_n = \langle c \rangle Q/S \quad \textbf{(2)}$$

where

$\langle C \rangle$ = coefficient of fluence correction [17] = 4

Q [neutron. s^{-1}] = a target's total neutron production rate in seconds

S [cm^2] = the vault's total internal surface area.

4. THE OPTIMIZATION CALCULATIONS METHOD

The major objective of the optimization computation is to lower the overall cost, which is made up of the shielding cost and the radiological health cost. The function of cost for this optimization problem is expressed mathematically as follows:

$$C = X + Y \quad \textbf{(3)}$$

where C is the overall cost and is determined by engineering parameters, cyclotron operational parameters, and pertinent financial values. X represents the radiation protection cost, which includes the costs of radiological shielding, real estate, and the management of radioactive waste. The engineering parameter and pertinent financial values determine this cost. Y is the radiological health cost that depends on operational cyclotron parameters and financial pertinent values [4].

Equation (3) is referred to as the objective function, and it is constrained by the following:

$$H_t \le H_L \quad \textbf{(4)}$$

where H_t is the total dose equivalent (mSv/year) at the exterior of the concrete shielding delivered to individuals at contact with the shielding thickness. and H_L is equivalent limit of the permissible average collective dose (mSv/year) [6].

4.1 Evaluation of the radiation protection cost

The following formulas are used to determine the area of total internal wall surface (S) of the target vault wall, total surface area (F), and net volume (V) of the shielding concrete.

$$V\left[m^3\right] = \left((a+x).(b+x) - ab\right)h \quad \textbf{(5)}$$

$$F\left[m^2\right] = (a+x).(b+x) \quad \textbf{(6)}$$

$$S\left[m^2\right] = 2(ab + ah + bh) \quad \textbf{(7)}$$

where a[m] = the target vault length,

b [m] = the target vault breadth,

h [m]= the target vault height,

x = thickness of shielding.

Then the shielding cost (C_S), real estate (C_E) and disposal of radioactive waste (C_W) are determined as follows:

$$C_S [\$] = Vs \quad \textbf{(8)}$$

When we insert the value of V from equation (5) into equation (8) we obtain

$$C_S [\$] = \left((a+x).(b+x) - ab\right)hs \quad \textbf{(9)}$$

since s = cost of 1 m^3 of the concrete shielding ($).

$$C_E [\$] = Fl \quad \textbf{(10)}$$

When we insert the value of F from equation (6) into equation (10) we obtain

$$C_E [\$] = (a+x).(b+x)l \quad \textbf{(11)}$$

since l = real estate rate of 1 m^2 of floor space ($).

$$C_W [\$] = A_{SAT}P \quad \textbf{(12)}$$

where A_{SAT} is the saturation activity of ^{59}Fe that is existing in the target station's structure steel and P is radioactive waste disposal cost [$GBq-1]· A_{SAT} of ^{59}Fe is computed as:

$$A_{SAT}[GBq] = N\phi_n\sigma \quad \textbf{(13)}$$

where N = total amount atoms of ^{59}Fe in the steel structure, ϕ_n = rate of neutron fluence, σ = cross section of reaction $^{58}Fe[n,\gamma]^{59}Fe$ = $1.15\times10^{-24}\left[\mathrm{cm}^2\right]$ [18].

The total amount atoms of ^{58}Fe (N) could be detemined from the steel structure's weight and the abundance of isotopic of ^{58}Fe

$$N = (0.01KL).\left(\frac{W}{A_{Fe}}\right) \quad \textbf{(14)}$$

where K = abundance of isotopic of ^{58}Fe =0.28 %,

L = Avogadro's number = $6.022\text{x}10^{23}$ atom.mol^{-1}

A_{Fe} = Iron atomic weight = 55.85.

The target current could be used to express the rate of neutron fluence ϕ_n [neutron.cm^{-2}.s^{-1}] given in equation(2) so:

$$\phi_n = \langle c\rangle qI/S \quad \textbf{(15)}$$

where q = rate of neutron production of target's solid copper = 6.4 x 10^{10} (neutron. μA^{-1}) [2].

