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ORIGINAL ARTICLE

Relations for moments of k-th record values from exponential-Weibull lifetime distribution and a characterization

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Abstract In this note we give some recurrence relations satisfied by single and product moments of k-th upper record values from the exponential-Weibull lifetime distribution. Using a recurrence relation for single moments we obtain a characterization of exponential-Weibull, Weibull, exponential, Rayleigh and two parameter linear failure rate distributions.

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1. Introduction

Let $\{X_n, n \geqslant 1\}$ be a sequence of independent and identically distributed (*iid*) random variables with distribution function (*df*) $F(x)$ and probability density function (*pdf*) $f(x)$. The j-th order statistic of a sample $X_1, X_2, \ldots, X_n$ is denoted by $X_{j:n}$. For a fixed $k \geqslant 1$ we define the sequences $\{U_n^{(k)}, n \geqslant 1\}$ of k-th upper record times of $\{X_n, n \geqslant 1\}$ as follows:

$$U_1^{(k)} = 1$$

$$U_{n+1}^{(k)} = \min\left\{j > U_n^{(k)} : X_{j:j+k-1} > X_{U_n^{(k)}:U_n^{(k)}+k-1}\right\}.$$

For $k = 1$ and $n = 1, 2, \ldots,$ we write $U_n^{(1)} = U_n$. Then $\{U_n, n \geqslant 1\}$ is the sequence of record times of $\{X_n, n \geqslant 1\}$. The sequence $\{Y_n^{(k)}, n \geqslant 1\}$, where $Y_n^{(k)} = X_{U_n^{(k)}}$ is called the sequence of k-th upper record values of $\{X_n, n \geqslant 1\}$. For convenience, we shall also take $Y_0^{(k)} = 0$. Note that for $k = 1$ we have $Y_n^{(1)} = X_{U_n}, n \geqslant 1$, which are the record values of $\{X_n, n \geqslant 1\}$ (Ahsanullah [1]). Moreover, we see that $Y_1^{(k)} = \min(X_1, X_2, \ldots, X_k) = X_{1:k}$. Then the *pdf* of $Y_n^{(k)}$ and the joint *pdf* of $Y_m^{(k)}$ and $Y_n^{(k)}$ are as follows:

$$f_{Y_n^{(k)}}(x) = \frac{k^n}{(n-1)!}[-\ln \overline{F}(x)]^{n-1}[\overline{F}(x)]^{k-1} f(x), \quad n \geqslant 1, \qquad (1.1)$$

$$f_{Y_m^{(k)}, Y_n^{(k)}}(x, y) = \frac{k^n}{(m-1)!(n-m-1)!}[-\ln \overline{F}(x)]^{m-1} \times \frac{f(x)}{\overline{F}(x)}[\ln \overline{F}(x) - \ln \overline{F}(y)]^{n-m-1} \times [\overline{F}(y)]^{k-1} f(y), \quad x < y, \quad 1 \leqslant m < n, \quad n \geqslant 2, \qquad (1.2)$$

where $\overline{F}(x) = 1 - F(x)$.

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(Dziubdziela and Kopociński [2], Grudzień [3]).

Record values arise naturally in many real life applications involving data related to economic, sports, weather and life testing problems. The statistical study of record values started with Chandler [4] and has now spread in various directions. The various developments on record values and related topics are extensively studied in the literature. See for instance Glick [5], Nevzorov [6], Resnick [7], Nagaraja [8], Balakrishnan and Ahsanullah [9,10], Grudzień and Szynal [11], Arnold et al. [12], Pawlas and Szynal [13,14], Raqab and Ahsanullah [15], Saran and Pushkarna [16], Saran and Singh [17].

In this paper we establish some recurrence relations for single and product moments of k-th upper record values from exponential-Weibull lifetime distribution. These relations are deduced for moments of record values. Further, two theorems for characterizing this distribution based on a recurrence relation for single moments of k-th upper record values are given.

