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ORIGINAL ARTICLE

# Numerical computation of nonlinear fractional Zakharov–Kuznetsov equation arising in ion-acoustic waves

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**Abstract** The main aim of the present work is to propose a new and simple algorithm for fractional Zakharov–Kuznetsov equations by using homotopy perturbation transform method (HPTM). The Zakharov–Kuznetsov equation was first derived for describing weakly nonlinear ion-acoustic waves in strongly magnetized lossless plasma in two dimensions. The homotopy perturbation transform method is an innovative adjustment in Laplace transform algorithm (LTA) and makes the calculation much simpler. HPTM is not limited to the small parameter, such as in the classical perturbation method. The method gives an analytical solution in the form of a convergent series with easily computable components, requiring no linearization or small perturbation. The numerical solutions obtained by the proposed method indicate that the approach is easy to implement and computationally very attractive.

**2010 MATHEMATICS SUBJECT CLASSIFICATION:** 34A08; 35A20; 35A22



## 1. Introduction

In recent years, fractional differential equations have gained importance and popularity, mainly due to its demonstrated applications in numerous seemingly diverse fields of physics and engineering. Many important phenomena in electromagnetics, acoustics, viscoelasticity, electrochemistry and material science, probability and statistics, electrochemistry of corrosion, chemical physics, and signal processing are well described by differential equations of fractional order [1–15]. Hence, great attention has been given to find solutions of fractional

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differential equations. In general, it is difficult to obtain an exact solution for a fractional differential equation. So numerical methods attracted the interest of researchers, the perturbation method is one of these. But the perturbation methods have some limitations e.g., the approximate solution involves series of small parameters which poses difficulty since majority of nonlinear problems have no small parameters at all. Although appropriate choices of small parameters some time lead to ideal solution but in most of the cases unsuitable choices lead to serious effects in the solutions. The homotopy perturbation method (HPM) was first introduced by researcher He in 1998 and was developed by him [16–18]. The homotopy perturbation method was also studied by many authors to handle linear and nonlinear equations arising in physics and engineering [19–24]. In recent years, many authors have paid attention to study the solutions of linear and nonlinear partial differential equations by using various methods combined with the Laplace transform. Among these are Laplace decomposition method (LDM) [25–28] and homotopy perturbation transform method (HPTM) [29–31].

In this paper, we consider the following fractional Zakharov–Kuznetsov equations (FZK($p$, $q$, $r$)) of the form:

$$D_t^{\beta}u + a(u^p)_x + b(u^q)_{xxx} + c(u^r)_{xyy} = 0, \tag{1}$$

where $u = u(x, y, t)$, $\beta$ is parameter describing the order of the fractional derivative ($0 < \beta \leqslant 1$). $a$, $b$, and $c$ are arbitrary constants and $p$, $q$, and $r$ are integers and $p, q, r \neq 0$ governs the behavior of weakly nonlinear ion acoustic waves in a plasma comprising cold ions and hot isothermal electrons in the presence of a uniform magnetic field [32,33]. The Zakharov–Kuznetsov equation was first derived for describing weakly nonlinear ion-acoustic waves in strongly magnetized lossless plasma in two dimensions [34]. The FZK equations have been studied previously by using VIM [35] and HPM [36].

In the present article, further we apply the homotopy perturbation transform method (HPTM) to solve the FZK equations. The HPTM is combined form of Laplace transform, HPM and He's polynomials. The advantage of this technique is its capability of combining two powerful methods for obtaining exact and approximate analytical solutions for nonlinear equations. It is worth mentioning that the proposed approach is capable of reducing the volume of the computational work as compared to the classical methods while still maintaining the high accuracy of the numerical result; the size reduction amounts to an improvement of the performance of the approach.