According to the numerical values of K, c, q, L, σ and A_{Fe}, equations (14) and (15), then the calculation of A_{SAT} of ^{59}Fe in equation (13) can be written as:

$$A_{SAT}\,(GBq) = 9.83IW/S \quad \textbf{(16)}$$

So A_{SAT} value from equation (16) and S value from equation (7) can be substituted in equation (12) and we obtain:

$$C_W\,[\$] = 4.92IWP/(ab+ah+bh) \quad \textbf{(17)}$$

where P = waste disposal cost [\$/GBq] for the activated ^{59}Fe.Then, the radiation protection total cost X is calculated as:

$$X(\$) = (C_S + C_E + C_W) \quad (\$)$$

Then

$$X(\$) = ((a+x).(b+x)-ab)hs + (a+x).(b+x)\ l + 4.92IWP/(ab+ah+bh) \quad \textbf{(18)}$$

4.2 Evaluation of the radiological health cost

The following mathematical formula can be used to express the cost of radiological health:

$$Y = \beta\ \Lambda(x) \quad \textbf{(19)}$$

where

β = unit collective dose cost for radiation protection (\$/person.Sv).

Λ(x) = equivalent collective dose (μSv) for the shield thickness x.

The equivalent collective dose $\Lambda(x)$ can be written as follows[2]:

$$\Lambda(x) = 2.4\rho\eta\tau NTI\left(\exp\left(\left(-x/\lambda\right)\Big/(0.5a+x)^2\right)\right) \quad \textbf{(20)}$$

where

ρ = ratio of the average permissible dose equivalent H_L and maximum dose equivalent H_t,

η = occupancy factor,

τ = operation factor of cyclotron =8760 h/y,

N = number of people exposed,

T = the shielding installation expected life (y),

I = current of proton at target (μA).

So according to the value of $\Lambda(x)$ in equation (20), the total financial value of the radiological health is expressed as follows:

$$Y[\$] = 2.1\beta\rho\eta NTI\left(\exp\left(\left(-x/\lambda\right)\Big/(0.5a+x)^2\right)\right)\times 10^4 \quad \textbf{(21)}$$

Then by substituting X from equation (18) and Y from equation (21) in equation (3) we have:

$$C = ((a+x).(b+x)-ab)hs + (a+x).(b+x)l + 4.92IWP/(ab+ah+bh) + 2.1\beta\rho\eta NTI\left(\exp\left(\left(-x/\lambda\right)\Big/(0.5a+x)^2\right)\right)\times 10^4 \quad \textbf{(22)}$$

5. INTERVAL OPTIMIZATION MODEL FOR SHIELDING COST

Now, we can formulate our interval optimization model as follows:

$$c(x) = \min\left((a+\tilde{x}).(b+\tilde{x})-ab\right)hs + (a+\tilde{x}).(b+\tilde{x})\,l + 4.92IWP/(ab+ah+bh) + 2.1\beta\rho\eta NTI\left(\exp\left(\left(-\tilde{x}/\lambda\right)\Big/(0.5a+\tilde{x})^2\right)\right)\times 10^4 \quad \textbf{(23)}$$

subject to

$$\widetilde{H}_t \le \widetilde{H}_L \quad \textbf{(24)}$$

$$\tilde{x} \ge 0 \quad \textbf{(25)}$$

By replacing H_t from equation (1) then equation (24) can be written as:

$$2.4\,I\,\exp\left(\left(-\tilde{x}/\lambda\right)\Big/(c+\tilde{x})^2\right) \le \widetilde{H}_L \quad \textbf{(26)}$$

where $\tilde{x} = \left[x^L, x^R\right]$ represent interval medical cyclotron thickness shielding in the objective function and constraint. Also $\widetilde{H}_L = \left[H_L^L, H_L^R\right]$ is the interval permissible average collective dose equivalent limit (mSv/year)

Therefore, problem (23) - (25) can be written as following:

$$c(x) = \min \left[\begin{array}{l} \left(\left(a+\left[x^L,x^R\right]\right)\left(b+\left[x^L,x^R\right]\right)-ab\right).hs+\left(a+\left[x^L,x^R\right]\right)\left(b+\left[x^L,x^R\right]\right).l+ \\ 4.92IWP\big/\left(ab+ah+bh\right)+\ 2.1\beta\rho\eta NTI\left(\exp\left(\left(-\left[x^L,x^R\right]\big/\lambda\right)\Big/\left(0.5a+\left[x^L,x^R\right]\right)^2\right)\right)\times 10^4 \end{array} \right] \quad \textbf{(27)}$$

subject to

$$2.4I\exp\left(\left(-\left[x^L,x^R\right]\big/\lambda\right)\Big/\left(c+\left[x^L,x^R\right]\right)^2\right) \le \left[H_L^L, H_L^R\right] \quad \textbf{(28)}$$

