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MODEL PARAMETERS ESTIMATION FROM RESIDUAL GRAVITY ANOMALY USING A LEAST-SQUARES INVERSION ALGORITHM

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حساب معاملات النموذج من الشاذة المتبقية التثاقلية
باستخدام اللوغاريتم العكسى بطريقة التربيع الأدنى

الخلاصـة: نقدم فى هذا البحث طريقة عكسية جديدة باستخدام طريقة التربيع الأدنى لتعيين العمق ومعامل الشكل ومعامل السعة للتركيب الجيولوجى المدفون باستخدام الشاذات التثاقلية المتبقية. وبمعرفة بعض النقط المميزة على البروفيل أمكن تحويل مشكلة تعيين العمق الى مشكلة حل معادلة واحدة غير خطية وبمعرفة العمق، يمكن حساب معامل الشكل ومعامل السعة. هذه الطريقة تم استخدامها للأشكال الهندسية المنتظمة مثل الأسطوانة الرأسية والأسطوانة الأفقية والكرة. وقد تم تطبيق الطريقة على بيانات نظرية بها أخطاء عشوائية وأخرى ليس بها أخطاء عشوائية لتوضيح كفاءتها وأيضاً على مثال حقلى من الولايات المتحدة الأمريكية. وفى كل الحالات، وجد أن العمق والمعاملات الأخرى المحسوبة تتفق مع البيانات الحقيقية.

ABSTRACT: *A new inversion technique using a quick least-squares method is developed to estimate, successively, the depth, the shape factor and the amplitude coefficient of a buried structure using residual gravity anomalies. By defining the anomaly value at different points on the profile, the problem of depth estimation is transformed into a problem of solving a non-linear equation of the form z = f(z). Knowing the depth, the shape factor can be estimated and finally, the amplitude coefficient can be estimated. This technique is applicable for a class of geometrically simple anomalous bodies including the semi-infinite vertical cylinder, the infinitely long horizontal cylinder and the sphere. The efficiency of this technique is demonstrated with gravity anomaly due to a theoretical model in each case with and without random errors. Finally, the applicability is illustrated using the residual gravity anomaly of Humble salt dome, Texas, USA. The interpreted depth and the other model parameters are in good agreement with the known actual values.*

INTRODUCTION

The problem in gravity exploration is the computation of the oretical anomalies caused by idealized models of known shapes. It is known that the gravity interpretation is non-unique where different subsurface causative targets may yield the same gravity anomaly. However, a priori information about the geometry of the causative target may lead to a unique solution (Roy et al. 2000; Aboud et al. 2004).

Several methods have been used to interpret the residual gravity anomalies. Among these methods, the half-g_{max} rule (Nettleton, 1962; Telford et al., 1976), simple method (Essa, 2007), Fourier transformation (Sharma and Geldart, 1968), Euler deconvolution (Reid et al., 1990), Mellin transform (Babu et al,. 1991), Hilbert transform (Sundararajan et al. 1983), Hartley transform (Sundararajan and Rama Brahmam, 1998), least-squares minimization (Salem et al., 2003), and Walsh transform (Al-Garni, 2008). In the above-mentioned methods, the geometry of causative target is assumed where the accuracy of the results depends on how close the assumed model from the real structure.

Some recent approaches have been developed to estimate the shape factor of the causative targets of gravity anomaly. Among these approaches is Walsh transform (Shaw and Agarwal, 1990), analytic signal (Nandi et al., 1997), and non-linear least-squares minimization (Abdelrahman and Sharafeldin, 1995; Abdelrahman et al., 2001), and derivative of a numerical formula (Aboud et al., 2004).

In this study, a quick least-squares inversion technique based on nonlinear equation z = f(z) to analyze gravity anomalies due to simple structures developed to estimate the model parameters (the depth, the shape factor and the amplitude coefficient).By defining the anomaly value at different points on the profile, the problem of depth estimation is transformed into a problem of solving a non-linear equation of the form z = f(z).The accuracy of the result obtained by this procedure depends upon the accuracy to which the residual anomaly can be separated from the Bouguer anomaly. The methodology is illustrated with theoretical models, in each case with and without random errors, and tested by the gravity anomaly of the Humble salt dome, Texas, USA.

