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On paranormed Zweier ideal convergent sequence spaces defined by Orlicz function

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Abstract In this article we introduce paranorm ideal convergent sequence spaces using Zweier transform and Orlicz function. We study some topological and algebraic properties. Further we prove some inclusion relations related to these new spaces.

MATHEMATICS SUBJECT CLASSIFICATION: 40A05; 40D25; 40G15

1. Introduction

Throughout the paper w, ℓ_∞, c, c_0 denote the classes of *all, bounded, convergent* and *null* sequence spaces, respectively. Each linear subspace of w, for example, λ, $\mu \subset w$ is called a sequence space.

A sequence space λ with linear topology is called a K-space provided each of maps $p_i \to \mathbb{C}$ defined by $p_i(x) = x_i$ is continuous for all $i \in \mathbb{N}$.

A K-space λ is called an FK-space provided λ is a complete linear metric space. An FK-space whose topology is normable is called a BK-space.

Let λ and μ be two sequence spaces and $A = (a_{nk})$ is an infinite matrix of real or complex numbers a_{nk}, where $n, k \in \mathbb{N}$. Then we say that A defines a matrix mapping from λ to μ, and we denote it by writing $A:\lambda \to \mu$.

If for every sequence $x = (x_k) \in \lambda$ the sequence $Ax = \{A(Ax)_n\} \in \mu$, then A transform of x is in μ, where

$$(Ax)_n = \sum_k a_{nk}x_k, \quad (n \in \mathbb{N})$$

By $(\lambda:\mu)$, we denote the class of matrices A such that $A:\lambda \to \mu$. Thus, $A \in (\lambda, \mu)$ if and only if $(Ax) \in \mu$ for every $x \in \lambda$.

The approach of constructing the new sequence spaces by means of the matrix domain of a particular limitation method has been recently employed by Malkowsky [1] and many others. Sengonul [2] defined the sequence $y = (y_i)$ which is frequently used as the Z^p-transformation of the sequence $x = (x_i)$, i.e.

$$y_i = px_i + (1-p)x_{i-1}$$

where $x_{-1} = 0$, $p \neq 0$, $1 < p < \infty$, and Z^p denotes the matrix $Z^p = (z_{ik})$ defined by

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$$z_{ik} = \begin{cases} p, & (i = k) \\ 1 - p, & (i - 1 = k); (i, k \in N) \\ 0, & \text{otherwise.} \end{cases}$$

Sengonul [2] introduced the Zweier sequence spaces $\mathcal{Z}$ and $\mathcal{Z}_0$ as follows

$\mathcal{Z} = \{x = (x_k) \in w : Z^p x \in c\}$

and $\mathcal{Z}_0 = \{x = (x_k) \in w : Z^p x \in c_0\}$.

Kostyrko et al. [3] was initially introduced the notion of $\mathcal{I}$-convergence based on the structure of the admissible ideal $\mathcal{I}$ of subset of natural numbers $\mathbb{N}$. Further details on ideal convergence, we refer to Cakalli and Hazarika [4], Dems [5], Esi and Hazarika [6], Hazarika and Savas [7], Hazarika [8–11], Hazarika et al. [12], Khan et al. [13], Lahiri and Das [14], Mursaleen and Mohiuddine [15], Salat et al. [16,17], Tripathy and Hazarika [18–21], Tripathy et al. [22] and references therein.

Let X be a non-empty set. Then a family of set $\mathcal{I} \subseteq 2^x$ (power sets of$X$) is said to be an *ideal* on X if and only if

(i) $\phi \in \mathcal{I}$
(ii) $\mathcal{I}$ is *additive*, i.e. $A, B \in \mathcal{I} \Rightarrow A \cup B \in \mathcal{I}$
(iii) $\mathcal{I}$ is *hereditary*, i.e. $A \in \mathcal{I}, B \subset A \Rightarrow B \in \mathcal{I}$.

A non-empty family of sets $F \subset 2^X$ is said to be a *filter* on X if and only if

(i) $\phi \notin F$
(ii) for each A, $B \in F$ we have $A \cap B \in F$
(iii) for each $A \in F$ and $B \supseteq A$ implies $B \in F$.

For each ideal $\mathcal{I}$, there is a filter $F(\mathcal{I})$ corresponding to $\mathcal{I}$, i.e.

$F = F(\mathcal{I}) = \{K \subseteq \mathbb{N} : \quad K^c \in \mathcal{I}\}$,

where $K^c = \mathbb{N} - K$.

An ideal $\mathcal{I} \subseteq 2^X$ is called non-trivial if $\mathcal{I} \neq 2^X$.

A non-trivial ideal $\mathcal{I} \subset 2^X$ is called admissible if $\mathcal{I} \supset \{\{x\} : x \in X\}$.

A non-trivial ideal $\mathcal{I}$ is *maximal* if there exist any non-trivial ideal $J \neq \mathcal{I}$ containing $\mathcal{I}$ as a subset.

An Orlicz function M:$[0,\infty) \to [0,\infty)$ which is continuous, non-decreasing and convex with $M(0) = 0$, $M(x) > 0$ for $x > 0$ and $M(x) \to \infty$ as $x \to \infty$. If the convexity of the regular function M is replaced by

$M(x + y) \leqslant M(x) + M(y)$,

Then this function is called modulus function. The notion of modulus function was introduced by Nakano [23]. Ruckle [24] and Maddox [25] further investigated the modulus functions with application to sequence spaces.

Remark 1. If M is an *Orlicz function*, then $M(\lambda x) \leqslant \lambda M(x)$ for all λ with $0 < \lambda < 1$.

An Orlicz function M is said to satisfy Δ_2-condition for all values of u if there exists a constant $K > 0$ such that $M(Lu) \leqslant KLM(u)$ for all values of $L > 1$ (see [26]).