**Definition 1.1.** The Laplace transform of function $f(t)$is defined by

$$F(s) = L[f(t)] = \int_0^{\infty} e^{-st}f(t)dt. \tag{2}$$

**Definition 1.2.** The Laplace transform $L[f(t)]$ of the Riemann–Liouville fractional integral is defined as [5]:

$$L\left[I_t^{\beta}f(t)\right] = s^{-\beta}F(s). \tag{3}$$

**Definition 1.3.** The Laplace transform $L[f(t)]$ of the Caputo fractional derivative is defined as [5]:

$$L\left[D_t^{\beta}f(t)\right] = s^{\beta}F(s) - \sum_{k=0}^{m-1} s^{(\beta-k-1)}f^{(k)}(0), \quad m-1 < \beta \leqslant m. \tag{4}$$

## 2. Basic Idea of newly homotopy perturbation transform method

To illustrate the basic idea of this method, we consider a general fractional nonlinear nonhomogeneous partial differential equation with the initial conditions of the form:

$$D_t^{\beta}u(x,t) + Ru(x,t) + Nu(x,t) = g(x,t), \quad 0 < \beta \leq 1, \tag{5}$$

$$u(x,0) = h(x), \tag{6}$$

where $D_t^{\beta}u(x,t)$ is the Caputo fractional derivative of the function $u(x, t)$, $R$ is the linear differential operator, $N$ represents the general nonlinear differential operator and $g(x, t)$ is the source term.

Applying the Laplace transform (denoted in this paper by $L$) on both sides of Eq. (5), we get

$$L\left[D_t^{\beta}u(x,t)\right] + L[Ru(x,t)] + L[Nu(x,t)] = L[g(x,t)]. \tag{7}$$

Using the property of the Laplace transform, we have

$$L[u(x,t)] = \frac{h(x)}{s} + \frac{1}{s^{\beta}}L[g(x,t)] - \frac{1}{s^{\beta}}L[Ru(x,t)] - \frac{1}{s^{\beta}}L[Nu(x,t)]. \tag{8}$$

Operating with the Laplace inverse on both sides of Eq. (8) gives

$$u(x,t) = G(x,t) - L^{-1}\left[\frac{1}{s^{\beta}}L[Ru(x,t) + Nu(x,t)]\right], \tag{9}$$

where $G(x, t)$ represents the term arising from the source term and the prescribed initial conditions. Now we apply the HPM

$$u(x,t) = \sum_{n=0}^{\infty} p^n u_n(x,t), \tag{10}$$

and the nonlinear term can be decomposed as

$$Nu(x,t) = \sum_{n=0}^{\infty} p^n H_n(u), \tag{11}$$

for some He's polynomials $H_n(u)$ [37,38] that are given by

$$H_n(u_0, u_1, \ldots, u_n) = \frac{1}{n!}\frac{\partial^n}{\partial p^n}\left[N\left(\sum_{i=0}^{n} p^i u_i\right)\right]_{p=0}, \quad n = 0, 1, 2, 3, \ldots \tag{12}$$

Using Eqs. (10) and (11) in Eq. (9), we get

$$\sum_{n=0}^{\infty} p^n u_n(x,t) = G(x,t) - p\left(L^{-1}\left[\frac{1}{s^{\beta}}L\left[R\sum_{n=0}^{\infty} p^n u_n(x,t) + \sum_{n=0}^{\infty} p^n H_n(u)\right]\right]\right), \tag{13}$$

which is the coupling of the Laplace transform and the HPM using He's polynomials. Comparing the coefficients of like powers of p, the following approximations are obtained.

$$\begin{aligned} p^0 &: u_0(x,t) = G(x,t), \\ p^1 &: u_1(x,t) = L^{-1}\left[\frac{1}{s^{\beta}}L[Ru_0(x,t) + H_0(u)]\right], \\ p^2 &: u_2(x,t) = L^{-1}\left[\frac{1}{s^{\beta}}L[Ru_1(x,t) + H_1(u)]\right], \\ p^3 &: u_3(x,t) = L^{-1}\left[\frac{1}{s^{\beta}}L[Ru_2(x,t) + H_2(u)]\right]. \end{aligned} \tag{14}$$

Proceeding in this same manner, the rest of the components $u_n(x, t)$ can be completely obtained and the series solution is thus entirely determined.