THE METHOD

The general mathematical expression of the gravity anomaly (g) produced by simple geometric-shaped bodies (a sphere, a horizontal cylinder and a semi-infinite vertical cylinder) at any point P along the x-axis of an arbitrary structure in a Cartesian coordinate system (Abdelrahman et al., 1989) is given by:

$$g(x_i, z, q) = K \frac{z^m}{(x_i^2 + z^2)^q} , \qquad (1)$$

where

$$\mathbf{K} = \begin{cases} \frac{4}{3}\pi G\sigma R^3 \\ 2\pi G\sigma R^2 \\ \pi G\sigma R^2 \end{cases}, \quad \mathbf{m} = \begin{cases} 1 \\ 1 \\ 0 \end{cases},$$

$$\mathbf{q} = \begin{cases} \frac{3}{2} & \text{for a sphere} \\ 1 & \text{for a horizontal cylinder} \\ \frac{1}{2} & \text{for a vertical cylinder } R << z \end{cases}.$$

In equation (1), z is the depth, q is the shape factor, e.g., the shape factors for the semi-infinite vertical cylinder (3D),horizontal cylinder (2D), and sphere (3D) are 0.5, 1.0, and1.5, respectively. Also, the shape factor for the finite vertical cylinder is approximately 1 (Abdelrahman and El-Araby, 1993). The shape factor (q) approaches zero as the structure becomes a nearly horizontal bed, and approaches 1.5 as the structure becomes a perfect sphere (point mass).x_i is the position coordinate, σ is the density contrast, G is the universal gravitational constant, and R is the radius.

At the origin (x_i = 0), the equation (1) gives the following relationship:

$$K = g(0)z^{2q-m}. \quad (2)$$

Using equation (1), we obtain the following normalized equation at x_i = ± N and x_i =± M where N = 1, 2, 3, … and M = 1, 2, 3,……

$$g(\pm N) = K\left(\frac{z^2}{N^2 + z^2}\right)^q, \quad (3)$$

$$g(\pm M) = K\left(\frac{z^2}{M^2 + z^2}\right)^q, \quad (4)$$

Let $\mathbf{T} = \left(\frac{\mathbf{g(N)}}{\mathbf{g(M)}}\right)$, then from equation (3) and equation (4), we get:

$$q = \frac{\ln T}{\ln Y}, \text{ and } \ln Y = \ln\left(\frac{M^2 + z^2}{N^2 + z^2}\right)^q \quad M \neq N \quad (5)$$

The equation (1) can be rewritten using equations (2) and (5) as follows

$$g(x_i, z) = g(0)\frac{z^{2\ln T/\ln Y}}{(x_i^2 + z^2)^{\ln T/\ln Y}}. \quad (6)$$

The unknown depth (z) in equation (6) can be obtained by minimizing

$$\varphi(z) = \sum_{i=-k}^{k}\left[L(x_i) - g(0)\frac{z^{2\ln T/\ln Y}}{(x_i^2 + z^2)^{\ln T/\ln Y}}\right]^2, \quad (7)$$

where $L(x_i)$ denotes the observed gravity anomaly at x_i.

Minimization of φ(z) in the least-squares sense, i.e., $\partial\varphi(\mathbf{z})/\partial\mathbf{z} = 0$, leads to the following equation:

$$z_f = \left[\frac{\sum_{i=-n}^{n} L(x_i)\frac{x_i^2}{(x_i^2 + z^2)^{1+\ln T/\ln Y}}}{g(0)\sum_{i=-n}^{n}\frac{x_i^2}{(x_i^2 + z^2)^{2\ln T/\ln Y + 1}}}\right]^{\ln Y/2\ln T}. \quad (8)$$

Equation (8) can be solved for z using the iterative fixed point method (Mustoe and Barry, 1998) and its iteration form can expressed as

$$z_f = f(z_i), \quad (10)$$

where z_f and z_i are the final and the initial depths, the iterative process executed when $|\mathbf{z_f} - \mathbf{z_i}| \leq \mathbf{e}$, where e is a small predetermined real number close to zero. Any initial supposition for z works well because there is only one global minimum, i.e., no limitation or optimal range are required for the initial guess of the depth parameter.