Lindenstrauss and Tzafriri [27] used the idea of Orlicz function to construct the sequence space

$$l_M = \left\{ x \in w : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho}\right) < \infty \text{ for some } \rho > 0 \right\}.$$

The space l_M becomes a Banach space with the norm

$$\|x\| = \inf\left\{ \rho > 0 : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho}\right) \leqslant 1 \right\},$$

which is called an *Orlicz sequence space*. The space l_M is closely related to the space l_p which is an Orlicz sequence space with $M(t) = t^p$ for $1 \leqslant p < \infty$.

Later on Orlicz sequence spaces were investigated by Parashar and Chaudhary [28], Esi [29], Tripathy et al. [30], Bhardwaj and Singh [31], Et [32], Esi and Et [33], Hazarika et al. [34], and many others.

Let X be a linear space. A function $g : X \to \mathbb{R}$ is called a *paranorm* if for all x, $y \in X$,

(i) $g(x) = 0$ if $x = \theta$ (θ is the zero element of X)
(ii) $g(-x) = g(x)$
(iii) $g(x + y) \leqslant g(x) + g(y)$
(iv) if (λ_n) is a sequence of scalars with $\lambda_n \to \lambda (n \to \infty)$ and x_n, $a \in X$ with $x_n \to a (n \to \infty)$ then $g(\lambda_n x_n - \lambda a) \to 0 (n \to \infty)$.

The concept of paranorm is closely related to linear metric spaces. It is a generalization of that of absolute value (see [35]).

The notion of paranormed sequence space was studied at the initial stage by Nakano [36] and Simons [37]. Later on it was further investigated by Maddox [35], Lascarides [38,39], Tripathy and Sen [40], and many others.

2. Definitions and preliminaries

We assume throughout this paper that the symbols $\mathbb{R}$ and $\mathbb{N}$ as the set of real and natural numbers, respectively. Throughout the paper, we also denote that $\mathcal{I}$ is an admissible ideal of subsets of $\mathbb{N}$, unless otherwise stated.

A sequence $(x_k) \in w$ is said to be $\mathcal{I}$-convergent to the number L if for every $\varepsilon > 0, \{k \in \mathbb{N} : |x_k - L| \geqslant \varepsilon\} \in \mathcal{I}$. In this case we write $\mathcal{I} - \lim x_k = L$.

A sequence $(x_k) \in w$ is said to be $\mathcal{I}$-*null* if $L = 0$. In this case we write $\mathcal{I} - \lim x_k = 0$.

Let $\mathcal{I}$ be an admissible ideal. A sequence $(x_k) \in w$ is said to be $\mathcal{I}$-*Cauchy* if for every $\varepsilon > 0$ there exists a number $m = m(\varepsilon)$ such that $\{k \in \mathbb{N} : |x_k - x_m| \geqslant \varepsilon\} \in \mathcal{I}$.

A sequence $(x_k) \in w$ is said to be $\mathcal{I}$-*bounded* if there exists $M > 0$ such that $\{k \in \mathbb{N} : |x_k| > M\} \in \mathcal{I}$.

Let (x_k) and (y_k) be two sequences. We say that $x_k = y_k$ for *almost all k relative to I (a.a.k.r.I)*, if $\{k \in \mathbb{N} : x_k \neq y_k\} \in \mathcal{I}$.

A sequence space E is said to be *solid (or normal)* if $(\alpha_k x_k) \in E$ whenever $(x_k) \in E$ and for all sequence (α_k) of scalars with $|\alpha_k| \leqslant 1$ for all $k \in \mathbb{N}$.

A sequence space E is said to be *symmetric* if $(x_k) \in E$ implies $(x_{\pi(k)}) \in E$, where π is a permutation of $\mathbb{N}$.

A sequence space E is said to be *sequence algebra* if $(x_k)*(y_k) = (x_k y_k) \in E$ whenever (x_k), $(y_k) \in E$.

A sequence space E is said to be *convergence free* if $(y_k) \in E$ whenever $(x_k) \in E$ and $x_k = 0$ implies $y_k = 0$.

Let $K = \{k_1 < k_2 < \cdots\} \subset \mathbb{N}$ and let E be a sequence space. A K-step space of E is a sequence space $\lambda_K^E = \{(x_{k_n}) \in w : (x_n) \in E\}$.

A *canonical preimage* of a sequence $(x_{k_n}) \in \lambda_K^E$ is a sequence $(y_n) \in w$ defined by

$$y_n = \begin{cases} x_n, & \text{if } n \in K \\ 0, & \text{otherwise.} \end{cases}$$

A *canonical preimage* of a *step space* λ_K^E is a set of canonical preimages of all the elements in λ_K^E, i.e. y is in the canonical preimage of λ_K^E if and only if y is a *canonical preimage* of some $x \in \lambda_K^E$.

A sequence space E is said to be *monotone* if it contain the canonical preimages of its step spaces.

Throughout the article $\mathcal{Z}^I, \mathcal{Z}_0^I, \mathcal{Z}_\infty^I, m_{\mathcal{Z}}^I$ and $m_{\mathcal{Z}_0}^I$ represents Zweier $\mathcal{I}$-convergent, Zweier $\mathcal{I}$-null, Zweier bounded $\mathcal{I}$-convergent and Zweier bounded $\mathcal{I}$-null sequence space, respectively.

The following results will be used for establishing some results of this article.

