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Some double sequence spaces of interval numbers defined by Orlicz function

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Abstract In this paper we introduce some interval valued double sequence spaces defined by Orlicz function and study different properties of these spaces like inclusion relations, solidity, etc. We establish some inclusion relations among them. Also we introduce the concept of double statistical convergence for interval number sequences and give an inclusion relation between interval valued double sequence spaces.

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1. Introduction

The idea of statistical convergence for single sequences was introduced by Fast [1] in 1951. Schoenberg [2] studied statistical convergence as a summability method and listed some of elementary properties of statistical convergence. Both of these authors noted that if bounded sequence is statistically convergent, then it is Cesaro summable. Existing work on statistical convergence appears to have been restricted to real or complex sequence, but several authors extended the idea to apply to sequences of fuzzy numbers and also introduced and discussed the concept of statistically sequences of fuzzy numbers.

Interval arithmetic was first suggested by Dwyer [3] in 1951. Development of interval arithmetic as a formal system and evidence of its value as a computational device was provided by Moore [4] in 1959 and Moore and Yang [5] 1962. Furthermore, Moore and others [6–9] have developed applications to differential equations.

Chiao in [10] introduced sequence of interval numbers and defined usual convergence of sequences of interval number. Şengönül and Eryilmaz in [11] introduced and studied bounded and convergent sequence spaces of interval numbers and showed that these spaces are complete metric space. Recently Esi in [12,13] introduced and studied strongly almost λ–convergence and statistically almost λ–convergence of interval numbers and lacunary sequence spaces of interval numbers, respectively.

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A set consisting of a closed interval of real numbers x such that $a \leqslant x \leqslant b$ is called an interval number. A real interval can also be considered as a set. Thus we can investigate some properties of interval numbers, for instance arithmetic properties or analysis properties.We denote the set of all real valued closed intervals by $\mathrm{I}\mathbb{R}$. Any elements of $\mathrm{I}\mathbb{R}$ is called closed interval and denoted by $\bar{x}$. That is $\bar{x} = \{x \in \mathbb{R} : a \leqslant x \leqslant b\}$. An interval number $\bar{x}$ is a closed subset of real numbers [10]. Let x_l and x_r be first and last points of $\bar{x}$ interval number, respectively. For $\bar{x}_1, \bar{x}_2 \in \mathrm{I}\mathbb{R}$, we have $\bar{x}_1 = \bar{x}_2 \iff x_{1_l} = x_{2_l}$, $x_{1_r} = x_{2_r}$. $\bar{x}_1 + \bar{x}_2 = \{x \in \mathbb{R} : x_{1_l} + x_{2_l} \leqslant x \leqslant x_{1_r} + x_{2_r}\}$, and if $\alpha \geqslant 0$, then $\alpha\bar{x} = \{x \in \mathbb{R} : \alpha x_{1_l} \leqslant x \leqslant \alpha x_{1_r}\}$ and if $\alpha < 0$, then $\alpha\bar{x} = \{x \in \mathbb{R} : \alpha x_{1_r} \leqslant x \leqslant \alpha x_{1_l}\}$,

$$\bar{x}_1.\bar{x}_2 = \left\{ \begin{array}{l} x \in \mathbb{R} : \min\{x_{1_l}.x_{2_l}, x_{1_l}.x_{2_r}, x_{1_r}.x_{2_l}, x_{1_r}.x_{2_r}\} \leqslant x \\ \leqslant \max\{x_{1_l}.x_{2_l}, x_{1_l}.x_{2_r}, x_{1_r}.x_{2_l}, x_{1_r}.x_{2_r}\} \end{array} \right\}.$$

The set of all interval numbers $\mathrm{I}\mathbb{R}$ is a complete metric space defined by

$$d(\bar{x}_1, \bar{x}_2) = \max\{|x_{1_l} - x_{2_l}|, |x_{1_r} - x_{2_r}|\} \quad [4].$$

In the special case $\bar{x}_1 = [a, a]$ and $\bar{x}_2 = [b, b]$, we obtain usual metric of $\mathbb{R}$.

Let us define transformation $f : \mathbb{N} \to \mathbb{R}$ by $k \to f(k) = \bar{x}$, $\bar{x} = (\bar{x}_k)$. Then $\bar{x} = (\bar{x}_k)$ is called sequence of interval numbers. The $\bar{x}_k$ is called kth term of sequence $\bar{x} = (\bar{x}_k)$. w^i denotes the set of all interval numbers with real terms and the algebric properties of w^i can be found in [10].

