



ORIGINAL ARTICLE

Recurrence relations for single and product moments of progressively Type-II right censored order statistics from doubly truncated exponentiated Pareto distribution



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Abstract In this paper, we establish recurrence relations for single and product moments of progressively Type-II right censored order statistics from doubly truncated exponentiated Pareto distribution. These relations may then be used to obtain recurrence relations for single and product moments of progressively Type-II right censored order statistics from exponentiated Pareto distribution for all sample size n and all censoring schemes (R_1, R_2, \dots, R_m) . Also, we can use it to compute all the means, variances and covariances of exponentiated Pareto progressive Type-II right censored order statistics.

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1. Introduction

Censored sample arises in a life-testing whenever the experimenter does not observe (either intentionally or unintention-

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ally) the failure times of all units placed on a life-test. Inference based on censored sampling has been studied during the past 50 years by numerous authors for a wide range of lifetime distribution such as normal, exponential and Pareto. Naturally, there are many different forms of censoring that have been discussed in the literature. In progressive Type-II censoring from a total n units placed on a Life-test only m are completely observed until failure. At the time of first failure R_1 of the $n - 1$ surviving units are randomly withdrawn (or censored) from the life-testing experiment. At the time of the next failure R_2 of the $n - R_1 - 2$ surviving units are censored, and so on. Finally, at the time of the m th failure, all the remaining $R_m = n - R_1 - R_2 - \dots - R_{m-1} - m$ surviving units

are censored. Progressively Type-II censored has been studied by many authors including [1–10].

Recurrence relations for single and product moments for any continuous distribution can be used to compute all means, variances and covariance of such a distribution. [11] have reviewed several recurrence relations and identities established for the single and product moments of order statistics from an arbitrary continuous distribution. Also [12] have reviewed several recurrence relations and identities established for the single and product moments of order statistics from specific continuous distribution. [13] have updated the reviews of [11,12] and discuss several recurrence relations and identities for single and product moments of order statistics. [14] have derived complete recurrence relations for single and product moments from standard exponential distribution. [15] generalized the results of [16] on the order statistics from the doubly truncated exponential distribution and have derived similar recurrence relations for moments of progressively Type-II right censored order statistics from the Pareto distribution and its truncated forms.[17] have established several recurrence relations for the single and product moments of progressively Type-II right censored order statistics from a half-logistic distribution.

Pareto distribution is found to coincide with many social, scientific, geophysical, actuarial, and various other types of observable phenomena. Some examples were the Pareto distribution gives good fit are the sizes of human settlements, the values of oil reserve in oil fields, the standardized price returns on individual stocks, sizes of meteorites. Pareto originally used this distribution to describe the allocation of wealth among individuals since it seemed to show rather well the way that a larger portion of the wealth of any society is owned by a smaller percentage of the people in that society. He also used it to describe distribution of income. The following examples are sometimes seen as approximately Pareto-distributed: Hard disk drive error rates, The values of oil reserves in oil fields (a few large fields, many small fields), The standardized price returns on individual stocks, Numbers of species per genus. It has relations to other distributions like the exponential distribution, the log-normal distribution and the generalized Pareto distribution. Complete recurrence relations for single and product moments of progressively Type-II right censored order statistics from the generalized Pareto distribution was established by [18]. A family of distribution, namely the exponentiated family of distribution, is defined like the exponentiated exponential distribution as a generalization of the standard exponential distribution, the exponentiated gamma, exponentiated weibull, exponentiated Gumbel and the exponentiated Pareto distribution. The exponentiated Pareto distribution is an extension generalization of the Pareto distribution which introduced by [19], he showed that the exponentiated Pareto distribution can be used quite effectively in analyzing many life time data. [20] studied several exponentiated distributions, including exponentiated Pareto distribution, and discussed their properties. They showed that exponentiated Pareto distribution gives a good fit to the tail-distribution of NASDAQ data.

In this paper, we derive new recurrence relations satisfied by the single and product moments of progressively Type-II right censored order statistics from the doubly truncated exponentiated Pareto distribution.