Lemma 1 [41]. *The sequence space E is solid implies that E is monotone.*

Lemma 2 [16, Lemma 2.5]. *Let $K \in F(\mathcal{I})$ and $M \subseteq \mathbb{N}$. If $M \notin I$, then $M \cap K \notin \mathcal{I}$.*

3. Main results

In this article, we introduce following class of sequence spaces:

$$\mathcal{Z}^{\mathcal{I}}(M,p) = \left\{ x = (x_k) : \left\{ n \in \mathbb{N} : \left[M\left(\frac{|(Z^p x)_n - L|}{\rho} \right) \right]^{p_n} \geqslant \varepsilon \right\} \in \mathcal{I} \right\} \text{ for some } L \in \mathbb{C}$$

$$\mathcal{Z}_0^{\mathcal{I}}(M,p) = \left\{ x = (x_k) : \left\{ n \in \mathbb{N} : \left[M\left(\frac{|(Z^p x)_n|}{\rho} \right) \right]^{p_n} \geqslant \varepsilon \right\} \in \mathcal{I} \right\}$$

$$\mathcal{Z}_\infty^{\mathcal{I}}(M,p) = \left\{ x = (x_k) : \left\{ n \in \mathbb{N} : \exists K > 0 \left[M\left(\frac{|(Z^p x)_n|}{\rho} \right) \right]^{p_n} \geqslant K \right\} \in \mathcal{I} \right\}.$$

Also we write

$$m_{\mathcal{Z}}^{\mathcal{I}}(M,p) = \mathcal{Z}^{\mathcal{I}}(M,p) \cap \mathcal{Z}_\infty^{\mathcal{I}}(M,p) \text{ and } m_{\mathcal{Z}_0}^{\mathcal{I}}(M,p) = \mathcal{Z}_0^{\mathcal{I}}(M,p) \cap \mathcal{Z}_\infty^{\mathcal{I}}(M,p),$$

where $p = (p_k)$ is a sequence of positive real numbers.

Theorem 1. *The class of sequences $\mathcal{Z}^{\mathcal{I}}(M,p), \mathcal{Z}_0^{\mathcal{I}}(M,p)$, $m_{\mathcal{Z}}^{\mathcal{I}}(M,p)$ and $m_{\mathcal{Z}_0}^{\mathcal{I}}(M,p)$ are linear spaces.*

Proof. We shall prove the result for the space $\mathcal{Z}^{\mathcal{I}}(M,p)$. Let $x = (x_k), y = (y_k) \in \mathcal{Z}^{\mathcal{I}}(M,p)$ and let α, β be scalars. For given $\varepsilon > 0$, we denote

$$A_1 = \left\{ x = (x_k) : \left\{ n \in \mathbb{N} : \left[M\left(\frac{|(Z^p x)_n - L_1|}{\rho_1} \right) \right]^{p_n} \geqslant \frac{\varepsilon}{2} \right\} \in \mathcal{I} \right\} \text{ for some } L_1 \in \mathbb{C}$$

$$A_2 = \left\{ y = (y_k) : \left\{ n \in \mathbb{N} : \left[M\left(\frac{|(Z^p y)_n - L_2|}{\rho_2} \right) \right]^{p_n} \geqslant \frac{\varepsilon}{2} \right\} \in \mathcal{I} \right\} \text{ for some } L_2 \in \mathbb{C}.$$

Let $\rho_3 = \max\{2|\alpha|\rho_1, 2|\beta|\rho_2\}$. Since M is non-decreasing and convex function, we have

$$\begin{aligned}
&\left[M\left(\frac{|(\alpha(Z^p x)_n + \beta(Z^p y)_n) - (\alpha L_1 + \beta L_2)|}{\rho_3} \right) \right]^{p_n} \\
&\leqslant \left[M\left(\frac{|\alpha||(Z^p x)_n - L_1|}{\rho_3} \right) \right]^{p_n} + \left[M\left(\frac{|\beta||(Z^p y)_n - L_2|}{\rho_3} \right) \right]^{p_n} \\
&\leqslant \left[M\left(\frac{|(Z^p x)_n - L_1|}{\rho_1} \right) \right]^{p_n} + \left[M\left(\frac{|(Z^p y)_n - L_2|}{\rho_2} \right) \right]^{p_n}
\end{aligned}$$

$$\begin{aligned}
&\left\{ n \in \mathbb{N} : \left[M\left(\frac{|(\alpha(Z^p x)_n + \beta(Z^p y)_n) - (\alpha L_1 + \beta L_2)|}{\rho_3} \right) \right]^{p_n} \geqslant \varepsilon \right\} \\
&\subseteq \left\{ n \in \mathbb{N} : \left[M\left(\frac{|\alpha||(Z^p x)_n - L_1|}{\rho_1} \right) \right]^{p_n} \geqslant \frac{\varepsilon}{2} \right\} \cup \left\{ n \in \mathbb{N} : \left[M\left(\frac{|\beta||(Z^p y)_n - L_2|}{\rho_2} \right) \right]^{p_n} \geqslant \frac{\varepsilon}{2} \right\} \\
&\subseteq A_1 \cup A_2 \in \mathcal{I}
\end{aligned}$$

Therefore $(\alpha(Z^p x)_n + \beta(Z^p y)_n) \in \mathcal{Z}^{\mathcal{I}}(M,p)$. Hence $\mathcal{Z}^{\mathcal{I}}(M,p)$ is a linear space.

The proof for other spaces will follow similarly. □

Theorem 2. *The spaces $m_{\mathcal{Z}}^{\mathcal{I}}(M,p)$ and $m_{\mathcal{Z}_0}^{\mathcal{I}}(M,p)$ are paranormed spaces, with the paranorm g defined by*

$$g(x) = \inf\left\{ \rho^{\frac{p_n}{H}} : \sup_n M\left(\frac{|(Z^p x)_n|}{\rho} \right) \leqslant 1, \text{ for some } \rho > 0 \right\},$$

where $H = \max\{1, \sup_n p_n\}$.