Once, the depth (z) is known, the shape factor (q) can be estimated from the equation (5). Finally, knowing the depth (z) and the shape factor (q), the amplitude coefficient (K) can be estimated from equation (2).

For each N and M value, we compute the values of the model parameters (z, q and K) from equations (8), (5) and (2), respectively. Theoretically, the anomaly values at the origin and any two N and M distances are just enough to determine the model parameters. However, in practice, it is recommended to use all possible combinations of N and M values to determine the most appropriate source parameters solutions from all gravity data. We then measure the goodness of fit between the observed and computed gravity data for each set of solutions. The simplest way to compare two gravity profiles is to compute the standard error (μ) between the observed values and the values computed from estimated values of z, q and A. The model parameters which give the least root-mean-sum-squares differences are the best. In this way, we can select the best-fit source parameters solutions from all gravity data.

The standard error (μ) is used in this paper as statistical preference criteria in order to compare the observed and calculated values. This μ is given by the following mathematical relationship:

$$\mu = \sqrt{\frac{\sum_{i=1}^{N}\left[g(x_i) - g_c(x_i)\right]^2}{N}}, \qquad (13)$$

where $g(x_i)$ is the observed gravity values and $g_c(x_i)$ is the calculated gravity values.

SYNTHETIC EXAMPLES

The newly introduced gravity inversion algorithm tested on several synthetic datasets of a semi-infinite vertical cylinder (3D), an infinitely long horizontal cylinder (2D), and a sphere (3D)causative body. In order to assess and analyze this algorithm better, we will examine in the following:

The effect of random noise in the data:

The residual gravity anomaly of a semi-infinite vertical cylinder model (K = 200 mGal, z = 2 km, q = 0.5) was used. Equations (8), (5) and (2) were applied to there sidual anomaly profiles, yielding the model parameters; the depth, the shape factor and the amplitude coefficient, respectively, solutions for all possible N and M points. The computed model parameters for the model are summarized in Table1, respectively.

Table 1 shows the standard error (μ) are equal zero for all model parameters (z, q, A) when using synthetic data without random noise for all combination of N and M. Otherwise, The standard error (μ) has different values when using synthetic data with 10% random noise which has been added to the synthetic data. The best fit model parameter (z = 1.81 km, q = 0.47, A = 181.66 mGal) was taken at the lowest μ which is used as a criterion for estimating the best fit model parameters.

Also, the gravity synthetic datasets of a horizontal cylinder (K = 800 mGal*km, q = 1.0, profile length = 30 km, N = 5 km, M = 8 km) was generated and inverted for various depths of burial at the Earth's subsurface. The obtained noise-free datasets have been inverted using equations (8), (5), and (2), and the suggested algorithm has recovered almost the true value of each gravity parameter (z, q, K) of the anomalous bodies. Each of the gravity datasets has been contaminated with 10%random noise, and then inverted. The maximum absolute errors in the inverted parameters (z, q, K; respectively) are found to be 3.2%,1.3%, and 5.8%, respectively (figure 1).

Table (1). Numerical results of the present method applied to the semi-infinite vertical cylinder synthetic example (q = 0.5,z = 2 km, K = 100 mGal, profile length = 20 km, sampling interval = 1 km) without and with 10% random noise.

N (km)	M (km)	Using synthetic data				Using data with 10% random errors			
		z (km)	q	K (mGal)	μ (mGal)	z (km)	q	K (mGal)	μ (mGal)
1.00	2.00	2.00	0.50	200.00	0.00	0.13	0.14	59.47	14.04
1.00	3.00	2.00	0.50	200.00	0.00	0.49	0.21	76.19	9.52
1.00	4.00	2.00	0.50	200.00	0.00	0.86	0.28	94.48	6.24
1.00	5.00	2.00	0.50	200.00	0.00	1.77	0.47	176.08	1.19
2.00	3.00	2.00	0.50	200.00	0.00	1.49	0.41	142.89	2.27
2.00	4.00	2.00	0.50	200.00	0.00	1.70	0.45	166.25	1.41
2.00	5.00	2.00	0.50	200.00	0.00	2.56	0.64	346.19	2.74
3.00	4.00	2.00	0.50	200.00	0.00	1.81	0.47	181.66	1.11
3.00	5.00	2.00	0.50	200.00	0.00	2.71	0.68	401.66	3.21
4.00	5.00	2.00	0.50	200.00	0.00	3.68	0.94	1198.43	5.71