A random variable X is said to have exponential-Weibull lifetime distribution (Cordeiro et al. [18]) if its *pdf* is of the form

$$f(x) = (\alpha + \beta\gamma x^{\gamma-1})e^{-(\alpha x+\beta x^{\gamma})}, \quad x > 0, \quad \alpha > 0, \quad \beta > 0, \quad \gamma > 0 \tag{1.3}$$

and the corresponding *df* is

$$F(x) = 1 - e^{-(\alpha x+\beta x^{\gamma})}, \quad x > 0, \quad \alpha > 0, \quad \beta > 0, \quad \gamma > 0. \tag{1.4}$$

It is easy to see that

$$f(x) = (\alpha + \beta\gamma x^{\gamma-1})\overline{F}(x), \tag{1.5}$$

where

$$\overline{F}(x) = 1 - F(x).$$

It is observed in Cordeiro et al. [18] that the exponential-Weibull distribution can be used as an alternative model to other models available in the literature for modelling positive real data in many areas such as engineering, survival analysis, hydrology and economics.

The relation in (1.5) will be exploited in this paper to derive some recurrence relations for the moments of k-th upper record values from the exponential-Weibull distribution and to give a characterization of the exponential-Weibull distribution.

2. Relations for single moments

Theorem 2.1. *Fix a positive integer $k \geqslant 1$, for $n \geqslant 1$ and $j = 0, 1, \ldots,$*

$$E\left(Y_n^{(k)}\right)^j = \frac{\alpha k}{j+1}\left[E\left(Y_n^{(k)}\right)^{j+1} - E\left(Y_{n-1}^{(k)}\right)^{j+1}\right] + \frac{\beta\gamma k}{j+\gamma}\left[E\left(Y_n^{(k)}\right)^{j+\gamma} - E\left(Y_{n-1}^{(k)}\right)^{j+\gamma}\right]. \tag{2.1}$$

Proof. For $n \geqslant 1$ and $j = 0, 1, \ldots,$ we have from (1.1) and (1.5)

$$E\left(Y_n^{(k)}\right)^j = \frac{k^n}{(n-1)!}\left\{\alpha\int_0^{\infty} x^j[-\ln\overline{F}(x)]^{n-1}[\overline{F}(x)]^k dx + \beta\gamma\int_0^{\infty} x^{j+\gamma-1}[-\ln\overline{F}(x)]^{n-1}[\overline{F}(x)]^k dx\right\} = (\alpha I_j + \beta\gamma I_{j+\gamma-1}), \tag{2.2}$$

where

$$I_r = \frac{k^n}{(n-1)!}\int_0^{\infty} x^r[-\ln\overline{F}(x)]^{n-1}[\overline{F}(x)]^k dx.$$

Integrating by parts, taking x^r as the part to be integrated, we get

$$I_r = \frac{k}{r+1}\left[E\left(Y_n^{(k)}\right)^{r+1} - E\left(Y_{n-1}^{(k)}\right)^{r+1}\right].$$

Substituting for I_j and $I_{j+\gamma-1}$ in (2.2) and simplifying the resulting expression, we derive the relation given in (2.1). □

Remarks.

(i) Putting $\beta = 0$ in (2.1), we deduce the recurrence relation for single moments of k-th upper record values from the exponential distribution with parameter $\alpha > 0$, established by Pawlas and Szynal [13].

(ii) Setting $\alpha = 0$ in (2.1), we get the recurrence relation for single moments of k-th upper record values from the Weibull distribution, obtained by Pawlas and Szynal [19].

(iii) Setting $\gamma = 2$ in (2.1), the result for single moments is deduced for linear failure rate distribution as

$$E\left(Y_n^{(k)}\right)^{j+2} = E\left(Y_{n-1}^{(k)}\right)^{j+2} + \frac{j+2}{2\beta k}E\left(Y_n^{(k)}\right)^j - \frac{(j+2)\alpha}{2\beta(j+1)}\left[E\left(Y_n^{(k)}\right)^{j+1} - E\left(Y_{n-1}^{(k)}\right)^{j+1}\right].$$