Let X_1, X_2, \dots, X_n denote a random sample from the doubly truncated exponentiated Pareto distribution $EP(\theta, \alpha)$ with probability density function (pdf)

$$f(x) = \frac{\alpha\theta}{P-Q} [1 - (1+x)^{-\alpha}]^{\theta-1} (1+x)^{-\alpha-1}, \theta, \alpha > 0 \quad (1.1)$$

where $Q_1 < x < P_1$, $Q = [1 - (1 + Q_1)^{-\alpha}]^\theta$ and $P = [1 - (1 + P_1)^{-\alpha}]^\theta$ and cumulative distribution function (cdf) is given by

$$F(x) = \frac{1}{P-Q} \{ [1 - (1+x)^{-\alpha}]^\theta - Q \} \quad (1.2)$$

also, the characterizing differential equations are given by:

$$f(x) = \frac{\alpha\theta}{P-Q} \sum_{r=0}^{(1/\alpha)+1} \binom{(1/\alpha)+1}{r} (-1)^r \sum_{s=0}^{1-1/\theta+r/\theta} \binom{1-1/\theta+r/\theta}{s} (-1)^s \times (P-Q)^s P^{1-1/\theta+r/\theta-s} [1-F(x)]^s \quad (1.3)$$

Making use of (1.1) and (1.2) the following recurrence relations for the single and product moments of progressively Type-II right censored order statistics from doubly truncated exponentiated Pareto distribution have been derived.

2. Recurrence relations for single moments

Let $X_{1:m:n}, X_{2:m:n}, \dots, X_{m:m:n}$ be the progressively type-II right censored order statistics of size m from the sample of size n with censoring scheme (R_1, R_2, \dots, R_m) be from the doubly truncated exponentiated Pareto distribution whose probability function is given by (1.1) and distribution function is given by (1.2). For any continuous distribution the single moments of the progressively type-II can be written as:

$$\begin{aligned} \mu_{i:m:n}^{(R_1, R_2, \dots, R_m)(k)} &= E \left\{ \left[X_{i:m:n}^{(R_1, R_2, \dots, R_m)} \right]^k \right\} \\ &= A(n, m-1) \int \dots \int_{Q_1 < x_1 < x_2 < \dots < x_m < P_1} x_i^k f(x_1) [1-F(x_1)]^{R_1} \\ &\quad \times f(x_2) [1-F(x_2)]^{R_2} \dots f(x_m) [1-F(x_m)]^{R_m} dx_1 \dots dx_m \end{aligned}$$

where

$$A(n, m-1) = n(n-R_1-1)(n-R_1-R_2-2) \dots \times (n-R_1-R_2-\dots-R_{m-1}-m+1)$$

See [15]

Theorem 2.1. For $2 \leq m \leq n-1$, $R_1 \geq 1$ and $k > -1$

$$\begin{aligned} \mu_{1:m:n}^{(R_1, R_2, \dots, R_m)(k)} &= \frac{\alpha\theta}{(k+1)(P-Q)} \sum_{r=0}^{(1/\alpha)+1} \binom{(1/\alpha)+1}{r} (-1)^r \\ &\quad \times \sum_{s=0}^{1-1/\theta+r/\theta} \binom{1-1/\theta+r/\theta}{s} (-1)^s (P-Q)^s P^{1-1/\theta+r/\theta-s} \\ &\quad \times \left\{ \frac{n(n-R_1-1)}{(n+s-1)} \mu_{1:m-1:n+s-1}^{(R_1+R_2+s, R_3, \dots, R_m)(k+1)} \right. \\ &\quad \left. - Q_1^{(k+1)} + \frac{n(R_1+s)}{(n+s-1)} \mu_{1:m:n+s-1}^{(R_1+s-1, R_2, \dots, R_m)(k+1)} \right\} \quad (2.1) \end{aligned}$$