Now we give the definition of convergence of interval numbers:

Definition 1.1 [10]. A sequence $\bar{x} = (\bar{x}_k)$ of interval numbers is said to be convergent to the interval number $\bar{x}_o$ if for each $\varepsilon > 0$ there exists a positive integer k_o such that $d(\bar{x}_k, \bar{x}_o) < \varepsilon$ for all $k \geqslant k_o$ and we denote it by $\lim_k \bar{x}_k = \bar{x}_o$.

Thus, $\lim_k \bar{x}_k = \bar{x}_o \iff \lim_k x_{k_l} = x_{o_l}$ and $\lim_k x_{k_r} = x_{o_r}$.

Recall in [14,15] that an Orlicz function M is continuous, convex, nondecreasing function define for $x > 0$ such that $M(0) = 0$ and $M(x) > 0$. If convexity of Orlicz function is replaced by $M(x + y) \leqslant M(x) + M(y)$ then this function is called the modulus function and characterized by Ruckle [17]. An Orlicz function M is said to satisfy Δ_2-*condition* for all values u, if there exists $K > 0$ such that $M(2u) \leqslant KM(u)$, $u \geqslant 0$. Subsequently, the notion of Orlicz function was used to defined sequence spaces by Altin et al [18], Tripathy and Mahanta [19], Tripathy et al [20], Tripathy and Sarma [21] and many others.

Let's define transformation f from $\mathbb{N} \times \mathbb{N}$ to $\mathbb{R}$ by $i, j \to f(i, j) = \bar{x}$, $\bar{x} = (\bar{x}_{i,j})$. Then $\bar{x} = (\bar{x}_{i,j})$ is called sequence of double interval numbers. The $\bar{x}_{i,j}$ is called (i, j)th term of sequence $\bar{x} = (\bar{x}_{i,j})$.

Definition 2.2. An interval valued double sequence $\bar{x} = (\bar{x}_{i,j})$ is said to be convergent in the Pringsheim's sense or P-convergent to an interval number $\bar{x}_o$, if for every $\varepsilon > 0$, there exists $N \in \mathbb{N}$ such that

$$d(\bar{x}_{i,j}, \bar{x}_o) < \varepsilon \text{ for } i, j > N$$

and we denote it by $P - lim\bar{x}_{i,j} = \bar{x}_o$. The interval number $\bar{x}_o$ is called the Pringsheim limit of $\bar{x} = (\bar{x}_{i,j})$. More exactly, we say that a double sequence of interval numbers $\bar{x} = (\bar{x}_{i,j})$ converges to a finite interval number $\bar{x}_o$ if $\bar{x}_{i,j}$ tends to $\bar{x}_o$ as both i and j tend to infinity independently of each another. We denote by $\bar{c}^2$ the set of all double convergent interval numbers of double interval numbers.

Definition 2.3. An interval valued double sequence $\bar{x} = (\bar{x}_{i,j})$ is bounded if there exists a positive number M such that $d(\bar{x}_{i,j}, \bar{x}_o) \leqslant M$ for all $i, j \in \mathbb{N}$. We will denote the set of all bounded double interval number sequences by $\bar{l}^2_\infty$. It should be noted that, similarly to the case of double sequences, $\bar{c}^2$ is not the subset of $\bar{l}^2_\infty$.

Let $p = (p_{i,j})$ be a double sequence of positive real numbers. If $0 < p_{i,j} \leqslant \sup_{i,j} p_{i,j} = H < \infty$ and $D = \max(1, 2^{H-1})$, then for $a_{i,j}, b_{i,j} \in \mathbb{R}$ for all $i, j \in \mathbb{N}$, we have

$$|a_{i,j} + b_{i,j}|^{p_{i,j}} \leqslant D(|a_{i,j}|^{p_{i,j}} + |b_{i,j}|^{p_{i,j}}).$$

2. Results

In this paper, we define new double sequence spaces for interval sequences as follows.