(iv) Putting $\alpha = 0$ and $\gamma = 2$ in (2.1), we get the recurrence relation for Rayleigh distribution in the form

$$E\left(Y_n^{(k)}\right)^{j+2} = E\left(Y_{n-1}^{(k)}\right)^{j+2} + \frac{j+2}{2\beta k}E\left(Y_n^{(k)}\right)^j.$$

Corollary 2.1. *The recurrence relation for single moments of upper record values from the exponential-Weibull lifetime distribution has the form*

$$EX_{U_n}^j = \frac{\alpha}{j+1}\left[EX_{U_n}^{j+1} - EX_{U_{n-1}}^{j+1}\right] + \frac{\beta\gamma}{j+\gamma}\left[EX_{U_n}^{j+\gamma} - EX_{U_{n-1}}^{j+\gamma}\right].$$

Remarks.

(i) If $\beta = 0$ we get the recurrence relation for single moments of record values from exponential distribution with parameter $\alpha > 0$, obtained by Balakrishnan and Ahsanullah [20].

(ii) If $\alpha = 0$, the result for single moments of record values obtained by Pawlas and Szynal [19] for Weibull distribution is deduced.

(iii) If $\gamma = 2$, the result for single moments of record values is deduced for linear failure rate distribution.

(iv) If $\alpha = 0$ and $\gamma = 2$, the result for single moments of record values is deduced for Rayleigh distribution.

3. Relations for product moments

Theorem 3.1. *For $m \geqslant 1$ and $i, j = 0, 1, \ldots,$*

$$E\left[\left(Y_m^{(k)}\right)^i\left(Y_{m+1}^{(k)}\right)^j\right] = \frac{\alpha k}{j+1}\left\{E\left[\left(Y_m^{(k)}\right)^i\left(Y_{m+1}^{(k)}\right)^{j+1}\right] - E\left(Y_m^{(k)}\right)^{i+j+1}\right\} + \frac{\beta\gamma k}{j+\gamma}\left\{E\left[\left(Y_m^{(k)}\right)^i\left(Y_{m+1}^{(k)}\right)^{j+\gamma}\right] - E\left(Y_m^{(k)}\right)^{i+j+\gamma}\right\} \quad (3.1)$$

and for $1 \leqslant m \leqslant n-2, i, j = 0, 1, \ldots,$

$$E\left[\left(Y_m^{(k)}\right)^i\left(Y_n^{(k)}\right)^j\right] = \frac{\alpha k}{j+1}\left\{E\left[\left(Y_m^{(k)}\right)^i\left(Y_n^{(k)}\right)^{j+1}\right] - E\left[\left(Y_m^{(k)}\right)^i\left(Y_{n-1}^{(k)}\right)^{j+1}\right]\right\} + \frac{\beta\gamma k}{j+\gamma}\left\{E\left[\left(Y_m^{(k)}\right)^i\left(Y_n^{(k)}\right)^{j+\gamma}\right] - E\left[\left(Y_m^{(k)}\right)^i\left(Y_{n-1}^{(k)}\right)^{j+\gamma}\right]\right\}. \quad (3.2)$$

Proof. From (1.2) for $1 \leqslant m \leqslant n-1$ and $i, j = 0, 1, \ldots,$

$$E\left[\left(Y_m^{(k)}\right)^i\left(Y_n^{(k)}\right)^j\right] = \frac{k^n}{(m-1)!(n-m-1)!} \times \int_0^\infty x^i[-\ln \overline{F}(x)]^{m-1}\frac{f(x)}{\overline{F}(x)}I(x)dx, \quad (3.3)$$

where

$$I(x) = \int_x^\infty y^j[\ln \overline{F}(x) - \ln \overline{F}(y)]^{n-m-1}[\overline{F}(y)]^{k-1}f(y)dy.$$