Proof. Suppose that $(1/\alpha)$ and $(1 - \frac{1}{\theta} + \frac{r}{\theta})$ are positive integers and since we have

$$\begin{aligned} \mu_{1:m:n}^{(R_1, R_2, \dots, R_m)^{(k)}} &= A(n, m - 1) \\ &\times \iiint \dots \int_{Q_1 < x_2 < x_3 < \dots < x_m < P_1} \left(\int_{Q_1}^{x_2} x_1^k f(x_1) [1 - F(x_1)]^{R_1} dx_1 \right) \\ &\times f(x_2) [1 - F(x_2)]^{R_2} \dots f(x_m) [1 - F(x_m)]^{R_m} \\ &\times dx_1 \dots dx_{i-1} dx_{i+1} \dots dx_m \end{aligned} \tag{2.2}$$

By using (1.3) and integrating the innermost integral by parts, we get

$$\begin{aligned} \int_{Q_1}^{x_2} x_1^k f(x_1) [1 - F(x_1)]^{R_1} dx_1 &= \frac{\alpha\theta}{(k+1)(P-Q)} \sum_{r=0}^{(1/\alpha)+1} \binom{(1/\alpha)+1}{r} (-1)^r \\ &\times \sum_{s=0}^{1-1/\theta+r/\theta} \binom{1-1/\theta+r/\theta}{s} (-1)^s (P-Q)^s P^{(1-1/\theta+r/\theta-s)} \\ &\times \left\{ x_2^{k+1} (F(x_2))^{R_1+s} - Q_1^{(k+1)} + (R_1+s) \right. \\ &\times \left. \int_{Q_1}^{x_2} x_1^{k+1} f(x_1) [1 - F(x_1)]^{R_1+s-1} dx_1 \right\} \end{aligned}$$

Substituting the above expression into (2.2) we get (2.1). \square

Theorem 2.2. For $1 \leq i \leq m - 1$, $m \leq n - 1$, $R_i \geq 1$ and $k > -1$

$$\begin{aligned} \mu_{i:m:n}^{(R_1, R_2, \dots, R_m)^{(k)}} &= \frac{\alpha\theta}{(k+1)(P-Q)} \sum_{r=0}^{(1/\alpha)+1} \binom{(1/\alpha)+1}{r} (-1)^r \\ &\times \sum_{s=0}^{1-1/\theta+r/\theta} \binom{1-1/\theta+r/\theta}{s} (-1)^s (P-Q)^s P^{(1-1/\theta+r/\theta-s)} \\ &\times \left\{ \frac{A(n, i)}{A(n+s-1, i-1)} \mu_{i:m-1:n+s-1}^{(R_1, R_2, \dots, R_{i-1}, R_i+R_{i+1}+s, R_{i+2}, \dots, R_m)^{(k+1)}} \right. \\ &- \frac{A(n, i-1)}{A(n+s-1, i-2)} \mu_{i-1:m-1:n+s-1}^{(R_1, \dots, R_{i-2}, R_{i-1}+R_i+s, R_{i+1}, \dots, R_m)^{(k+1)}} \\ &+ (R_i+s) \frac{A(n, i-1)}{A(n+s-1, i-1)} \\ &\times \left. \mu_{i:m:n+s-1}^{(R_1, R_2, \dots, R_{i-1}, R_i+s-1, R_{i+1}, \dots, R_m)^{(k+1)}} \right\} \end{aligned} \tag{2.3}$$

Proof. Suppose that $(1/\alpha)$ and $(1 - \frac{1}{\theta} + \frac{r}{\theta})$ are positive integers and since we have

$$\begin{aligned} \therefore \mu_{i:m:n}^{(R_1, R_2, \dots, R_m)^{(k)}} &= A(n, m - 1) \iiint \dots \int_{Q_1 < x_1 < x_2 < \dots < x_{i-1} < x_{i+1} < \dots < x_m < P_1} \\ &\times \left(\int_{x_{i-1}}^{x_{i+1}} x_i^k f(x_i) [1 - F(x_i)]^{R_i} dx_i \right) \\ &\times f(x_1) [1 - F(x_1)]^{R_1} \dots f(x_{i-1}) [1 - F(x_{i-1})]^{R_{i-1}} \\ &\times f(x_{i+1}) [1 - F(x_{i+1})]^{R_{i+1}} \dots f(x_m) \\ &\times [1 - f(x_m)]^{R_m} dx_1 \dots dx_{i-1} dx_{i+1} \dots dx_m \end{aligned} \tag{2.4}$$