Finally, we approximate the analytical solution $u(x, t)$ by truncated series

$$u(x,t) = \lim_{N\to\infty}\sum_{n=0}^{N} u_n(x,t). \tag{15}$$

The above series solutions generally converge very rapidly. A classical approach of convergence of this type of series is already presented by Abbaoui and Cherruault [39].

### 3. Numerical examples

In this section, we discuss the implementation of our new proposed method and investigate its accuracy by applying the HPM with coupling of the Laplace transform. The simplicity and accuracy of the proposed algorithm is illustrated through the following numerical examples.

**Example 1.** In this example, we consider the following FZK (2, 2, 2) equation [36] as:

$$D_t^{\beta} u + (u^2)_x + \frac{1}{8}(u^2)_{xxx} + \frac{1}{8}(u^2)_{xyy} = 0, \tag{16}$$

where $0 < \beta \leqslant 1$. The exact solution to Eq. (16) when $\beta = 1$ and subject to the initial condition

$$u(x,y,0) = \frac{4}{3}\rho\sinh^2(x+y), \tag{17}$$

where $\rho$ is an arbitrary constant, was derived in [40] and is given as:

$$u(x,y,t) = \frac{4}{3}\rho\sinh^2(x+y-\rho t). \tag{18}$$

Applying the Laplace transform on both sides of Eq. (16), subject to initial condition (17), we have

$$L[u(x,y,t)] = \frac{4}{3s}\rho\sinh^2(x+y) - \frac{1}{s^{\beta}}L\left[(u^2)_x + \frac{1}{8}(u^2)_{xxx} + \frac{1}{8}(u^2)_{xyy}\right]. \tag{19}$$

Operation inverse Laplace transform in Eq. (19) implies that

$$u(x,y,t) = \frac{4}{3}\rho\sinh^2(x+y) - L^{-1}\left[\frac{1}{s^{\beta}}L\left[(u^2)_x + \frac{1}{8}(u^2)_{xxx} + \frac{1}{8}(u^2)_{xyy}\right]\right]. \tag{20}$$

Now applying the HPM [16], we get

$$\sum_{n=0}^{\infty} p^n u_n(x,y,t) = \frac{4}{3}\rho\sinh^2(x+y) - p\left(L^{-1}\left[\frac{1}{s^{\beta}}L\left[\left(\sum_{n=0}^{\infty}p^n H_n(u)\right) + \frac{1}{8}\left(\sum_{n=0}^{\infty}p^n H'_n(u)\right) + \frac{1}{8}\left(\sum_{n=0}^{\infty}p^n H''_n(u)\right)\right]\right]\right). \tag{21}$$

where $H_n(u)$, $H'_n(u)$ and $H''_n(u)$ are He's polynomials [37,38] that represents the nonlinear terms. So, the He's polynomials are given by

$$\sum_{n=0}^{\infty} p^n H_n(u) = (u^2)_x. \tag{22}$$

The first few, components of He's polynomials, are given by

$$H_0(u) = (u_0^2)_x,\ H_1(u) = (2u_0u_1)_x, \cdots \tag{23}$$

for $H'_n(u)$, we find that

$$\sum_{n=0}^{\infty} p^n H'_n(u) = (u^2)_{xxx},\ H'_0(u) = (u_0^2)_{xxx},\ H'_1(u) = (2u_0u_1)_{xxx}, \cdots \tag{24}$$

and for $H''_n(u)$, we find that

$$\sum_{n=0}^{\infty} p^n H''_n(u) = (u^2)_{xyy},\ H''_0(u) = (u_0^2)_{xyy},\ H''_1(u) = (2u_0u_1)_{xyy}, \cdots \tag{25}$$