Integrating $I(x)$ by parts and using (1.5) we obtain

$$\begin{aligned} I(x) = {} & \frac{\alpha k}{j+1}\int_x^\infty y^{j+1}[\ln \overline{F}(x) - \ln \overline{F}(y)]^{n-m-1}[\overline{F}(y)]^{k-1}f(y)dy \\ & - \frac{\alpha(n-m-1)}{j+1} \times \int_x^\infty y^{j+1}[\ln \overline{F}(x) - \ln \overline{F}(y)]^{n-m-2}[\overline{F}(y)]^{k-1}f(y)dy \\ & + \frac{\beta\gamma k}{j+\gamma}\int_x^\infty y^{j+\gamma}[\ln \overline{F}(x) - \ln \overline{F}(y)]^{n-m-1}[\overline{F}(y)]^{k-1}f(y)dy \\ & - \frac{\beta\gamma(n-m-1)}{j+\gamma} \times \int_x^\infty y^{j+\gamma}[\ln \overline{F}(x) - \ln \overline{F}(y)]^{n-m-2}[\overline{F}(y)]^{k-1}f(y)dy. \end{aligned}$$

Substituting this expression into (3.3) and simplifying, it leads to (3.2). Proceeding in a similar manner for the case $n = m+1$, the recurrence relation given in (3.1) can easily be established. □

Remarks.

(i) Putting $\beta = 0$ in (3.2), we get the recurrence relation for product moments of k-th upper record values from the exponential distribution with parameter $\alpha > 0$, established by Pawlas and Szynal [13].

(ii) Setting $\alpha = 0$ in (3.2), we get the recurrence relation for product moments of k-th upper record values as obtained by Pawlas and Szynal [19] for Weibull distribution.

(iii) If $\gamma = 2$ the result for single moments is deduced for linear failure rate distribution as

$$E\left[\left(Y_m^{(k)}\right)^i\left(Y_n^{(k)}\right)^j\right] = \frac{\alpha k}{j+1}\left\{E\left[\left(Y_m^{(k)}\right)^i\left(Y_n^{(k)}\right)^{j+1}\right] - E\left[\left(Y_m^{(k)}\right)^i\left(Y_{n-1}^{(k)}\right)^{j+1}\right]\right\} + \frac{2\beta k}{j+2}\left\{E\left[\left(Y_m^{(k)}\right)^i\left(Y_n^{(k)}\right)^{j+2}\right] - E\left[\left(Y_m^{(k)}\right)^i\left(Y_{n-1}^{(k)}\right)^{j+2}\right]\right\}.$$

(iv) Letting $\alpha = 0$ and $\gamma = 2$ in (3.2), we get the recurrence relation for Rayleigh distribution in the form

$$E\left[\left(Y_m^{(k)}\right)^i\left(Y_n^{(k)}\right)^j\right] = \frac{2\beta k}{j+2}\left\{E\left[\left(Y_m^{(k)}\right)^i\left(Y_n^{(k)}\right)^{j+2}\right] - E\left[\left(Y_m^{(k)}\right)^i\left(Y_{n-1}^{(k)}\right)^{j+2}\right]\right\}.$$

Corollary 3.1. *The recurrence relation for single moments of upper record values from the exponential-Weibull lifetime distribution has the form*

$$E\left(X_{U_m}^i X_{U_n}^j\right) = \frac{\alpha}{j+1}\left\{E\left(X_{U_m}^i X_{U_n}^{j+1}\right) - E\left(X_{U_m}^i X_{U_{n-1}}^{j+1}\right)\right\} + \frac{\beta\gamma}{j+\gamma}\left\{E\left(X_{U_m}^i X_{U_n}^{j+\gamma}\right) - E\left(X_{U_m}^i X_{U_{n-1}}^{j+\gamma}\right)\right\}.$$

Remarks.

(i) If $\beta = 0$ we get the recurrence relation for product moments of record values from exponential distribution with parameter $\alpha > 0$, established by Balakrishnan and Ahsanullah [20].

(ii) If $\alpha = 0$, the result for product moments of record values obtained by Pawlas and Szynal [19] for Weibull distribution is deduced.

(iii) If $\gamma = 2$, the result for product moments of record values is deduced for linear failure rate distribution.

(iv) If $\alpha = 0$ and $\gamma = 2$, the result for product moments of record values is deduced for Rayleigh distribution.