By using (1.3) and integrating the innermost integral by parts, we get

$$\begin{aligned} \int_{x_{i-1}}^{x_{i+1}} x_i^k f(x_i) [1 - F(x_i)]^{R_i} dx_i &= \frac{\alpha\theta}{(k+1)(P-Q)} \sum_{r=0}^{(1/\alpha)+1} \binom{(1/\alpha)+1}{r} (-1)^r \\ &\times \sum_{s=0}^{1-1/\theta+r/\theta} \binom{1-1/\theta+r/\theta}{s} (-1)^s (P-Q)^s \\ &\times P^{(1-1/\theta+r/\theta-s)} \times \left\{ x_{i+1}^{k+1} (F(x_{i+1}))^{R_i+s} - x_{i-1}^{k+1} (F(x_{i-1}))^{R_i+s} \right. \\ &+ (R_i+s) \int_{x_{i-1}}^{x_{i+1}} x_i^{k+1} f(x_i) [1 - F(x_i)]^{R_i+s-1} dx_i \left. \right\} \end{aligned}$$

Substituting the above expression into (2.4) we get (2.3). \square

Theorem 2.3. For $2 \leq m \leq n - 1$, $R_m \geq 1$ and $k > -1$

$$\begin{aligned} \mu_{m:m:n}^{(R_1, R_2, \dots, R_m)^{(k)}} &= \frac{\alpha\theta}{(k+1)(P-Q)} \sum_{r=0}^{(1/\alpha)+1} \binom{(1/\alpha)+1}{r} (-1)^r \\ &\times \sum_{s=0}^{1-1/\theta+r/\theta} \binom{1-1/\theta+r/\theta}{s} (-1)^s (P-Q)^s P^{(1-1/\theta+r/\theta-s)} \\ &\times \left\{ -\frac{A(n, m-1)}{A(n+s-1, m-2)} \mu_{m-1:m-1:n+s-1}^{(R_1, R_2, \dots, R_{m-1}+R_m+s)^{(k+1)}} \right. \\ &+ (R_m+s) \frac{A(n, m-1)}{A(n+s-1, m-1)} \mu_{m:m:n+s-1}^{(R_1, R_2, \dots, R_m+s-1)^{(k+1)}} \left. \right\} \end{aligned} \tag{2.5}$$

Proof. Suppose that $(1/\alpha)$ and $(1 - \frac{1}{\theta} + \frac{r}{\theta})$ are positive integers and since we have

$$\begin{aligned} \therefore \mu_{m:m:n}^{(R_1, R_2, \dots, R_m)^{(k)}} &= A(n, m - 1) \iiint \dots \int_{Q_1 < x_1 < x_2 < \dots < x_{m-1} < P_1} \\ &\times \left(\int_{x_{m-1}}^{P_1} x_m^k f(x_m) [1 - F(x_m)]^{R_m} dx_m \right) \\ &\times f(x_1) [1 - F(x_1)]^{R_1} \dots f(x_{m-1}) \\ &\times [1 - F(x_{m-1})]^{R_{m-1}} dx_1 \dots dx_{m-1} \end{aligned} \tag{2.6}$$

By using (1.3) and integrating the innermost integral by parts, we get

$$\begin{aligned} \int_{x_{m-1}}^{P_1} x_m^k f(x_m) [1 - F(x_m)]^{R_m} dx_m &= \frac{\alpha\theta}{(k+1)(P-Q)} \sum_{r=0}^{(1/\alpha)+1} \binom{(1/\alpha)+1}{r} (-1)^r \\ &\times \sum_{s=0}^{1-1/\theta+r/\theta} \binom{1-1/\theta+r/\theta}{s} (-1)^s (P-Q)^s P^{(1-1/\theta+r/\theta-s)} \\ &\times \left\{ -x_{m-1}^{k+1} (F(x_{m-1}))^{R_m+s} + (R_m+s) \int_{x_{m-1}}^{P_1} x_m^k f \right. \\ &\times \left. (x_m) [1 - F(x_m)]^{R_m+s-1} dx_m \right\} \end{aligned}$$