Comparing the coefficients of like powers of p, we have

$$\begin{aligned}
p^0 &: u_0(x,y,t) = \frac{4}{3}\rho\sinh^2(x+y),\\
p^1 &: u_1(x,y,t) = -L^{-1}\left[\frac{1}{s^{\beta}}L\left[H_0(u) + \frac{1}{8}H'_0(u) + \frac{1}{8}H''_0(u)\right]\right]\\
&= -\left[\frac{224}{9}\rho^2\sinh^3(x+y)\cosh(x+y) + \frac{32}{3}\rho^2\sinh(x+y)\cosh^3(x+y)\right]\frac{t^{\beta}}{\Gamma(\beta+1)},\\
p^2 &: u_2(x,y,t) = -L^{-1}\left[\frac{1}{s^{\beta}}L\left[H_1(u) + \frac{1}{8}H'_1(u) + \frac{1}{8}H''_1(u)\right]\right]\\
&= \frac{64}{27}\rho^3\left[2400\cosh^6(x+y) - 4160\cosh^4(x+y) + 1936\cosh^2(x+y) - 158\right]\frac{t^{2\beta}}{\Gamma(2\beta+1)}.
\end{aligned} \tag{26}$$

Proceeding in the same manner the rest of components of the HPTM solution can be obtained. Thus the solution $u(x, y, t)$ of the Eq. (16) is given as:

$$\begin{aligned}
u(x,y,t) &= \lim_{N\to\infty}\sum_{n=0}^{N}u_n(x,y,t) = \frac{4}{3}\rho\sinh^2(x+y)\\
&- \left[\frac{224}{9}\rho^2\sinh^3(x+y)\cosh(x+y) + \frac{32}{3}\rho^2\sinh(x+y)\cosh^3(x+y)\right]\frac{t^{\beta}}{\Gamma(\beta+1)}\\
&+ \frac{64}{27}\rho^3\left[2400\cosh^6(x+y) - 4160\cosh^4(x+y) + 1936\cosh^2(x+y) - 158\right]\frac{t^{2\beta}}{\Gamma(2\beta+1)} + \cdots.
\end{aligned} \tag{27}$$

The results for the exact solution (18) and the approximate solution (27) obtained by using the HPTM, for the special case $\beta = 1$, are shown in Fig. 1. It can be seen from the Fig. 1 that the solution obtained by the HPTM is nearly identical with the exact solution. The approximate solutions when $\beta = 0.5$ and $\beta = 0.75$ are shown by Figs. 2a and b respectively. It is to be noted that only the second order term of the HPTM was used in evaluating the approximate solutions for Fig. 2. It is evident that the efficiency of the present method can be dramatically enhanced by computing further terms of $u(x, y, t)$ when the HPTM is used.