Proof. Clearly $g(-x) = g(x)$ and $g(0) = 0$. Let $x = (x_k)$ and $y = (y_k)$ be two elements in $m_{\mathcal{Z}}^{\mathcal{I}}(M,p)$. Now for ρ_1, $\rho_2 > 0$, we denote

$$A_1 = \left\{ \rho_1 : \sup_n M\left(\frac{|(Z^p x)_n|}{\rho_1} \right) \leqslant 1 \right\}$$

and

$$A_2 = \left\{ \rho_2 : \sup_n M\left(\frac{|(Z^p y)_n|}{\rho_2} \right) \leqslant 1 \right\}.$$

Let us take $\rho = \rho_1 + \rho_2$. Then using the convexity of Orlicz functions M, we obtain

$$M\left(\frac{|(Z^p(x+y))_n|}{\rho} \right) \leqslant \frac{\rho_1}{\rho_1 + \rho_2} M\left(\frac{|(Z^p x)_n|}{\rho_1} \right) + \frac{\rho_2}{\rho_1 + \rho_2} M\left(\frac{|(Z^p y)_n|}{\rho_2} \right),$$

which in terms give us,

$$\sup_n \left[M\left(\frac{|(Z^p(x+y))_n|}{\rho} \right) \right]^{p_n} \leqslant 1$$

and

$$\begin{aligned}
g(x+y) &= \inf\left\{ (\rho_1 + \rho_2)^{\frac{p_n}{H}} : \rho_1 \in A_1, \rho_2 \in A_2 \right\} \\
&\leqslant \inf\left\{ \rho_1^{\frac{p_n}{H}} : \rho_1 \in A_1 \right\} + \inf\left\{ \rho_2^{\frac{p_n}{H}} : \rho_2 \in A_2 \right\} \\
&= g(x) + g(y).
\end{aligned}$$

Let $t^m \to L$, where $t^m, L \in \mathbb{C}$ and let $g(x^m - x) \to 0$ as $m \to \infty$. To prove that $g(t^m x^m - Lx) \to 0$ as $m \to \infty$. We put

$$A_3 = \left\{ \rho_m > 0 : \sup_n \left[M\left(\frac{|(Z^p x^m)_n|}{\rho_m} \right) \right]^{p_n} \leqslant 1 \right\}$$

and

$$A_4 = \left\{ \rho_s > 0 : \sup_n \left[M\left(\frac{|(Z^p(x^m - x))_n|}{\rho_s} \right) \right]^{p_n} \leqslant 1 \right\}.$$

By the continuity M, we observe that

$$\begin{aligned}
M\left(\frac{|(Z^p(t^m x^m - Lx))_n|}{|t^m - L|\rho_m + |L|\rho_s} \right) &\leqslant M\left(\frac{|(Z^p(t^m x^m - Lx^m))_n|}{|t^m - L|\rho_m + |L|\rho_s} \right) + M\left(\frac{|(Z^p(Lx^m - Lx))_n|}{|t^m - L|\rho_m + |L|\rho_s} \right) \\
&\leqslant \frac{|t^m - L|\rho_m}{|t^m - L|\rho_m + |L|\rho_s} M\left(\frac{|(Z^p(x^m))_n|}{\rho_m} \right) + \frac{|L|\rho_s}{|t^m - L|\rho_m + |L|\rho_s} M\left(\frac{|(Z^p(x^m - x))_n|}{\rho_s} \right).
\end{aligned}$$

From the above inequality it follows that

$$\sup_n\left[M\left(\frac{|(Z^p(t^m x^m - Lx))_n|}{|t^m - L|\rho_m + |L|\rho_s}\right)\right]^{p_n} \leqslant 1$$

and consequently,

$$\begin{aligned} g(t^m x^m - Lx) &= \inf\left\{[|t^m - L|\rho_m + |L|\rho_s]^{\frac{p_n}{H}} : \rho_m \in A_3, \rho_s \in A_4\right\} \\ &\leqslant |t^m - L|^{\frac{p_n}{H}} \inf\left\{(\rho_m)^{\frac{p_n}{H}} : \rho_m \in A_3\right\} \\ &+ |L|^{\frac{p_n}{H}} \inf\left\{(\rho_s)^{\frac{p_n}{H}} : \rho_s \in A_4\right\} \\ &\leqslant \max\left\{1, |t^m - L|^{\frac{p_n}{H}}\right\} g(x^m) \\ &+ \max\left\{1, |L|^{\frac{p_n}{H}}\right\} g(x^m - x) \qquad (1) \end{aligned}$$

Note that $g(x^m) \leqslant g(x) + g(x^m - x)$ for all $m \in \mathbb{N}$.

Hence by our assumption the right hand side of the relation (1) tends to 0 as $m \to \infty$ and the result follows. This completes the proof. □

Theorem 3. *$m_{\mathcal{Z}}^{\mathcal{I}}(M,p)$ is a closed subspace of $l_\infty(M,p)$.*

Proof. Let $\left(x_k^{(i)}\right)$ be a Cauchy sequence in $m_{\mathcal{Z}}^{\mathcal{I}}(M,p)$ such that $x^{(i)} \to x$. We show that $x \in m_{\mathcal{Z}}^{\mathcal{I}}(M,p)$.

Since $(x_k^{(i)}) \in m_{\mathcal{Z}}^{\mathcal{I}}(M,p)$, then there exists a_n such that

$$\left\{n \in \mathbb{N} : \left[M\left(\frac{|(Z^p x^{(i)})_n - a_n|}{\rho}\right)\right]^{p_n} \geqslant \varepsilon\right\} \in \mathcal{I}$$

We need to show that

(i) (a_i) converges to a.
(ii) If $U = \left\{x = x_k : \left\{n \in \mathbb{N} : \left[M\left(\frac{|(Z^p x)_n - L|}{\rho}\right)\right]^{p_n} < \varepsilon\right\}\right\}$, then $U^c \in \mathcal{I}$.