Substituting the above expression into (2.6) we get (2.5). \square

3. Recurrence relations for product moments

In this section we have obtained the recurrence relation for product moments of progressively Type-II right censored order statistics from doubly truncated exponentiated Pareto distribution. We can write the (i, j) th product moment of progressively Type-II right censored order statistics as

$$\begin{aligned} \mu_{i,j:m:n}^{(R_1, R_2, \dots, R_m)} &= E \left[X_{i:m:n}^{(R_1, R_2, \dots, R_m)} X_{j:m:n}^{(R_1, R_2, \dots, R_m)} \right] \\ &= A(n, m - 1) \int \dots \int_{Q_1 < x_1 < \dots < x_m < P_1} x_i x_j f(x_1) [1 - F(x_1)]^{R_1} \\ &\times f(x_2) [1 - F(x_2)]^{R_2} \dots f(x_m) [1 - F(x_m)]^{R_m} dx_1 \dots dx_m \end{aligned}$$

See [15]

Theorem 3.1. For $1 \leq i \leq j \leq m - 1$ and $m \leq n$,

$$\begin{aligned} \mu_{i,j;m;n}^{(R_1, R_2, \dots, R_m)} &= \frac{\alpha\theta}{(P-Q)} \sum_{r=0}^{(1/\alpha)+1} \binom{(1/\alpha)+1}{r} (-1)^r \sum_{s=0}^{1-\frac{1}{\theta}+\frac{r}{\theta}} \binom{1-\frac{1}{\theta}+\frac{r}{\theta}}{s} \\ &\times (-1)^s (P-Q)^s P^{(1-\frac{1}{\theta}+\frac{r}{\theta}-s)} \\ &\times \left\{ \frac{A(n, j)}{A(n+s-1, j-1)} \mu_{i,j;m-1;n+s-1}^{(R_1, \dots, R_{j-1}, R_j+R_{j+1}+s, R_{j+2}, \dots, R_m)} \right. \\ &- \frac{A(n, j-1)}{A(n+s-1, j-2)} \mu_{i,j-1;m-1;n+s-1}^{(R_1, \dots, R_{j-2}, R_{j-1}+R_j+s, R_{j+1}, \dots, R_m)} \\ &\left. + (R_j+s) \frac{A(n, j-1)}{A(n+s-1, j-1)} \mu_{i,j;m;n+s-1}^{(R_1, \dots, R_{j-1}, R_j+s-1, R_{j+1}, \dots, R_m)} \right\} \end{aligned} \quad (3.1)$$

Proof. Suppose that $(1/\alpha)$ and $(1 - \frac{1}{\theta} + \frac{r}{\theta})$ are positive integers and since we have

$$\begin{aligned} \therefore \mu_{i,m;n}^{(R_1, R_2, \dots, R_m)} &= A(n, m-1) \iiint \dots \int_{Q_1 < x_1 < x_2 < \dots < x_{j-1} < x_{j+1} < \dots < x_m < P_1} x_i \\ &\times \left(\int_{x_{j-1}}^{x_{j+1}} x_j^0 f(x_j) [1 - F(x_j)]^{R_j} dx_j \right) \\ &\times f(x_1) [1 - F(x_1)]^{R_1} \dots f(x_{j-1}) [1 - F(x_{j-1})]^{R_{j-1}} \\ &\times f(x_{j+1}) [1 - F(x_{j+1})]^{R_{j+1}} \dots f(x_m) [1 - F(x_m)]^{R_m} \\ &\times dx_1 \dots dx_{j-1} dx_{j+1} \dots dx_m \end{aligned} \quad (3.2)$$

By using (1.3) and integrating the innermost integral by parts, we get

$$\begin{aligned} \int_{x_{j-1}}^{x_{j+1}} x_j^0 f(x_j) [1 - F(x_j)]^{R_j} dx_j &= \alpha\theta \sum_{r=0}^{(1/\alpha)+1} \binom{(1/\alpha)+1}{r} (-1)^r \\ &\times \sum_{s=0}^{1-1/\theta+r/\theta} \binom{1-1/\theta+r/\theta}{s} (-1)^s (P-Q)^s P^{(1-\frac{1}{\theta}+\frac{r}{\theta}-s)} \\ &\times \{ x_{j+1} (F(x_{j+1}))^{R_j+s} - x_{j-1} (F(x_{j-1}))^{R_j+s} + (R_j+s) \\ &\times \int_{x_{j-1}}^{x_{j+1}} x_j f(x_j) [1 - F(x_j)]^{R_j+s-1} dx_j \} \end{aligned}$$