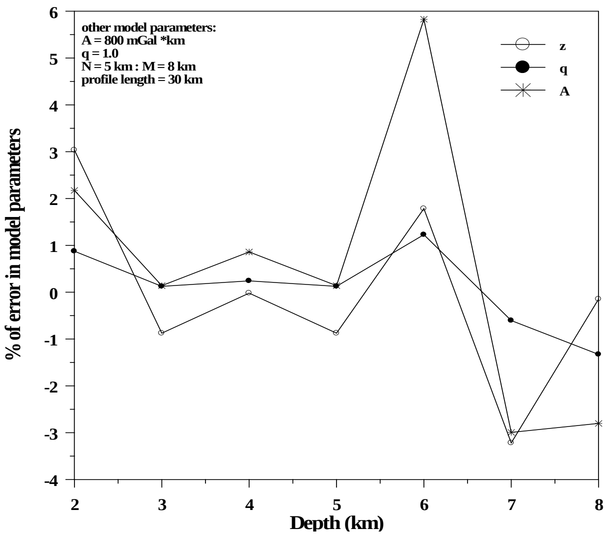

Fig. (1): Error response in model parameters estimates when using synthetic with 10% random errors. Abscissa: model depth. Ordinate: % of error in model parameters.

These results show that the suggested method is stable with respect to the noise added, and can recover the gravity model parameters with an acceptable accuracy when applied to noisy data.

Effect of errors in T and g(0):

In studying the error response of the present algorithm, synthetic example of a semi-infinite vertical cylinder model (q = 0.5, z = 3 km, K = 200mGal, N = 4 km, M = 9 km, and profile length = 60 km) were considered in which errors of ±1%,±2%, …, ±5% were supposed in both T and g(0). Following the same interpretation method, values of the three model parameters (z, q, K) were computed and the percentage of errors in the model parameters were mapped in case using synthetic data without random noise (Figs. 2-4) and in case using synthetic data with random noise (Figs. 5-7).

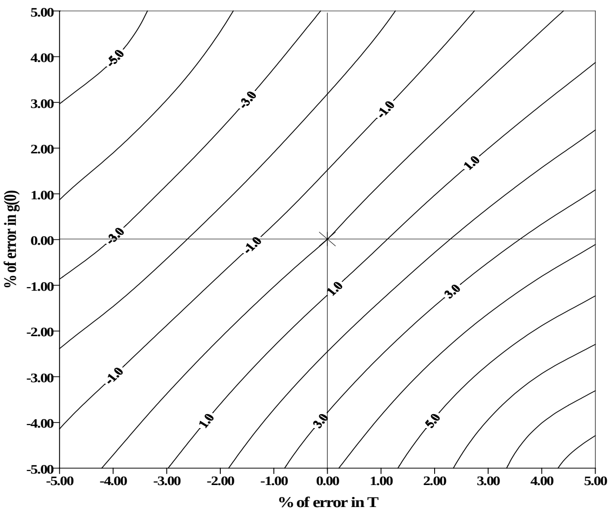

Fig. (2): A map showing error response in depth parameter estimates for synthetic data (q = 0.5, z=3 km, K = 200 mGal). Abscissa: percent error in T. Ordinate: percent error in g(0). C.I. = 1%.