(i) Since $(x_k^{(i)})$ is a Cauchy sequence in $m_{\mathcal{Z}}^{\mathcal{I}}(M,p)$ then for a given $\varepsilon > 0$, there exists $k_0 \in \mathbb{N}$ such that

$\sup_n \left[M\left(\frac{|(Z^p x^{(i)})_n - (Z^p x^{(j)})_n|}{\rho}\right)\right]^{p_n} < \frac{\varepsilon}{3}$, for all $i,j \geqslant k_0$.

For a given $\varepsilon > 0$, we have

$$B_{ij} = \left\{n \in \mathbb{N} : \left[M\left(\frac{|(Z^p x^{(i)})_n - (Z^p x^{(j)})_n|}{\rho}\right)\right]^{p_n} < \frac{\varepsilon}{3}\right\}$$

$$B_i = \left\{n \in \mathbb{N} : \left[M\left(\frac{|(Z^p x^{(i)} - a_i)_n|}{\rho}\right)\right]^{p_n} < \frac{\varepsilon}{3}\right\}$$

$$B_j = \left\{n \in \mathbb{N} : \left[M\left(\frac{|(Z^p x^{(j)} - a_j)_n|}{\rho}\right)\right]^{p_n} < \frac{\varepsilon}{3}\right\}.$$

Then $B_{ij}, B_i, B_j \in \mathcal{I}$.

Let $B^c = B_{ji}^c \cup B_i^c \cup B_j^c$, where $B = \left\{n \in \mathbb{N} : \left[M\left(\frac{|a_i - a_j|}{\rho}\right)\right]^{p_n} < \varepsilon\right\}$.

Then $B^c \in \mathcal{I}$. We choose $n_0 \in B^c$, then for each $j, i \geqslant n_0$, we have

$$\begin{aligned} \left\{n \in \mathbb{N} : \left[M\left(\frac{|a_i - a_n|}{\rho}\right)\right]^{p_n} < \varepsilon\right\} &\supseteq \left\{n \in \mathbb{N} : \left[M\left(\frac{|(Z^p x^{(i)} - a_i)_n|}{\rho}\right)\right]^{p_n} < \frac{\varepsilon}{3}\right\} \\ &\cap \left\{n \in \mathbb{N} : \left[M\left(\frac{|(Z^p x^{(i)} - a_i)_n|}{\rho}\right)\right]^{p_n} < \frac{\varepsilon}{3}\right\} \\ &\cap \left\{n \in \mathbb{N} : \left[M\left(\frac{|(Z^p x^{(j)} - a_j)_n|}{\rho}\right)\right]^{p_n} < \frac{\varepsilon}{3}\right\}. \end{aligned}$$

Then (a_j) is a Cauchy sequence of scalars in $\mathbb{C}$ and so there exists a scalar $a \in \mathbb{C}$ such that $a_j \to a$ as $j \to \infty$.

(ii) Let $0 < \delta < 1$ be given. We show that if $U = \left\{x = (x_k) : \left\{n \in \mathbb{N} : \left[M\left(\frac{|(Z^p x - a)_n|}{\rho}\right)\right]^{p_n} < \delta\right\}\right\}$, then $U^c \in \mathcal{I}$. Since $x^{(i)} \to x$, then there exists $q_0 \in \mathbb{N}$ such that

$$P = \left\{n \in \mathbb{N} : \left[M\left(\frac{|(Z^p x^{(q_0)})_n - (Z^p x)_n|}{\rho}\right)\right]^{p_n} < \left(\frac{\delta}{3D}\right)^H\right\} \qquad (2)$$

which implies $P^c \in \mathcal{I}$.

The number q_0 can be so chosen that together with (2), we have

$$Q = \left\{n \in \mathbb{N} : \left[M\left(\frac{|a_{q_0} - a|}{\rho}\right)\right]^{p_n} < \left(\frac{\delta}{3D}\right)^H\right\}.$$

Then we have $Q^c \in \mathcal{I}$.

Since $\left\{n \in \mathbb{N} : \left[M\left(\frac{|(Z^p x^{(q_0)} - a_{q_0})_n|}{\rho}\right)\right]^{p_n} \geqslant \delta\right\} \in \mathcal{I}$. Then we have a subset $S \in \mathbb{N}$ such that $S^c \in \mathcal{I}$, where $S = \left\{n \in \mathbb{N} : \left[M\left(\frac{|(Z^p x^{(q_0)} - a_{q_0})_n|}{\rho}\right)\right]^{p_n} < \left(\frac{\delta}{3D}\right)^H\right\}$.

Let $U^c = P^c \cup Q^c \cup S^c$, where $U = \left\{n \in \mathbb{N} : \left[M\left(\frac{|(Z^p x - a)_n|}{\rho}\right)\right]^{p_n} < \delta\right\}$.

Therefore for each $k \in U^c$, we have

$$\begin{aligned} \left\{n \in \mathbb{N} : \left[M\left(\frac{|(Z^p x - a)_n|}{\rho}\right)\right]^{p_n} < \delta\right\} &\supseteq \left\{n \in \mathbb{N} : \left[M\left(\frac{|(Z^p x^{(q_0)})_n - (Z^p x)_n|}{\rho}\right)\right]^{p_n} < \left(\frac{\delta}{3D}\right)^H\right\} \\ &\cap \left\{n \in \mathbb{N} : \left[M\left(\frac{|(Z^p x^{(q_0)} - a_{q_0})_n|}{\rho}\right)\right]^{p_n} < \left(\frac{\delta}{3D}\right)^H\right\} \\ &\cap \left\{n \in \mathbb{N} : \left[M\left(\frac{|a_{q_0} - a|}{\rho}\right)\right]^{p_n} < \left(\frac{\delta}{3D}\right)^H\right\} \end{aligned}$$

Then the result follows.