$$x^L, x^R \ge 0 \quad \textbf{(29)}$$

By applying the interval treatment technique that was described in section 2, the deterministic optimization model of shielding cost (27) – (29) will be as follow:

$$\begin{array}{l} \min c(x) = \dfrac{1}{2}\alpha_1 \left[\begin{array}{l} \left(\left(a+x^L\right)\left(b+x^L\right)-ab\right).hs+\left(a+x^L\right)\left(b+x^L\right).l+4.92IWP\big/\left(ab+ah+bh\right)+ \\ 2.1\beta\rho\eta NTI\left(\exp\left(\left(-x^L\big/\lambda\right)\Big/\left(0.5a+x^L\right)^2\right)\right)\times 10^4+\left(\left(a+x^R\right)\left(b+x^R\right)-ab\right).hs+ \\ \left(a+x^R\right)\left(b+x^R\right).l+4.92IWP\big/\left(ab+ah+bh\right)+\ 2.1\beta\rho\eta NTI\left(\exp\left(\left(-x^R\big/\lambda\right)\Big/\left(0.5a+x^R\right)^2\right)\right)\times 10^4 \end{array} \right]+ \\ \dfrac{1}{2}\alpha_2 \left[\begin{array}{l} \left(\left(a+x^R\right)\left(b+x^R\right)-ab\right).hs+\left(a+x^R\right)\left(b+x^R\right).l+4.92IWP\big/\left(ab+ah+bh\right)+ \\ 2.1\beta\rho\eta NTI\left(\exp\left(\left(-x^R\big/\lambda\right)\Big/\left(0.5a+x^R\right)^2\right)\right)\times 10^4-\left(\left(a+x^L\right)\left(b+x^L\right)-ab\right).hs- \\ \left(a+x^L\right)\left(b+x^L\right).l-4.92IWP\big/\left(ab+ah+bh\right)-\ 2.1\beta\rho\eta NTI\left(\exp\left(\left(-x^L\big/\lambda\right)\Big/\left(0.5a+x^L\right)^2\right)\right)\times 10^4 \end{array} \right] \end{array} \quad \textbf{(30)}$$

subject to

$$\frac{-2.4I\exp\left(\left(-x^L\big/\lambda\right)\Big/\left(c+x^L\right)^2\right)+H_L^L}{2.4I\exp\left(\left(-x^R\big/\lambda\right)\Big/\left(c+x^R\right)^2\right)-H_L^R-2.4I\exp\left(\left(-x^L\big/\lambda\right)\Big/\left(c+x^L\right)^2\right)+H_L^L} \ge \theta \quad \textbf{(31)}$$

$$x^L, x^R \ge 0 \quad \textbf{(32)}$$

$$\alpha_1, \alpha_2 \ge 0 \quad \textbf{(33)}$$

$$\alpha_1 + \alpha_2 = 1 \quad \textbf{(34)}$$

$$0 \le \theta \le 1 \quad \textbf{(35)}$$

Definition. Points (x^{L*}, x^{R*}) which satisfy the set of constraints (31)-(35) are considered to be the optimal solutions of problem (30)-(35), if and only if there are no other solutions (x^{L}, x^{R}) that satisfy the set of constraints (31)-(35) such that:

$$\frac{1}{2}\alpha_1\left[\begin{array}{l}\left(\left(a+x^L\right)\left(b+x^L\right)-ab\right).hs+\left(a+x^L\right)\left(b+x^L\right).l+4.92IWP\Big/\left(ab+ah+bh\right)+\\ 2.1\beta\rho\eta NTI\left(\exp\left(\left(-x^L\Big/\lambda\right)\Big/\left(0.5a+x^L\right)^2\right)\right)\times 10^4+\left(\left(a+x^R\right)\left(b+x^R\right)-ab\right).hs+\\ \left(a+x^R\right)\left(b+x^R\right).l+4.92IWP\Big/\left(ab+ah+bh\right)+\ 2.1\beta\rho\eta NTI\left(\exp\left(\left(-x^R\Big/\lambda\right)\Big/\left(0.5a+x^R\right)^2\right)\right)\times 10^4\end{array}\right]+$$