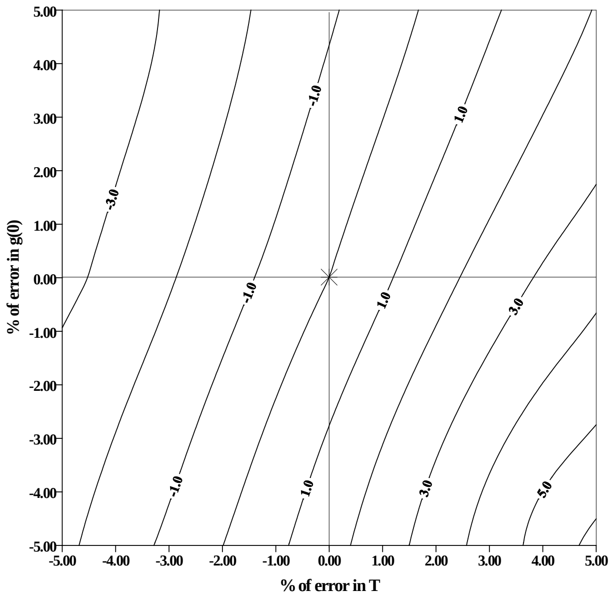

Fig. (3): A map showing error response in shape factor parameter estimates for synthetic data (q = 0.5, z=3 km, K = 200 mGal). Abscissa: percent error in T. Ordinate: percent error in g(0). C.I. = 1%.

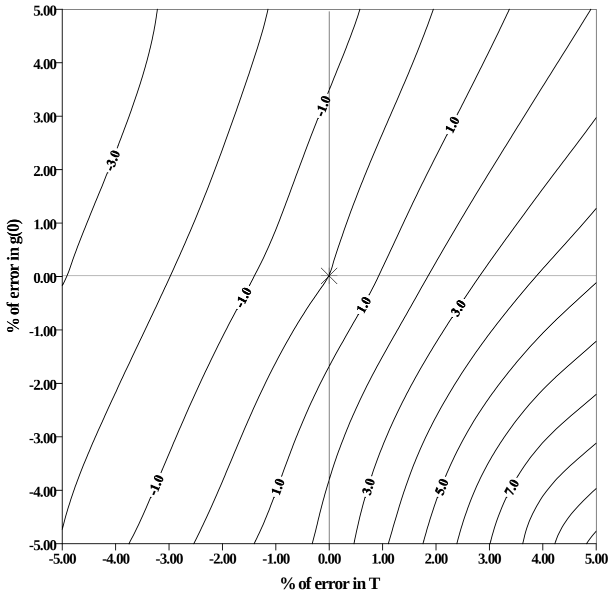

Fig. (4): A map showing error response in amplitude coefficient estimates for synthetic data (q = 0.5, z=3 km, K = 200 mGal). Abscissa: percent error in T. Ordinate: percent error in g(0). C.I. = 1%.

Figures 2 and 4 show that the maximum error in depth is about 13% when both T and g(0) have errors of 5% and -5%. The maximum error in shape factor is about 11% when T and g(0) have 5% and -5% errors, respectively (Figs. 3 and5). On the other hand, the maximum error in amplitude coefficient (K) is about 190% when T and g(0) have 23% and-5% errors, respectively (Figs. 5 and 7). Finally, when T or g(0) is kept undisturbed, the percentage of error in model parameters is slightly smaller or greater than the imposed error. This demonstrates that the proposed method will give reliable model parameters solution even when both T and g(0) are not correct and noisy.

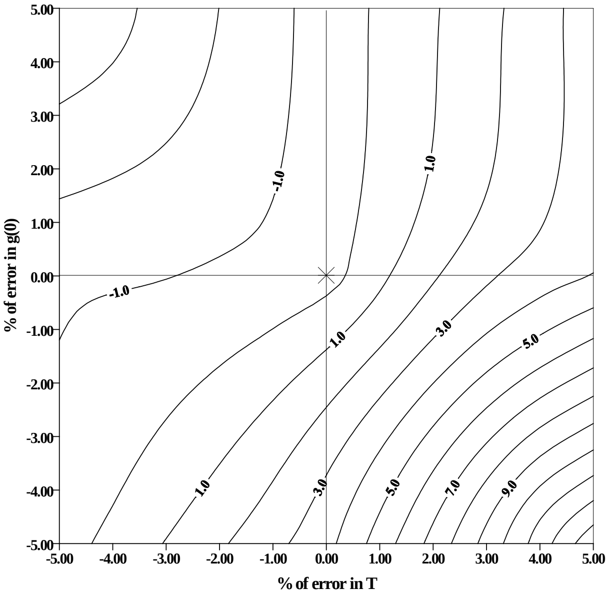

Fig. (5): A map showing error response in depth parameter estimates for noisy synthetic data (q = 0.5, z=3 km, K = 200 mGal). Abscissa: percent error in T. Ordinate: percent error in g(0). C.I. = 1%.