Since the inclusions $m_{\mathcal{Z}}^{\mathcal{I}}(M,p) \subset l_\infty(M,p)$ and $m_{\mathcal{Z}_0}^{\mathcal{I}}(M,p) \subset l_\infty(M,p)$ are strict, so in view of Theorem 3 we have the following results. □

Theorem 4. *The spaces $m_{\mathcal{Z}}^{\mathcal{I}}(M,p)$ and $m_{\mathcal{Z}_0}^{\mathcal{I}}(M,p)$ are nowhere dense subsets of $l_\infty(M,p)$.*

Theorem 5. *The spaces $m_{\mathcal{Z}}^{\mathcal{I}}(M,p)$ and $m_{\mathcal{Z}_0}^{\mathcal{I}}(M,p)$ are not separable.*

Proof. We shall prove the result for the space $m_{\mathcal{Z}}^{\mathcal{I}}(M,p)$.

Let A be an infinite subset of $\mathbb{N}$ of increasing natural numbers such that $A \in \mathcal{I}$. Let

$$p_n = \begin{cases} 1, \text{if } n \in A \\ 2, \text{otherwise.} \end{cases}$$

Let $P_0 = \{(x_n): x_n = 0 \text{ or } 1, n \in A \text{ and } x_n = 0, \text{ otherwise}\}$. Since A is infinite, so P_0 is uncountable.

Consider the class of open balls $B_1 = \left\{B\left(z, \frac{1}{2}\right) : z \in P_0\right\}$. Let C_1 be an open cover of $m_{\mathcal{Z}_0}^{\mathcal{I}}(M,p)$ and $m_{\mathcal{Z}}^{\mathcal{I}}(M,p)$ containing B_1. Since B_1 is uncountable, so C_1 cannot be reduced to a countable sub cover for $m_{\mathcal{Z}_0}^{\mathcal{I}}(M,p)$ as well as $m_{\mathcal{Z}}^{\mathcal{I}}(M,p)$. Thus $m_{\mathcal{Z}_0}^{\mathcal{I}}(M,p)$ and $m_{\mathcal{Z}}^{\mathcal{I}}(M,p)$ are not separable. □

Theorem 6. *Let $G = sup_n p_n < \infty$ and $\mathcal{I}$ an admissible ideal. Then the following are equivalent:*

(a) $x = (x_k) \in \mathcal{Z}^{\mathcal{I}}(M,p)$;
(b) *there exists* $y = (y_k) \in \mathcal{Z}(M,p)$ *such that* $x_k = y_k$ *for a.a. k.r.*$\mathcal{I}$*;*
(c) *there exists* $y = (y_k) \in \mathcal{Z}(M,p)$ *and* $z = (z_k) \in \mathcal{Z}_0^{\mathcal{I}}(M,p)$ *such that* $x_k = y_k + z_k$ *for all* $n \in \mathbb{N}$ *and* $\left\{y = (y_k) : \left\{n \in \mathbb{N} : \left[M\left(\frac{|(Z^p y)_n - L|}{\rho}\right)\right]^{p_n} \geqslant \varepsilon\right\}\right\} \in \mathcal{I}$,
(d) *there exists a subset* $K = \{n_1 < n_2 < \cdots\}$ *of* $\mathbb{N}$ *such that* $K \in \mathcal{F}(\mathcal{I})$ *and* $\lim_{n\to\infty}\left[M\left(\frac{|(Z^p x)_n - L|}{\rho}\right)\right]^{p_n} = 0$.

Proof. (a) implies (b). Let $x = (x_k) \in \mathcal{Z}^{\mathcal{I}}(M,p)$. Then there exists $L \in \mathbb{C}$ such that

$$\left\{x = (x_k) : \left\{n \in \mathbb{N} : \left[M\left(\frac{|(Z^p x)_n - L|}{\rho}\right)\right]^{p_n} \geqslant \varepsilon\right\} \in \mathcal{I}\right\}$$

Let (m_t) be an increasing sequence with $m_t \in \mathbb{N}$ such that

$$\left\{x = (x_k) : \left\{n \leqslant m_t : \left[M\left(\frac{|(Z^p x)_n - L|}{\rho}\right)\right]^{p_n} \geqslant t^{-1}\right\} \in \mathcal{I}\right\}.$$

Define a sequence $y = (y_k)$ as $y_k = x_k$, for all $k \leqslant m_1$.

For

$$m_t < k \leqslant m_{t+1}, t \in \mathbb{N}, y_k = \begin{cases} x_k, & \text{for } \left[M\frac{|(Z^p x)_n - L|}{\rho}\right]^{P_n} < t^{-1} \\ L, & \text{otherwise.} \end{cases}.$$

Then $(y_k) \in \mathcal{Z}(M,p)$ and form the following inclusion

$$\{k \leqslant m_t : x_k \neq y_k\} \subseteq \left\{k \leqslant m_t : \left[M\left(\frac{|(Z^p x)_n - L|}{\rho}\right)\right]^{p_n} \geqslant \varepsilon\right\} \in \mathcal{I}.$$

We get $x_k = y_k$, for a.a.k.r. $\mathcal{I}$.

(b) implies (c). For $(x_k) \in \mathcal{Z}^{\mathcal{I}}(M,p)$. Then there exists $(y_k) \in \mathcal{Z}(M,p)$ such that $x_k = y_k$, for a.a. k.r.$\mathcal{I}$.

Let $K = \{k \in \mathbb{N} : x_k \neq y_k\}$, then $k \in \mathcal{I}$.

Defined a sequence (z_k)as $z_k = \begin{cases} x_k - y_k, if k \in K \\ 0, otherwise. \end{cases}$.