**Example 2.** Next, we consider the following FZK (3, 3, 3) equation [36] as:

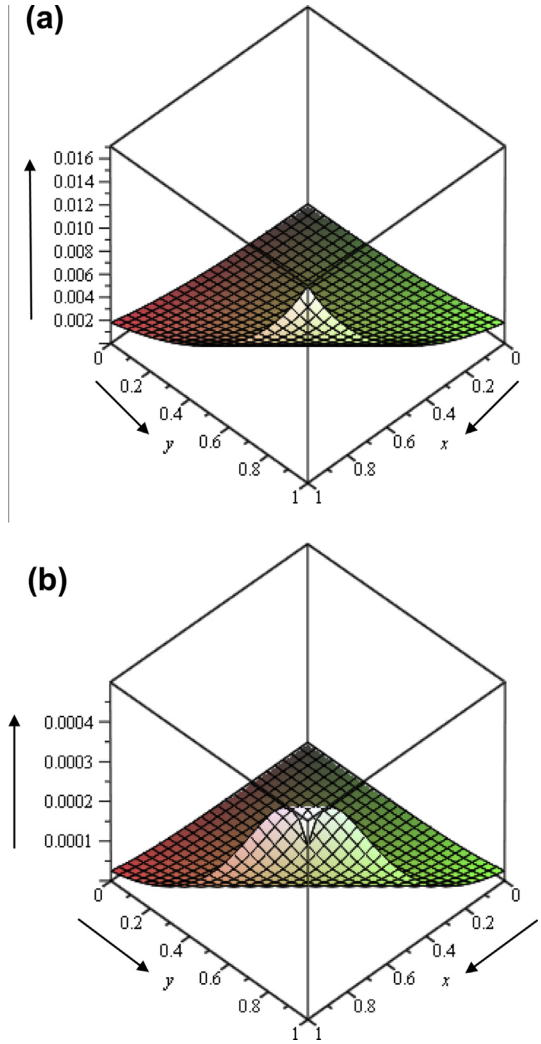


**Figure 1** The surface shows the solution $u(x, y, t)$ for Eqs. (16) and (17) when $\beta = 1$, $\rho = 0.001$, $t = 0.5$ (a) Approximate solution (27) and (b) $|u_{ex} - u_{app}|$.

$$D_t^{\beta} u + (u^3)_x + 2(u^3)_{xxx} + 2(u^3)_{xyy} = 0, \tag{28}$$

where $0 < \beta \leqslant 1$. The exact solution to Eq. (28) when $\beta = 1$ and subject to the initial condition

$$u(x, y, 0) = \frac{3}{2}\rho \sinh\left[\frac{1}{6}(x + y)\right], \tag{29}$$

where $\rho$ is an arbitrary constant, was derived in [40] and is given as:

$$u(x, y, t) = \frac{3}{2}\rho \sinh\left[\frac{1}{6}(x + y - \rho t)\right]. \tag{30}$$

In a similar way as above, we have

$$\sum_{n=0}^{\infty} p^n u_n(x,y,t) = \frac{3}{2}\rho\sinh\left[\frac{1}{6}(x+y)\right] - p\left(L^{-1}\left[\frac{1}{s^{\beta}}L\left[\left(\sum_{n=0}^{\infty} p^n H_n(u)\right) + 2\left(\sum_{n=0}^{\infty} p^n H'_n(u)\right) + 2\left(\sum_{n=0}^{\infty} p^n H''_n(u)\right)\right]\right]\right). \tag{31}$$

Comparing the coefficients of like powers of p, we have

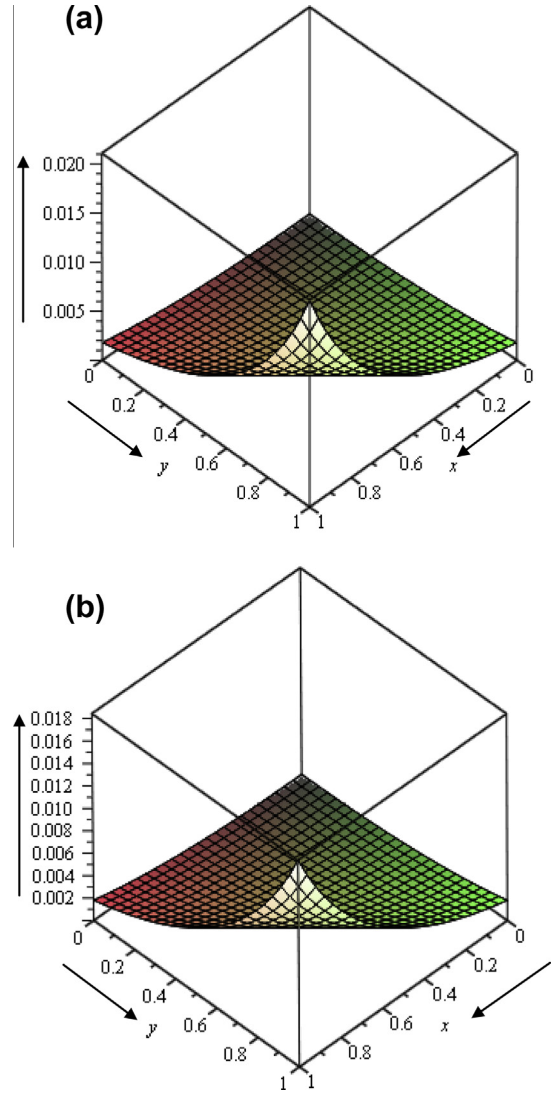


**Figure 2** The surface shows the solution $u(x, y, t)$ for Eqs. (16) and (17) when $\rho = 0.001$, $t = 0.5$: (a) $\beta = 0.5$ and (b) $\beta = 0.75$.