Let M be an Orlicz function and $p = (p_{i,j})$ be a double sequence of strictly positive real numbers. We introduce the following sequence spaces:

$${}_2\bar{w}(M, p) = \left\{ \begin{array}{l} \bar{x} = (\bar{x}_{i,j}) : P - \lim_{m,n} \frac{1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} \left[M\left(\frac{d(\bar{x}_{i,j}, \bar{x}_o)}{\rho} \right) \right]^{p_{i,j}} = 0, \\ \textit{for some } \rho > 0 \end{array} \right\},$$

$${}_2\bar{w}_o(M, p) = \left\{ \begin{array}{l} \bar{x} = (\bar{x}_{i,j}) : P - \lim_{m,n} \frac{1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} \left[M\left(\frac{d(\bar{x}_{i,j}, \bar{0})}{\rho} \right) \right]^{p_{i,j}} = 0, \\ \textit{for some } \rho > 0 \end{array} \right\}$$

and

$${}_2\bar{w}_\infty(M, p) = \left\{ \begin{array}{l} \bar{x} = (\bar{x}_{i,j}) : \sup_{m,n} \frac{1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} \left[M\left(\frac{d(\bar{x}_{i,j}, \bar{0})}{\rho} \right) \right]^{p_{i,j}} < \infty, \\ \textit{for some } \rho > 0 \end{array} \right\}.$$

Theorem 2.1. (a) *If* $0 < p_{i,j} < q_{i,j}$ *and* $\left(\frac{p_{i,j}}{q_{i,j}} \right)$ *is bounded, then* ${}_2\bar{w}(M, p) {}_2\bar{w}(M, q)$

(b) ${}_2\bar{w}(M, p) {}_2\bar{w}_\infty(M, p)$ *and* ${}_2\bar{w}_o(M, p) {}_2\bar{w}_\infty(M, p)$.

Proof. (a) If we take $\left[M\left(\frac{d(\bar{x}_{i,j}, \bar{x}_o)}{\rho} \right) \right]^{p_{i,j}} = w_{i,j}$ for all i,j $\in \mathbb{N}$, then using the same technique employed in the proof of Theorem 2.9 from [16], we get the result.

(b) It is easy, so omitted. □

Theorem 2.2. (a) *If* $0 < \inf_{i,j} p_{i,j} \leqslant p_{i,j} < 1$, *then* ${}_2\bar{w}(M, p) \subset {}_2\bar{w}(M)$,

(b) *If* $1 < p_{i,j} \leqslant \sup_{i,j} p_{i,j} < \infty$, *then* ${}_2\bar{w}(M) \subset {}_2\bar{w}(M, p)$.

Proof. The first part of the result follows from the inequality

$$\frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n}M\left(\frac{d(\bar{x}_{i,j},\bar{x}_o)}{\rho}\right)\leqslant\frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n}\left[M\left(\frac{d(\bar{x}_{i,j},\bar{x}_o)}{\rho}\right)\right]^{p_{i,j}}$$

and the second part of the result follows from the inequality

$$\frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n}\left[M\left(\frac{d(\bar{x}_{i,j},\bar{x}_o)}{\rho}\right)\right]^{p_{i,j}}\leqslant\frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n}M\left(\frac{d(\bar{x}_{i,j},\bar{x}_o)}{\rho}\right).$$

This completes the proof. □

Theorem 2.3. *Let M_1 and M_2 be two Orlicz functions. Then* ${}_2\bar{w}(M_1,p)\cap{}_2\bar{w}(M_2,p)\subset{}_2\bar{w}(M_1+M_2,p)$.

Proof. Let $(\bar{x}_{i,j})\in{}_2\bar{w}(M_1,p)\cap{}_2\bar{w}(M_2,p)$. Then

$$P-\lim_{m,n}\frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n}\left[M_1\left(\frac{d(\bar{x}_{i,j},\bar{x}_o)}{\rho_1}\right)\right]^{p_{i,j}}=0,\ \text{for some } \rho_1>0$$

and

$$P-\lim_{m,n}\frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n}\left[M_2\left(\frac{d(\bar{x}_{i,j},\bar{x}_o)}{\rho_2}\right)\right]^{p_{i,j}}=0,\ \text{for some } \rho_2>0.$$

Let $\rho=\max\{\rho_1,\rho_2\}$. The result follows from the following inequality