4. Characterizations

Before coming to the main results we require the following result of Lin [21].

Proposition. *Let n_0 be any fixed non-negative integer, $-\infty \leqslant a < b \leqslant \infty$, and $g(x) \geqslant 0$ an absolutely continuous function with $g'(x) \neq 0$ a.e. on (a, b). Then the sequence of functions $\{[g(x)]^n e^{-g(x)}, n \geqslant n_0\}$ is complete in $L(a, b)$ iff $g(x)$ is strictly monotone on (a, b).*

Using the above Proposition we get a stronger version of Theorem 2.1.

Theorem 4.1. *Fix a positive integer $k \geqslant 1$ and let j be a non-negative integer. A necessary and sufficient condition for a random variable X to be distributed with pdf given by (1.3) is that*

$$E\left(Y_n^{(k)}\right)^j = \frac{\alpha k}{j+1}\left[E\left(Y_n^{(k)}\right)^{j+1} - E\left(Y_{n-1}^{(k)}\right)^{j+1}\right] + \frac{\beta\gamma k}{j+\gamma}\left[E\left(Y_n^{(k)}\right)^{j+\gamma} - E\left(Y_{n-1}^{(k)}\right)^{j+\gamma}\right] \quad (4.1)$$

for $n = 1, 2, \ldots$.

Proof. The necessary part follows immediately from (4.1). On the other hand if the recurrence relation (4.1) is satisfied, then on rearranging the terms in (4.1) and using (1.1), we have

$$\frac{k^n}{(n-1)!}\int_0^\infty x^j[-\ln\overline{F}(x)]^{n-1}[\overline{F}(x)]^{k-1}f(x)dx$$
$$=\frac{\alpha k^{n+1}}{(n-1)!(j+1)}\times\int_0^\infty x^{j+1}[-\ln\overline{F}(x)]^{n-2}[\overline{F}(x)]^{k-1}f(x)\left\{[-\ln\overline{F}(x)]-\frac{n-1}{k}\right\}dx$$
$$+\frac{\beta\gamma k^{n+1}}{(n-1)!(j+\gamma)}\times\int_0^\infty x^{j+\gamma}[-\ln\overline{F}(x)]^{n-2}[\overline{F}(x)]^{k-1}f(x)\left\{[-\ln\overline{F}(x)]-\frac{n-1}{k}\right\}dx. \quad (4.2)$$

Let

$$h(x)=-\frac{1}{k}[-\ln\overline{F}(x)]^{n-1}[\overline{F}(x)]^k. \quad (4.3)$$

Differentiating both the sides of (4.3), we get

$$h'(x)=[-\ln\overline{F}(x)]^{n-2}[\overline{F}(x)]^{k-1}f(x)\left\{[-\ln\overline{F}(x)]-\frac{n-1}{k}\right\}.$$

Thus

$$\frac{k^n}{(n-1)!}\int_0^\infty x^j[-\ln\overline{F}(x)]^{n-1}[\overline{F}(x)]^{k-1}f(x)dx$$
$$=\frac{\alpha k^{n+1}}{(n-1)!(j+1)}\int_0^\infty x^{j+1}h'(x)dx+\frac{\beta\gamma k^{n+1}}{(n-1)!(j+\gamma)}\times\int_0^\infty x^{j+\gamma}h'(x)dx. \quad (4.4)$$

Integrating RHS in (4.4) by parts and using the value of $h(x)$from (4.3), we find that

$$\frac{k^n}{(n-1)!}\int_0^\infty x^j[-\ln\overline{F}(x)]^{n-1}[\overline{F}(x)]^{k-1}[f(x)-\alpha\overline{F}(x)-\beta\gamma x^{\gamma-1}\overline{F}(x)]dx=0.$$

It now follows from the above Proposition with $g(x)=-\ln\overline{F}(x)$ that

$$f(x)=[\alpha+\beta\gamma x^{\gamma-1}]\overline{F}(x)$$

which proves that $f(x)$ has the form as in (1.3). □

Remark. Theorem 4.1 can be used to characterize the exponential, Weibull, linear failure rate and Rayleigh distributions by setting $\beta=0,\alpha=0,\gamma=2$ and $\alpha=0,\gamma=2$ respectively.