$$\frac{1}{2}\alpha_2\left[\begin{array}{l}\left(\left(a+x^R\right)\left(b+x^R\right)-ab\right).hs+\left(a+x^R\right)\left(b+x^R\right).l+4.92IWP\Big/\left(ab+ah+bh\right)+\\ 2.1\beta\rho\eta NTI\left(\exp\left(\left(-x^R\Big/\lambda\right)\Big/\left(0.5a+x^R\right)^2\right)\right)\times 10^4-\left(\left(a+x^L\right)\left(b+x^L\right)-ab\right).hs-\\ \left(a+x^L\right)\left(b+x^L\right).l-4.92IWP\Big/\left(ab+ah+bh\right)-\ 2.1\beta\rho\eta NTI\left(\exp\left(\left(-x^L\Big/\lambda\right)\Big/\left(0.5a+x^L\right)^2\right)\right)\times 10^4\end{array}\right]\geq$$

$$\frac{1}{2}\alpha_1\left[\begin{array}{l}\left(\left(a+x^{L*}\right)\left(b+x^{L*}\right)-ab\right).hs+\left(a+x^{L*}\right)\left(b+x^{L*}\right).l+4.92IWP\Big/\left(ab+ah+bh\right)+\\ 2.1\beta\rho\eta NTI\left(\exp\left(\left(-x^{L*}\Big/\lambda\right)\Big/\left(0.5a+x^{L*}\right)^2\right)\right)\times 10^4+\left(\left(a+x^{R*}\right)\left(b+x^{R*}\right)-ab\right).hs+\\ \left(a+x^{R*}\right)\left(b+x^{R*}\right).l+4.92IWP\Big/\left(ab+ah+bh\right)+\ 2.1\beta\rho\eta NTI\left(\exp\left(\left(-x^{R*}\Big/\lambda\right)\Big/\left(0.5a+x^{R*}\right)^2\right)\right)\times 10^4\end{array}\right]+$$

$$\frac{1}{2}\alpha_2\left[\begin{array}{l}\left(\left(a+x^{R*}\right)\left(b+x^{R*}\right)-ab\right).hs+\left(a+x^{R*}\right)\left(b+x^{R*}\right).l+4.92IWP\Big/\left(ab+ah+bh\right)+\\ 2.1\beta\rho\eta NTI\left(\exp\left(\left(-x^{R*}\Big/\lambda\right)\Big/\left(0.5a+x^{R*}\right)^2\right)\right)\times 10^4-\left(\left(a+x^{L*}\right)\left(b+x^{L*}\right)-ab\right).hs-\\ \left(a+x^{L*}\right)\left(b+x^{L*}\right).l-4.92IWP\Big/\left(ab+ah+bh\right)-\ 2.1\beta\rho\eta NTI\left(\exp\left(\left(-x^{L*}\Big/\lambda\right)\Big/\left(0.5a+x^{L*}\right)^2\right)\right)\times 10^4\end{array}\right]$$

6. APPLICATION OF INTERVAL PROGRAMMING IN SHIELDING THICKNESS OPTIMIZATION

Considering the financial values of the radiation protection and radiological health (β, s, l, P), engineering design parameters (a, b, h, λ, T, W), and cyclotron radiological and operational parameters (N, η, I, ρ) that were used in this optimization study. The current optimization calculation's primary goal is to determine the concrete shielding wall thickness to achieve highest radiation safety at least operating and material costs by changing the parameters of engineering design.

Table 1 summarizes the classification and possible values for the input parameters utilized in the shielding optimization study:

Table (1): shows the input parameter values used during the optimization procedure [2].

Item	Description (unit)	value
β	Radiation protection cost [$/person.Sv]	400000
s	Shielding concrete cost [$m-3]	300
l	Real estate cost (surface area) [$m-2]	1000
P	Radioactive waste disposal cost [$GBq-1]	100
a	The vault length [m]	5-10
b	The vault breadth [m]	5-10
h	The vault height [m]	5-10
λ	Concrete neutron attenuation length [m]	0.126
T	Expected life of the shielding [y]	20-50
W	Weight of iron parts [kg]	50-200
N	Number of people exposed	10-50
η	Occupancy Factor	0.2-1
I	Current of proton beam [µA]	100-400
ρ	Allowable Dose Equivalent / Maximum Dose Equivalent	2-1

Win QSB program [19] was used to perform the optimization computation. It was done using a computer with 2.1 MHz Pentium processor and 4 MB of RAM. The significant results for various values of $\rho = 0.4, 0.8, 1$, $\eta = 0.2, 0.4, 0.7$, $\theta = 0.2, 0.7, 0.8$ and $\alpha_1 = 0.4, \alpha_2 = 0.6$, are displayed in tables 2, 3, and 4 for a proton beam current of 400 A.

Table (2): The outcomes of the genetic approach's optimization shielding calculations [2].