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\left[(M_1+M_2)\left(\frac{d(\bar{x}_{i,j},\bar{x}_o)}{\rho}\right)\right]^{p_{i,j}}$$
$$\leqslant D\left(\sum_{i=1}^{m}\sum_{j=1}^{n}\left[M_1\left(\frac{d(\bar{x}_{i,j},\bar{x}_o)}{\rho_1}\right)\right]^{p_{i,j}}+\sum_{i=1}^{m}\sum_{j=1}^{n}\left[M_2\left(\frac{d(\bar{x}_{i,j},\bar{x}_o)}{\rho_2}\right)\right]^{p_{i,j}}\right).$$

This completes the proof. □

Theorem 2.4. *The double sequence space ${}_2\bar{w}_\infty(M,p)$ is solid and hence monotone.*

Proof. Let $(\bar{x}_{i,j})\in{}_2\bar{w}_\infty(M,p)$ and $(\alpha_{i,j})$ be a scalar sequence such that $|\alpha_{i,j}|\leqslant 1$ for all $i,j\in\mathbb{N}$. Then

$$M\left(\frac{d(\alpha_{i,j}\bar{x}_{i,j},\bar{0})}{\rho}\right)\leqslant M\left(\frac{d(\bar{x}_{i,j},\bar{0})}{\rho}\right)$$
$$\Rightarrow\sup_{m,n}\frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n}\left[M\left(\frac{d(\alpha_{i,j}\bar{x}_{i,j},\bar{0})}{\rho}\right)\right]^{p_{i,j}}$$
$$\leqslant\sup_{m,n}\frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n}\left[M\left(\frac{d(\bar{x}_{i,j},\bar{0})}{\rho}\right)\right]^{p_{i,j}}<\infty.$$

This completes the proof. □

Now we give the definition of double statistical convergence for interval numbers as follows:

Definition 2.4. The double interval sequence $\bar{x}=(\bar{x}_{i,j})$ is said to be double statistical convergent to an interval $\bar{x}_o$ provided that for every $\varepsilon>0$

$$P-\lim_{m,n}\frac{1}{mn}|\{i\leqslant m,j\leqslant n:d(\bar{x}_{i,j},\bar{x}_o)\geqslant\varepsilon\}|=0.$$

In this case, we write $\bar{s}_2-\lim\bar{x}_{i,j}=\bar{x}_o$ and denote the set of all double statistically convergent sequences of interval numbers by $\bar{s}_2$. We shall now establish an inclusion theorem between $\bar{s}_2$ and ${}_2\bar{w}(M,p)$.

Theorem 2.5. *Let M be an Orlicz function and $0<h\leqslant\inf_{i,j}p_{i,j}\leqslant\sup_{i,j}p_{i,j}=H<\infty$, then ${}_2\bar{w}(M,p)\subset\bar{s}_2$.*

Proof. Let $\bar{x}=(\bar{x}_{i,j})\in{}_2\bar{w}(M,p)$. Then there exists $r>0$ such that

$$\frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n}\left[M\left(\frac{d(\bar{x}_{i,j},\bar{x}_o)}{r}\right)\right]^{p_{i,j}}\to 0$$

as $(i,j)\to\infty$ in the Pringsheim sense. If $\varepsilon>0$, then we obtain the following:

$$\frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n}\left[M\left(\frac{d(\bar{x}_{i,j},\bar{x}_o)}{r}\right)\right]^{p_{i,j}}$$
$$=\frac{1}{mn}\sum_{\substack{i=1\\ d(\bar{x}_{i,j},\bar{x}_o)<\varepsilon}}^{m}\sum_{j=1}^{n}\left[M\left(\frac{d(\bar{x}_{i,j},\bar{x}_o)}{r}\right)\right]^{p_{i,j}}$$
$$+\frac{1}{mn}\sum_{\substack{i=1\\ d(\bar{x}_{i,j},\bar{x}_o)\geqslant\varepsilon}}^{m}\sum_{j=1}^{n}\left[M\left(\frac{d(\bar{x}_{i,j},\bar{x}_o)}{r}\right)\right]^{p_{i,j}}$$
$$\geqslant\frac{1}{mn}\sum_{\substack{i=1\\ d(\bar{x}_{i,j},\bar{x}_o)\geqslant\varepsilon}}^{m}\sum_{j=1}^{n}\left[M\left(\frac{d(\bar{x}_{i,j},\bar{x}_o)}{r}\right)\right]^{p_{i,j}}$$
$$\geqslant\frac{1}{mn}\sum_{\substack{i=1\\ d(\bar{x}_{i,j},\bar{x}_o)\geqslant\varepsilon}}^{m}\sum_{j=1}^{n}\left[M\left(\frac{\varepsilon}{r}\right)\right]^{p_{k,l}}$$
$$\geqslant\frac{1}{mn}\sum_{\substack{i=1\\ d(\bar{x}_{i,j},\bar{x}_o)\geqslant\varepsilon}}^{m}\sum_{j=1}^{n}\min\left\{M\left(\frac{\varepsilon}{r}\right)^h,M\left(\frac{\varepsilon}{r}\right)^H\right\}$$
$$\geqslant\frac{1}{mn}|\{(i,j)\in\mathbb{N}\times\mathbb{N}:d(\bar{x}_{i,j},\bar{x}_o)\geqslant\varepsilon\}|\min\left\{M\left(\frac{\varepsilon}{r}\right)^h,M\left(\frac{\varepsilon}{r}\right)^H\right\}.$$