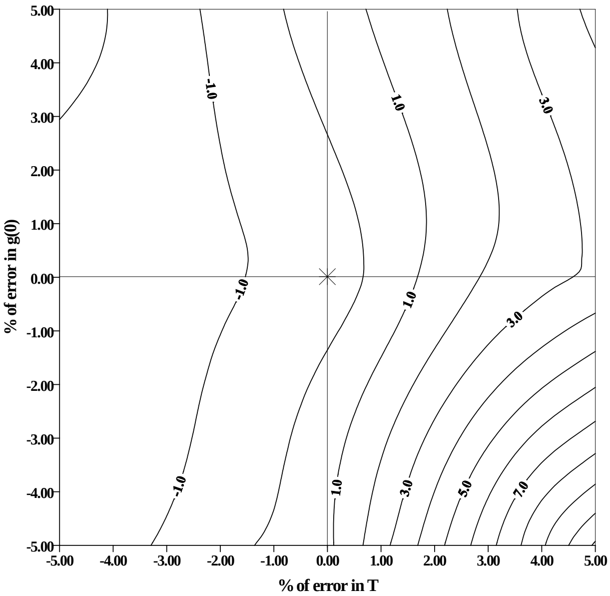

Fig. (6): A map showing error response in shape factor parameter estimates for noisy synthetic data (q = 0.5, z=3 km, K = 200 mGal). Abscissa: percent error in T. Ordinate: percent error in g(0). C.I. = 1%.

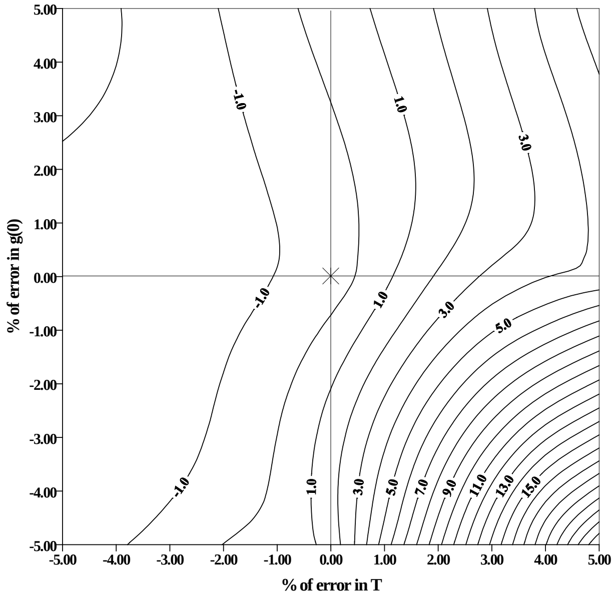

Fig. (7): A map showing error response in amplitude coefficient estimates for noisy synthetic data (q = 0.5, z=3 km, K = 200 mGal). Abscissa: percent error in T. Ordinate: percent error in g(0). C.I. = 1%.

FIELD EXAMPLE

A residual gravity field anomaly taken from USA has been interpreted using the proposed technique in order to examine its applicability and stability.

Humble salt dome:

The residual gravity anomaly profile over the Humble salt dome, Texas, USA (Nettleton, 1976) was digitized at an interval of 0.427 km (Fig. 8).

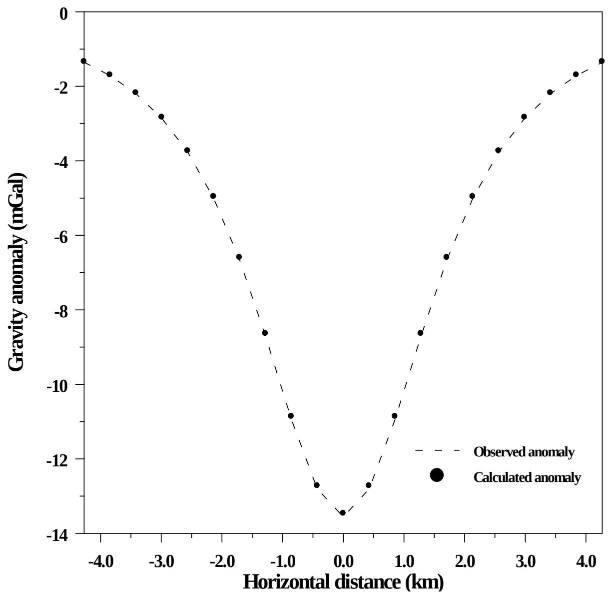

Fig. (8): The residual gravity anomaly (dashes lines) over the Humble salt dome, Texas, USA and the estimated response(black circles) computed from the present inversion method.