Group 1			Group 2		
$N = 20, T = 25\text{y}, W = 100\text{Kg}$ $H_L = 1\text{mSv/y}, \eta = 0.2$			$N = 15, T = 25\text{y}, W = 120\text{Kg}$ $H_L = 0.5\text{mSv/y}, \eta = 0.4$		
ρ	$x(\text{m})$	$X(\$)$	ρ	$x(\text{m})$	$X(\$)$
0.4	2.96	3.823 E+05	0.8	2.99	3.851 E+05

Group 3		
$N = 10, T = 25\text{y}, W = 200\text{Kg}$ $H_L = 0.8\text{mSv/y}, \eta = 0.7$		
ρ	$x(\text{m})$	$X(\$)$
1	3.18	3.853 E+05

Table (3:) The outcomes of the fuzzy approach's optimization shielding calculations [1].

Group 1				Group 2			
$N = 20, T = 25\text{y}, W = 100\text{Kg}$ $H_L = 0.2\text{mSv/y}, \eta = 0.2$				$N = 15, T = 25\text{y}, W = 120\text{Kg}$ $H_L = 0.5\text{mSv/y}, \eta = 0.4$			
α	ρ	$x(\text{m})$	$X(\$)$	α	ρ	$x(\text{m})$	$X(\$)$
0.2	0.4	2.9	3.815 E+05	0.7	0.8	3.05	3.850 E+05

Group 3			
$N = 10, T = 25\text{y}, W = 200\text{Kg}$ $H_L = 0.8\text{mSv/y}, \eta = 0.7$			
α	ρ	$x(\text{m})$	$X(\$)$
0.8	1.0	3.18	3.851 E+05

Table (4): The outcomes of the proposed approach's optimization shielding calculations.

Group 1						Group 2					
$N = 20, T = 25\text{y}, W = 100\text{Kg}$ $H_L = [0.2, 0.8]\text{mSv/y}, \eta = 0.2$						$N = 15, T = 25\text{y}, W = 120\text{Kg}$ $H_L = [0.2, 0.8]\text{mSv/y}, \eta = 0.4$					
α_1	α_2	θ	ρ	$\tilde{x}(\text{m})$	$X(\$)$	α_1	α_2	θ	ρ	$\tilde{x}(\text{m})$	$X(\$)$
0.4	0.6	0.2	0.4	[0 , 2.2918]	1.50173E+05	0.4	0.6	0.7	0.8	[0 , 2.2918]	1.71165E+05

Group 3					
$N = 10, T = 25\text{y}, W = 200\text{Kg}$ $H_L = [0.2, 0.8]\text{mSv/y}, \eta = 0.7$					
α_1	α_2	θ	ρ	$\tilde{x}(\text{m})$	$X(\$)$
0.4	0.6	0.8	1	[0 , 2.2918]	2.55133E+05

It is evident from tables 2, 3, and 4 that our interval strategy outperforms Mukherjee's [2] genetic and Samir's [1] fuzzy approaches in terms of the results of objective function values. Objective function values of Mukherjee's [2] are 3.823 E+05, 3.851 E+05 and 3.853 E+05 while objective function values of Samir's [1] are 3.815 E+05, 3.850 E+05 and 3.851 E+05. In the proposed approach the objective function values are 1.50173E+05, 1.71165E+05 and 2.55133E+05. Since the genetic technique looks for a "near optimal" answer instead of the optimal one, which nearly never coincides with the optimal solution in most circumstances. Also for fuzzy approach specifying an appropriate membership function is sometimes difficult. The proposed approach demands the optimal solution rather than a "near optimal" in genetic approach.

7.CONCLUSION

In this paper, we have demonstrated that an interval optimization approach is a suitable method for the radiological shielding design of concrete vaults holding cyclotron targets to generate medicinal radioisotopes. Additionally, we have demonstrated how interval programming works well in situations when there are various treatment options for each value of $(\alpha_1, \alpha_2, \theta)$. For the purpose of resolving this problem we suggested that the cyclotron shielding thickness is an interval parameter. The advantage of this strategy is that the problem is made simpler and the illustration is more accurate and useful. This is due to the fact that we are handling a problem that is uncertain and poorly defined. The proposed approach demanded the optimal solution rather than a "near optimal" in genetic approach. So that an interval optimization approach is a suitable and more accurate method for the radiological shielding design of medical cyclotron. Future studies must apply an optimization strategy to choose the shielding thickness as rough interval parameter. Also kind of shielding materials that used to construct medical devices requires further investigation using an optimization method.

DATA AND MATERIALS AVAILABILITY

All the data's available in the manuscript.

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