Hence $\bar{x}=(\bar{x}_{i,j})\in\bar{s}_2$. This completes the proof. □

Theorem 2.6. *Let M be an Orlicz function, $0<h\leqslant\inf_{k,l}p_{k,l}\leqslant\sup_{k,l}p_{k,l}=H<\infty$ and $\bar{x}=(\bar{x}_{i,j})$ a bounded sequence of interval numbers. Then $\bar{s}_2\subset\bar{w}(M,p)$.*

Proof. Let $\bar{x}=(\bar{x}_{i,j})\in\bar{l}^2_\infty$ and $\bar{s}_2-\lim\bar{x}_{i,j}=\bar{x}_o$. Since $\bar{x}=(\bar{x}_{i,j})\in\bar{l}^2_\infty$, then there is a constant integer $M>0$ such that $d(\bar{x}_{i,j},\bar{x}_o)\leqslant M$ for all $i,j\in\mathbb{N}$. Given $\varepsilon>0$, we have

$$\frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n}\left[M\left(\frac{d(\bar{x}_{i,j},\bar{x}_o)}{r}\right)\right]^{p_{i,j}}$$
$$=\frac{1}{mn}\sum_{\substack{i=1\\ d(\bar{x}_{i,j},\bar{x}_o)<\varepsilon}}^{m}\sum_{j=1}^{n}\left[M\left(\frac{d(\bar{x}_{i,j},\bar{x}_o)}{r}\right)\right]^{p_{i,j}}$$
$$+\frac{1}{mn}\sum_{\substack{i=1\\ d(\bar{x}_{i,j},\bar{x}_o)\geqslant\varepsilon}}^{m}\sum_{j=1}^{n}\left[M\left(\frac{d(\bar{x}_{i,j},\bar{x}_o)}{r}\right)\right]^{p_{i,j}}$$
$$\leqslant\frac{1}{mn}\sum_{\substack{i=1\\ d(\bar{x}_{i,j},\bar{x}_o)<\varepsilon}}^{m}\sum_{j=1}^{n}\left[M\left(\frac{\varepsilon}{r}\right)\right]^{p_{i,j}}$$
$$+\frac{1}{mn}\sum_{\substack{i=1\\ d(\bar{x}_{i,j},\bar{x}_o)\geqslant\varepsilon}}^{m}\sum_{j=1}^{n}\max\left\{M\left(\frac{M}{r}\right)^h,M\left(\frac{M}{r}\right)^H\right\}$$
$$\leqslant\max\left\{M\left(\frac{\varepsilon}{r}\right)^h,M\left(\frac{\varepsilon}{r}\right)^H\right\}$$
$$+\frac{\max\left\{M\left(\frac{M}{r}\right)^h,M\left(\frac{M}{r}\right)^H\right\}}{mn}|\{(i,j)\in\mathbb{N}\times\mathbb{N}:d(\bar{x}_{i,j},\bar{x}_o)\geqslant\varepsilon\}|.$$

Hence $\bar{x} = (\bar{x}_{i,j}) \in \bar{w}(M, p)$. This completes the proof. □

The following corollary follows directly from Theorem 2.5 and Theorem 2.6.

Corollary 2.7. $\bar{s}_2 \cap \bar{l}^2_\infty = \bar{l}^2_\infty \cap \bar{w}(M, p)$.

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