Table (2): Numerical results of the field example.

N (km)	**M (km)**	**z (km)**	**q**	**K (mGal)**	**µ (mGal)**
0.43	0.85	0.13	0.11	-8.45	2.804
0.43	1.28	0.53	0.20	-10.61	2.173
0.43	1.71	1.41	0.36	-17.42	1.337
0.43	2.14	4.02	1.07	-267.23	21.596
0.43	2.56	4.81	1.37	-206.93	0.028
0.43	2.99	4.88	1.39	-230.65	0.018
0.43	3.42	4.90	1.40	-239.32	0.015
0.43	3.84	4.92	1.41	-243.93	0.014
0.43	4.27	4.93	1.41	-247.52	0.012
0.85	1.28	1.58	0.40	-12.38	3.022
0.85	1.71	3.60	0.93	-146.68	0.319
0.85	2.14	4.76	1.35	-909.33	27.216
0.85	2.56	4.87	1.39	-227.87	0.019
0.85	2.99	4.91	1.40	-239.37	0.014
0.85	3.42	4.92	1.41	-244.03	0.013
0.85	3.84	4.92	1.41	-246.90	0.013
0.85	4.27	4.93	1.41	-248.81	0.012
1.28	1.71	4.74	1.34	-184.10	0.041
1.28	2.14	4.88	1.39	-228.03	0.018
1.28	2.56	4.91	1.40	-239.66	0.014
1.28	2.99	4.92	1.41	-244.74	0.013
1.28	3.42	4.93	1.41	-247.71	0.012
1.28	3.84	4.93	1.41	-249.59	0.012
1.28	4.27	4.94	1.41	-251.37	0.011
1.71	2.14	4.91	1.40	-239.82	0.015
1.71	2.56	4.92	1.41	-245.28	0.013
1.71	2.99	4.93	1.41	-248.34	0.012
1.71	3.42	4.93	1.41	-250.22	0.012
1.71	3.84	4.94	1.42	-251.95	0.011
1.71	4.27	4.94	1.42	-252.85	0.010
2.14	2.56	4.93	1.41	-249.40	0.011
2.14	2.99	4.94	1.41	-251.39	0.011
2.14	3.42	4.94	1.42	-252.02	0.011
2.14	3.84	4.94	1.42	-252.95	0.011
2.14	4.27	4.94	1.42	-253.37	0.011
2.56	2.99	4.94	1.42	-252.60	0.010
2.56	3.42	4.94	1.42	-253.61	0.010
2.56	3.84	4.94	1.42	-254.08	0.010
2.56	4.27	4.94	1.42	-254.22	0.010
2.99	3.42	4.94	1.42	-254.40	0.009
2.99	3.84	4.94	1.42	-254.62	0.010
2.99	**4.27**	**4.95**	**1.42**	**-255.40**	**0.008**
3.42	3.84	4.94	1.42	-254.79	0.010
3.42	4.27	4.95	1.42	-255.47	0.009
3.84	4.27	4.95	1.42	-255.34	0.010

Equations (8), (5) and (2) were used to determine the depth, the shape factor and the amplitude coefficient parameters, respectively, using all possible combination of N- and M-values. Then we computed the standard error (μ) between the observed values and the values computed from estimated parameters z, q and K for each N and M value. The results are shown in Table 2 for the cases of N and M values where the μ difference between the modeled and observed data is 0.008mGal.

The results are summarized in Table 2. Table 2 shows the optimum set is given at N = 2.99km and M = 4.27km. The best-fit-model parameters are z = 4.95km, q = 1.42 and K = -255.4 mGal (Fig. 8). The obtained depth agrees very well with the results obtained from drilling and seismic information (4.97 km; Nettleton, 1976).