Substituting the above expression into (3.2) we get (3.1). \square

Theorem 3.2. For $1 \leq i \leq m - 1$ and $m \leq n$,

$$\begin{aligned} \mu_{i,j;m;n}^{(R_1, R_2, \dots, R_m)} &= \frac{\alpha\theta}{(P-Q)} \sum_{r=0}^{(1/\alpha)+1} \binom{(1/\alpha)+1}{r} (-1)^r \sum_{s=0}^{1-\frac{1}{\theta}+\frac{r}{\theta}} \binom{1-\frac{1}{\theta}+\frac{r}{\theta}}{s} \\ &\times (-1)^s (P-Q)^s P^{(1-\frac{1}{\theta}+\frac{r}{\theta}-s)} \left\{ -\frac{A(n, m-1)}{A(n+s-1, m-2)} \right. \\ &\times \mu_{i,m-1;m-1;n+s-1}^{(R_1, R_2, \dots, R_{m-1}+R_m+s)} + (R_m+s) \frac{A(n, m-1)}{A(n+s-1, m-1)} \\ &\left. \times \mu_{i,m;m;n+s-1}^{(R_1, R_2, \dots, R_m+s-1)} \right\} \end{aligned} \quad (3.3)$$

Proof. Suppose that $(1/\alpha)$ and $(1 - \frac{1}{\theta} + \frac{r}{\theta})$ are positive integers and since we have

$$\begin{aligned} \therefore \mu_{i,m;n}^{(R_1, R_2, \dots, R_m)} &= A(n, m-1) \iiint \dots \int_{Q_1 < x_1 < x_2 < \dots < x_{m-1} < P_1} x_i \\ &\times \left(\int_{x_{m-1}}^{P_1} x_m^0 f(x_m) [1 - F(x_m)]^{R_m} dx_m \right) \\ &\times f(x_1) [1 - F(x_1)]^{R_1} \dots f(x_{m-1}) [1 - F(x_{m-1})]^{R_{m-1}} \\ &\times dx_1 \dots dx_{j-1} dx_{j+1} \dots dx_{m-1} \end{aligned} \quad (3.4)$$

By using (1.3) and integrating the innermost integral by parts, we get

$$\begin{aligned} \int_{x_{m-1}}^{P_1} x_m^0 f(x_m) [1 - F(x_m)]^{R_m} dx_m &= \alpha\theta \sum_{r=0}^{(1/\alpha)+1} \binom{(1/\alpha)+1}{r} (-1)^r \\ &\times \sum_{s=0}^{1-\frac{1}{\theta}+\frac{r}{\theta}} \binom{1-\frac{1}{\theta}+\frac{r}{\theta}}{s} (-1)^s (P-Q)^s P^{(1-\frac{1}{\theta}+\frac{r}{\theta}-s)} \\ &\times \left\{ -x_{m-1} (F(x_{m-1}))^{R_m+s} + (R_m+s) \int_{x_{m-1}}^{P_1} x_m f(x_m) \right. \\ &\left. \times [1 - F(x_m)]^{R_m+s-1} dx_m \right\} \end{aligned}$$

Substituting the above expression into (3.4) we get (3.3). \square

Remark 3.1. Setting $P = 1$ and $Q = 0$ the recurrence relations in Theorems 2.1, 2.2 and 2.3 in our paper obtain similar expressions for single moments of progressively Type-II right censored order statistics from exponentiated Pareto distribution.

Remark 3.2. Setting $P = 1$ and $Q = 0$ the recurrence relations in Theorems (3.1) and (3.2) in our paper obtain similar expressions for the product moments of progressively Type-II right censored order statistics from exponentiated Pareto distribution.

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