$$\begin{aligned}
p^0 &: u_0(x,y,t) = \frac{3}{2}\rho\sinh\left[\frac{1}{6}(x+y)\right],\\
p^1 &: u_1(x,y,t) = -3\rho^3\left[\sinh^2\left[\frac{1}{6}(x+y)\right]\cosh\left[\frac{1}{6}(x+y)\right] + \frac{1}{8}\cosh^3\left[\frac{1}{6}(x+y)\right]\right]\frac{t^{\beta}}{\Gamma(\beta+1)},\\
p^2 &: u_2(x,y,t) = \frac{3}{64}\rho^5\sinh\left[\frac{1}{6}(x+y)\right]\left[1530\cosh^4\left[\frac{1}{6}(x+y)\right] - 1458\cosh^2\left[\frac{1}{6}(x+y)\right] + 182\right]\frac{t^{2\beta}}{\Gamma(2\beta+1)}.
\end{aligned} \tag{32}$$

In this manner the rest of components of the HPTM solution can be obtained. Thus the solution $u(x, y, t)$ of the Eq. (28) is given as:

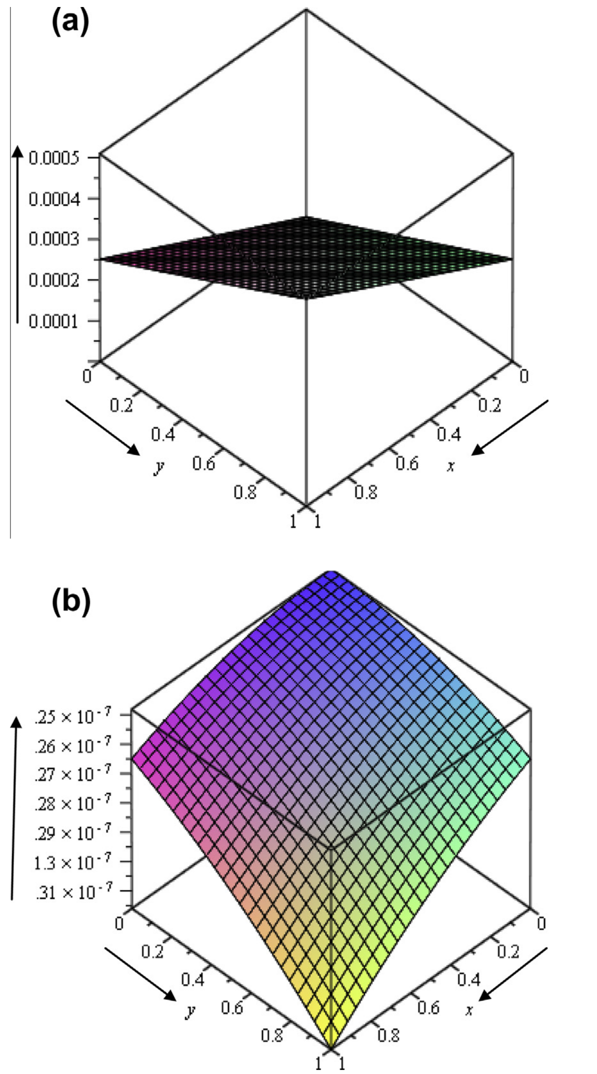


**Figure 3** The surface shows the solution $u(x, y, t)$ for Eqs. (28) and (29) when $\beta = 1$, $\rho = 0.001$, $t = 0.5$ (a) Approximate soution (33) and (b) $|u_{ex} - u_{app}|$.