Then $(z_k) \in \mathcal{Z}_0^{\mathcal{I}}(M,p)$ and $(y_k) \in \mathcal{Z}(M,p)$.

(c) implies (d). Suppose (c) holds. Let $\varepsilon > 0$ be given.

Let

$$P_1 = \left\{z = (z_k) : \left\{n \in \mathbb{N} : \left[M\left(\frac{|(Z^p z)_n|}{\rho}\right)\right]^{p_n} \geqslant \varepsilon\right\}\right\} \in \mathcal{I}$$

and $K = P_1^c = \{k_1 < k_2 < \ldots..\} \in \mathcal{F}(\mathcal{I})$.

Then we have $\lim_{n\to\infty}\left[M\left(\frac{|(Z^p x)_n - L|}{\rho}\right)\right]^{p_n} = 0$.

(d) implies (a). Let $K = \{k_1 < k_2 < \ldots..\} \in \mathcal{F}(\mathcal{I})$ and $\lim_{n\to\infty}\left[M\left(\frac{|(Z^p x)_n - L|}{\rho}\right)\right]^{p_n} = 0$.

Then for any $\varepsilon > 0$ and by Lemma 2, we have

$$\left\{x = (x_k) : \left\{n \in \mathbb{N} : \left[M\left(\frac{|(Z^p x)_n - L|}{\rho}\right)\right]^{p_n} \geqslant \varepsilon\right\}\right\}$$
$$\subseteq K^c \cup \left\{x = (x_k) : \left\{n \in K : \left[M\left(\frac{|(Z^p x)_n - L|}{\rho}\right)\right]^{p_n} \geqslant \varepsilon\right\}\right\} \in \mathcal{I}.$$

Thus $(x_k) \in \mathcal{Z}^{\mathcal{I}}(M,p)$. □

Theorem 7. *Let $h = inf_n q_n$ and $G = sup_n q_n$. Then the following results are equivalent.*

(i) $G < \infty$ *and* $h > 0$.
(ii) $\mathcal{Z}_0^{\mathcal{I}}(M,p) = \mathcal{Z}_0^{\mathcal{I}}$.

Proof. Suppose that $G < \infty$ and $h > 0$, then the inequalities $\min\{1, s^h\} \leqslant s^{p_n} \leqslant \max\{1, s^G\}$ hold for any $s > 0$ and for all $n \in \mathbb{N}$. □

Therefore the equivalence of (i) and (ii) is obvious.

Theorem 8. *Let M_1 and M_2 be two Orlicz functions that satisfy the Δ_2 – condition, then*

(i) $W(M_2,p) \subseteq W(M_1 M_2,p)$
(ii) $W(M_1,p) \cap W(M_2,p) \subseteq W(M_1 + M_2,p)$, *where* $W = \mathcal{Z}^{\mathcal{I}}, \mathcal{Z}_0^{\mathcal{I}}, m_{\mathcal{Z}}^{\mathcal{I}}, m_{\mathcal{Z}_0}^{\mathcal{I}}$.

Proof. Let $x = (x_k) \in \mathcal{Z}^{\mathcal{I}}(M_2,p)$. Let $\varepsilon > 0$ be given. For some $\rho > 0$, we have

$$\left\{(x_k) : \left\{n \in \mathbb{N} : \left[M_2\left(\frac{|(Z^p x)_n - L|}{\rho}\right)\right]^{p_n} \geqslant \varepsilon\right\} \in \mathcal{I}\right\}. \quad (3)$$

Let $\varepsilon > 0$ and choose $0 < \lambda < 1$ such that $M_1\ (t) \leqslant \varepsilon$ *for* $0 \leqslant t \leqslant \lambda$. We define

$$y_n = \left[M_2\left(\frac{|(Z^p x)_n - L|}{\rho}\right)\right]$$

and consider

$$\lim_{n\in\mathbb{N}:0\leqslant y_n\leqslant\lambda}[M_1(y_n)]^{p_n} = \lim_{n\in\mathbb{N}:y_n\leqslant\lambda}[M_1(y_n)]^{p_n} + \lim_{n\in\mathbb{N}:y_n>\lambda}[M_1(y_n)]^{p_n}.$$

We have

$$\lim_{n\in\mathbb{N}:y_n\leqslant\lambda}[M_1(y_n)]^{p_n} \leqslant [M_1(2)]^G + \lim_{n\in\mathbb{N}:y_n\leqslant\lambda}[(y_n)]^{p_n}, \quad G = \sup_n p_n. \quad (4)$$

For second summation (i.e. $y_n > \lambda$), we go through the following procedure. We have

$$y_n < \frac{y_n}{\lambda} < 1 + \frac{y_n}{\lambda}$$

Since M_1 is non-decreasing and convex, it follows that

$$M_1(y_n) < M_1\left(1 + \frac{y_n}{\lambda}\right) \leqslant \frac{1}{2}M_1(2) + \frac{1}{2}M_1\left(\frac{2y_n}{\lambda}\right).$$

Since M_1 satisfies Δ_2 – conditions, we can write

$$M_1(y_n) < \frac{1}{2}K\frac{y_n}{\lambda}M_1(2) + \frac{1}{2}K\frac{y_n}{\lambda}M_1(2) = K\frac{y_n}{\lambda}M_1(2).$$

We get the following estimates:

$$\lim_{n\in\mathbb{N}:y_n>\lambda}[M_1(y_n)]^{p_n} \leqslant \max\{1, (K\lambda^{-1}M_1(2))^H\} \lim_{n\in\mathbb{N}:y_n>\lambda}[y_n]^{p_n}. \quad (5)$$

From (3)–(5), it follows that

$$\left\{(x_k) : \left\{n \in \mathbb{N} : \left[M_1 M_2\left(\frac{|(Z^p x)_n - L|}{\rho}\right)\right]^{p_n} \geqslant \varepsilon\right\} \in \mathcal{I}\right\}.$$

Hence $\mathcal{Z}^{\mathcal{I}}(M_2,p) \subseteq \mathcal{Z}^{\mathcal{I}}(M_1M_2,p)$.