CONCLUSION

The problem of estimating the appropriate depth, shape factor and amplitude coefficient of a buried structure from the residual gravity data of a short or a long profile length can be solved using the present algorithm. A simple and rapid inversion approach is formulated to use the anomaly values at two pairs of measured data points(±N and ±M). The repetition of the method using all possible combinations of such pairs of measured points will lead to the best-fitting model. This happens when these two pairs of points contain the least amount of noise in the entire set of measured data. The advantages of this method over previous graphical and numerical techniques used to interpret gravity data are: (1) all the three model parameters can be obtained from all observed data, (2) the method is automatic, and (3) the method is works well even when gravity data was noisy.

REFERENCES

Abdelrahman, E.M. and H.M. El-Araby (1993), Shape and depth solutions from gravity data using correlation factors between successive least-squares residuals, Geophysics. 59, 1785-1791.

Abdelrahman E.M., Sharafeldin M.S. (1995) A least-squares minimization approach to shape determination from gravity data. Geophysics 60: 589-590

Abdelrahman, E.M., A.I. Bayoumi, Y.E. Abdelhady, M.M. Gobashy, and H.M. El-Araby (1989), Gravity interpretation using correlation factors between successive leastsquaresresidual anomalies, Geophysics.54, 1614-1621.

Abdelrahman, E.M., El-Araby, T.M., El-Araby, H.M., Abo-Ezz, E.R., 2001. A new method for shapeand depth determinations from gravity data. Geophysics 66, 1774-178.

Aboud E., Salem A., Elawadi E., Ushijima K. (2004) Estimation of shape factor of buried structure from residual gravity data. The7th SEGJ International Symposium, November 24–26, 2004.Tohoku University, Sendai

Al-Garni MA (2008) Walsh transforms for depth determination of afinite vertical cylinder from its residual gravity anomaly. SAGEEP 6–10:689–702

Babu L.A., Reddy K.G. and Mohan N.L. (1991) Gravity interpretation of vertical line element and slap–a Mellin transform method. Indian J. Pure Appl Math 22:439–447

Essa, K.S. (2007), A simple formula for shape and depth determination from residual gravity anomalies,ActaGeophysica.55, 182-190.

Mustoe L.R., Barry, M.D.J., 1998. Mathematics in Engineering and Science, Wiley, New York, 756pp.

Nandi, B.K., Shaw, R.K., Agarwal, N.P., 1997. A short note on identification of the shape of simple causative sources from gravity data. Geophysical Prospecting45, 513-520.

Nettleton, L.L. (1962), Gravity and magnetics for geologists and seismologists: AAPG, 46, 1815-1838.

Nettleton, L.L. (1976), Gravity and magnetic in oil prospecting, McGraw-Hill Book Co.

Reid, A.B. Allsop, J.M., Granser, H., Millet, A.J., and Somerton, I. W., 1990, Magnetic interpretation in three dimensions using Euler Deconvolution: Geophysics, 55, 80-91.

Roy, L., B.N.P. Agarwal, and R.K. Shaw (2000), A new concept in Euler deconvolution of isolated gravity anomalies, Geophysical Prospecting.16, 77-93.

Salem, A. and Elawadi, E, and K. Ushijima, 2003, Short note: Depth determination from residual gravity anomaly using a simple formula; Computer and Geosciences, 29, 801-804.

Sharma, B. and Geldart, L.P., 1968, Analysis of gravityanomalies of two-dimensional faults using Fouriertransforms: Geophysical Prospecting, 16, 77-93.

Shaw, R.K., Agarwal, P., 1990. The application of Walsh transforms to interpret gravity anomalies due to some simple geometrical shaped causative sources: A feasibility study. Geophysics55, 843-850.

Sundararajan N, Rama Brahmam G (1998) Spectral analysis of gravityanomalies caused by slab-like structures: A Hartley transformtechnique. J Appl Geophys 39: 53-61

Sundararajan, N., Mohan, N.L., Seshagiri Rao S.V. (1983) Interpretation of gravity anomalies due to some two-dimensional structures: a Hilbert transform technique. Journal of Earth System Science, 92, 179–188.

Telford, W.M., L.P. Geldart, R.E. Sheriff, and D.A. Key (1976), Applied Geophysics, London, Cambridge University Press.