Corollary 4.1. *Under the assumptions of Theorem 4.1 with $j=0$ the following equations*

$$E\left(Y_n^{(k)}\right)^\gamma=E\left(Y_{n-1}^{(k)}\right)^\gamma-\frac{\alpha}{\beta}\left[E\left(Y_n^{(k)}\right)-E\left(Y_{n-1}^{(k)}\right)\right]+\frac{1}{\beta k},\quad n=1,2,\ldots$$

characterize the exponential-Weibull lifetime distribution.

Remark. If $k=1$ we obtain the following characterization of the exponential-Weibull distribution

$$EX_{U_n}^\gamma=EX_{U_{n-1}}^\gamma-\frac{\alpha}{\beta}[EX_{U_n}-EX_{U_{n-1}}]+\frac{1}{\beta},\quad n=1,2,\ldots.$$

Now we shall show how Theorem 4.1 can be used in a characterization of the exponential-Weibull distribution in terms of moments of minimal order statistics. Putting $n=1$ in (4.1), we get

$$EX_{1:k}^j=\frac{\alpha k}{j+1}EX_{1:k}^{j+1}+\frac{\beta\gamma k}{j+\gamma}EX_{1:k}^{j+\gamma},$$

for any fixed integer $k\geqslant 1$. This result leads to the following theorem.

Theorem 4.2. *Let j be a non-negative integer. A necessary and sufficient condition for a random variable X to be distributed with pdf given by (1.3) is that*

$$EX_{1:k}^j=\frac{\alpha k}{j+1}EX_{1:k}^{j+1}+\frac{\beta\gamma k}{j+\gamma}EX_{1:k}^{j+\gamma} \quad (4.5)$$

for $k=1,2,\ldots$.

Proof. The necessary part follows immediately from (2.1). On the other hand if the recurrence relation (4.5) is satisfied, then

$$\int_0^\infty x^j[\overline{F}(x)]^{k-1}f(x)dx=\frac{\alpha k}{j+1}\int_0^\infty x^{j+1}[\overline{F}(x)]^{k-1}f(x)dx+\frac{\beta\gamma k}{j+\gamma}\int_0^\infty x^{j+\gamma}[\overline{F}(x)]^{k-1}f(x)dx.$$

Integrating the integrals on the right-hand side of the above expression by parts, we get

$$\int_0^\infty x^j[\overline{F}(x)]^{k-1}f(x)dx=\alpha\int_0^\infty x^j[\overline{F}(x)]^k dx+\beta\gamma\int_0^\infty x^{j+\gamma-1}[\overline{F}(x)]^k dx,$$

which further reduces to

$$\int_0^\infty x^j[\overline{F}(x)]^{k-1}[f(x)-\alpha\overline{F}(x)-\beta\gamma x^{\gamma-1}\overline{F}(x)]dx=0,\quad k=1,2,\ldots. \quad (4.6)$$

Now applying a generalization of the Müntz-Szász Theorem (see for example Hwang and Lin [22]), to Eq. (4.6), we obtain

$$f(x)=(\alpha+\beta\gamma x^{\gamma-1})\overline{F}(x),$$

which proves that

$$F(x)=1-e^{-(\alpha x+\beta x^\gamma)},\quad x>0,\quad \alpha>0,\quad \beta>0,\quad \gamma>0. \quad □$$

5. Conclusion

Above investigations demonstrate the recurrence relations between single and product moments of k-th record values in Section 2 and Section 3 respectively. Since recurrence relations reduce the amount of direct computation and hence reduce the time and labour. Therefore the relations under consideration may be useful in computing the moments of higher order from the exponential-Weibull lifetime distribution. In Section 4, characterization of exponential-Weibull lifetime, Weibull, exponential, Rayleigh and two parameter linear failure rate distributions are given through a recurrence relation for single moments of k-th record values.

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