(ii) Let $(x_k) \in \mathcal{Z}^{\mathcal{I}}(M_1,p) \cap \mathcal{Z}^{\mathcal{I}}(M_2,p)$. Let $\varepsilon > 0$ be given. Then there exists $\rho > 0$ such that

$$\left\{(x_k) : \left\{n \in \mathbb{N} : \left[M_1\left(\frac{|(Z^px)_n - L|}{\rho}\right)\right]^{p_n} \geqslant \varepsilon\right\} \in \mathcal{I}\right\}$$

and

$$\left\{(x_k) : \left\{n \in \mathbb{N} : \left[M_2\left(\frac{|(Z^px)_n - L|}{\rho}\right)\right]^{p_n} \geqslant \varepsilon\right\} \in \mathcal{I}\right\}.$$

The rest of the proof follows from the following relations.

$$\left\{n \in \mathbb{N} : \left[(M_1+M_2)\left(\frac{|(Z^px)_n - L|}{\rho}\right)\right]^{p_n} \geqslant \varepsilon\right\}$$
$$\subseteq \left\{n \in \mathbb{N} : \left[M_1\left(\frac{|(Z^px)_n - L|}{\rho}\right)\right]^{p_n} \geqslant \varepsilon\right\}$$
$$\cup \left\{n \in \mathbb{N} : \left[M_2\left(\frac{|(Z^px)_n - L|}{\rho}\right)\right]^{p_n} \geqslant \varepsilon\right\}.$$

Taking $M_2(x) = x$ and $M_1(x) = M(x)$ for all $x \in [0,\infty)$, we have the following result. □

Corollary 9. *$W \subseteq W(M,p)$ where $W = \mathcal{Z}^{\mathcal{I}}, \mathcal{Z}_0^{\mathcal{I}}, m_{\mathcal{Z}}^{\mathcal{I}}, m_{\mathcal{Z}_0}^{\mathcal{I}}$.*

Theorem 10. *Let $p = (p_n)$ and $r = (r_n)$ be two sequences of positive real numbers. Then $m_{\mathcal{Z}_0}^{\mathcal{I}}(M,p) \supseteq m_{\mathcal{Z}_0}^{\mathcal{I}}(M,r)$ if and only if $\lim_{n\in K}\inf\frac{p_n}{r_n} > 0$, where $K \subseteq \mathbb{N}$such that $K \in F(\mathcal{I})$.*

Proof. Let $\lim_{n\in K}\inf\frac{p_n}{r_n} > 0$ and $(x_n) \in m_{\mathcal{Z}_0}^{\mathcal{I}}(M,r)$. Then there exists $\beta > 0$ such that $p_n > \beta r_n$, for all sufficiently large $n \in K$.

Let $(x_k) \in m_{\mathcal{Z}_0}^{\mathcal{I}}(M,r)$ then for given $\varepsilon > 0$, we have

$$B = \left\{x = (x_k) : \left\{n \in \mathbb{N} : \left[M\left(\frac{|(Z^px)_n|}{\rho}\right)\right]^{r_n} \geqslant \varepsilon\right\} \in \mathcal{I}\right\}.$$

Let $G_0 = K^c \cup B$. Then we get $G_0 \in \mathcal{I}$.

Then for all sufficiently large $n \in G_0$,

$$\left\{(x_k) : \left\{n \in \mathbb{N} : \left[M\left(\frac{|(Z^px)_n|}{\rho}\right)\right]^{p_n} \geqslant \varepsilon\right\}\right\}$$
$$\subseteq \left\{(x_k) : \left\{n \in \mathbb{N} : \left[M\left(\frac{|(Z^px)_n|}{\rho}\right)\right]^{\beta r_n} \geqslant \varepsilon\right\} \in \mathcal{I}\right\},$$

i.e.

$$\left\{x = (x_k) : \left\{n \in \mathbb{N} : \left[M\left(\frac{|(Z^px)_n|}{\rho}\right)\right]^{r_n} \geqslant \varepsilon\right\} \in \mathcal{I}\right\}.$$

Therefore $(x_k) \in m_{\mathcal{Z}_0}^{\mathcal{I}}(M,p)$.

The converse part of the result follows obviously. □

Theorem 11. *Let $p = (p_n)$ and $r = (r_n)$ be two sequences of positive real numbers. Then $m_{\mathcal{Z}_0}^{\mathcal{I}}(M,r) \supseteq m_{\mathcal{Z}_0}^{\mathcal{I}}(M,p)$ if and only if $\lim_{n\in K}\inf\frac{r_n}{p_n} > 0$, where $K \subseteq \mathbb{N}$ such that $K \in F(\mathcal{I})$.*

Proof. The proof follows the Theorems 10 and 6 in [19]. □

Theorem 12. *Let $p = (p_n)$ and $r = (r_n)$ be two sequences of positive real numbers. Then $m_{\mathcal{Z}_0}^{\mathcal{I}}(M,r) = m_{\mathcal{Z}_0}^{\mathcal{I}}(M,p)$ if and only if $\lim_{n\in K}\inf\frac{p_n}{r_n} > 0$and $\lim_{n\in K}\inf\frac{r_n}{p_n} > 0$, where $K \subseteq \mathbb{N}$ such that $K^c \in \mathcal{I}$.*

Proof. The proof of the theorem follows from the Theorems 10 and 11 and the Corollary 7 in [19]. □

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