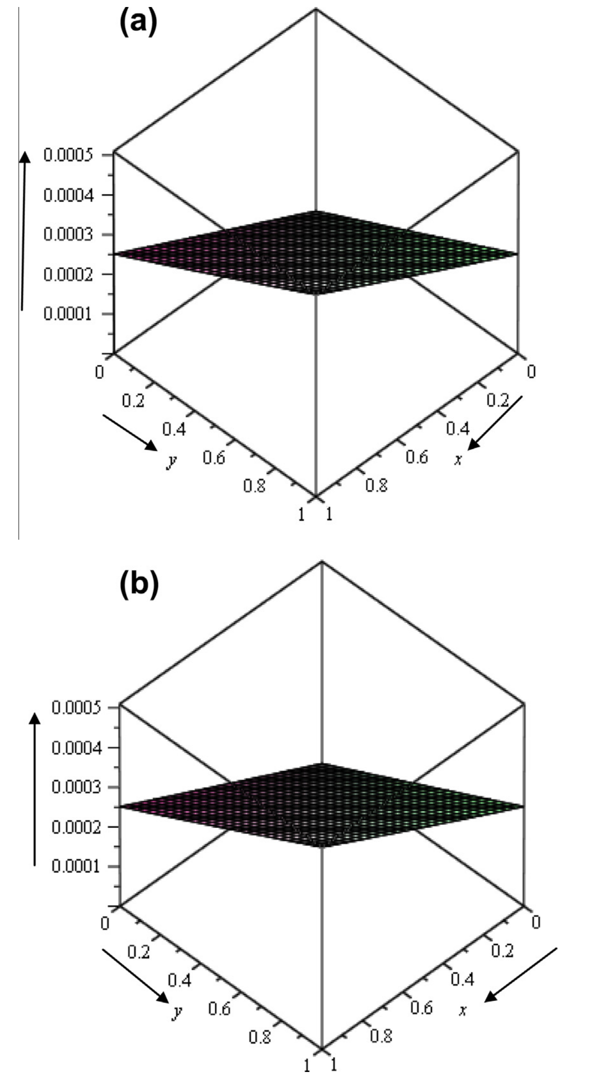


**Figure 4** The surface shows the solution $u(x, y, t)$ for Eqs. (28) and (29) when $\rho = 0.001$, $t = 0.5$: (a) $\beta = 0.5$ and (b) $\beta = 0.75$.

$$
\begin{aligned}
u(x,y,t) &= \lim_{N\to\infty}\sum_{n=0}^{N} u_n(x,y,t) \\
&= \frac{3}{2}\rho \sinh\left[\frac{1}{6}(x+y)\right] \\
&\quad - 3\rho^3\left[\sinh^2\left[\frac{1}{6}(x+y)\right]\cosh\left[\frac{1}{6}(x+y)\right]\right. \\
&\quad \left. +\frac{1}{8}\cosh^3\left[\frac{1}{6}(x+y)\right]\right]\frac{t^{\beta}}{\Gamma(\beta+1)} \\
&\quad +\frac{3}{64}\rho^5 \sinh\left[\frac{1}{6}(x+y)\right]\left[1530\cosh^4\left[\frac{1}{6}(x+y)\right]\right. \\
&\quad \left. -1458\cosh^2\left[\frac{1}{6}(x+y)\right]+182\right]\frac{t^{2\beta}}{\Gamma(2\beta+1)}+\cdots.
\end{aligned}
\tag{33}
$$

The results for the exact solution (30) and the approximate solution (33) obtained by using the HPTM, for the special case $\beta = 1$, are shown in Fig. 3. It can be seen from the Fig. 3 that the solution obtained by the HPTM is nearly identical with the exact solution. The approximate solutions when $\beta = 0.5$ and $\beta = 0.75$ are shown by Fig. 4a and b respectively.

## 4. Conclusions

In this paper, the homotopy perturbation transform method (HPTM) is successfully applied for solving FZK equations. It provides the solutions in terms of convergent series with easily computable components in a direct way without using linearization, perturbation or restrictive assumptions. The fact that the HPTM solves nonlinear problems without using Adomian's polynomials is a clear advantage of this technique over the decomposition method. Hence, we conclude that the HPTM is very powerful and efficient in finding analytical as well as numerical solutions for wide classes of linear and nonlinear fractional partial differential equations.

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