Journal of the Egyptian Mathematical Society (2016) 24, 49–53

Egyptian Mathematical Society

Journal of the Egyptian Mathematical Society

www.etms-eg.org
www.elsevier.com/locate/joems

ORIGINAL ARTICLE

Exact analytical solutions for 3D- Gross–Pitaevskii equation with periodic potential by using the Kudryashov method

Ahmad Neirameh

Department of Mathematics, Faculty of Sciences, Gonbad Kavous University, Gonbad, Iran

Received 6 May 2014; revised 27 July 2014; accepted 2 November 2014
Available online 7 February 2015

KEYWORDS

Kudryashov method;
3D- Gross–Pitaevskii equation;
Solitary wave solutions

Abstract This paper obtains solutions as well as other solutions to the 3D- Gross–Pitaevskii equation, which is called the non-linear Schrodinger equation under the conditions of Kudryashov method that appear in various areas of mathematical physics. This equation describes Bose–Einstein condensates in the low temperature regime. These new exact solutions will complement previous results and help further to understand the physical structures.

2000 MATHEMATICS SUBJECT CLASSIFICATION: 35Q53; 35Q80; 35Q55; 35G25

1. Introduction

In the recent years, seeking exact solutions of nonlinear partial differential equations (NLPDEs) was very important, because the nonlinear complex physical phenomena related to the NLPDEs are widely useful in many fields from physics, mechanics, biology, chemistry and engineering.

To this aim, a vast variety of powerful and direct methods to find the exact significant solutions of NLPDEs though they are difficult to find. Some of the most important methods are tanh- extended tanh method By Wakil [2], Fan [3] and Wazwaz [4], solitary wave ansatz method by Biswas [5–7], tanh method by Biswas [8,9], multiple exp-function method by Ma [10], Kudryashov method by Malfliet [11], Ma [12], Hirota's direct method by Kudryashov [13,14].

The Gross–Pitaevskii equation (GPE) is a classical nonlinear evolution equation. It is a variant of the famous nonlinear Schrodinger equation (NLSE), which is a universal model governing the evolution of complex field envelopes in nonlinear dispersive media. This article aims at considering the 3D- Gross–Pitaevskii equation with space and time modulated potential and nonlinearity by Manjun in [1],

$$i\frac{\partial}{\partial t}\hbar(s,t) = -\nabla\hbar(s,t) + U(x)\hbar(s,t) + g|\hbar|^2\hbar,$$
$$\nabla \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial y^2}, \tag{1}$$

where $s \in \mathbb{R}^3$; $t > 0$, ∇ stands for the Laplacian operator. The function $U(x)$ describes the potential of the trap to confine the condensate and $s = (x, y, z)$ is the propagation variable and t is

Peer review under responsibility of Egyptian Mathematical Society.

Production and hosting by Elsevier

http://dx.doi.org/10.1016/j.joems.2014.11.004
1110-256X

the transverse variable. The nonlinear coefficient $g(s,t)$ is the real-valued functions of time and spatial coordinates. We study nonlinear states for the NLS-type equation with additional periodic potential $U(x)$, also called the Gross–Pitaevskii equation, GPE, in theory of Bose–Einstein Condensate, BEC. In theoretical physics, the (one-dimensional) nonlinear Schrödinger equation (NLSE) is a nonlinear variation of the Schrödinger equation. It is a classical field equation whose principal applications are to the propagation of light in nonlinear optical fibers and planar waveguides and to Bose–Einstein condensates confined to highly anisotropic cigar-shaped traps, in the mean-field regime. Additionally, the equation appears in the studies of small-amplitude gravity waves on the surface of deep inviscid (zero-viscosity) water, the Langmuir waves in hot plasmas, the propagation of plane-diffracted wave beams in the focusing regions of the ionosphere, the propagation of Davydov's alpha-helix solitons, which are responsible for energy transport along molecular chains, and many others. More generally, the NLSE appears as one of universal equations that describe the evolution of slowly varying packets of quasi-monochromatic waves in weakly nonlinear media that have dispersion

2. Method applied

The purpose of this section is to present the algorithm of the modified Kudryashov method to find exact solutions of the nonlinear evolution equations. To do so we follow [15] by Malfliet, Ma [16,17] by Kudryashov.

Let us consider the nonlinear partial differential equation in the form

$$E(u_t, u_x, \ldots, x, t) = 0. \tag{2}$$

We use the following ansatz

$$u = U(\xi)e^{i(\alpha x+\beta t)}, \quad \xi = x - ct, \tag{3}$$

From Eq. (2) we obtain the ordinary nonlinear differential equation

$$\phi(-cU'(\xi)e^{i(\alpha x+\beta t)} + i\beta U e^{i(\alpha x+\beta t)}, U'(\xi)e^{i(\alpha x+\beta t)} + i\alpha U e^{i(\alpha x+\beta t)}, \ldots). \tag{4}$$

Now we show how one could obtain the exact solution of the Eq. (4) using the approach by modified Kudryashov method. This method is consisted of the following steps [15] by Malfliet and Ma [16].

1.2. Determination of the dominant term

To find dominant terms we substitute

$$U = \xi^p, \tag{5}$$

into all terms of Eq. (4). Then we compare degrees of all terms in Eq. (4) and choose two or more with the smallest degree. The minimum value of P define the pole of solution for Eq. (4) and we denote it as N. We have to point out that method can be applied when N is integer. If the value N is noninteger one can transform the equation not only study the procedure but also repeat it.

2.2. The solution structure

We look for exact solution of Eq. (4) in the form

$$U = a_0 + a_1 Q(\xi) + a_2 Q^2(\xi) + \cdots + a_N Q^N(\xi), \tag{6}$$

where a_i are unknown constants to be determined later, such that $a_N \neq 0$, while $Q(\xi)$ have the form

$$Q(\xi) = \frac{1}{1+e^{\xi}}, \tag{7}$$

These functions satisfy to the first order ordinary differential equations (Riccati equations)

$$Q'(\xi) = Q^2(\xi) - Q(\xi), \tag{8}$$

Eq. (8) are necessary to calculate the derivatives of functions $Q(\xi)$.

Remark 1. This Riccati equation also admits the following exact solutions:

$$\begin{aligned} Q_1(\xi) &= \frac{1}{2}\left(1 - \tanh\left[\frac{\xi}{2} - \frac{\varepsilon \ln \xi_0}{2}\right]\right), \quad \xi_0 > 0, \\ Q_2(\xi) &= \frac{1}{2}\left(1 - \coth\left[\frac{\xi}{2} - \frac{\varepsilon \ln \xi_0}{2}\right]\right), \quad \xi_0 < 0, \end{aligned} \tag{9}$$

3.2. Derivatives calculation

We should calculate all derivatives of functions $Q(\xi)$. One can do it by the computer algebra systems Maple or Mathematica. For example, we consider the general case when N is arbitrary. Differentiating the expressions (7) with respect to ξ taking into account (8) we have

$$\begin{aligned} Q'(\xi) &= \sum_{i=1}^{N} a_i i(Q-1)Q^i, \\ Q''(\xi) &= \sum_{i=1}^{N} a_i i((i+1)Q^2 - (2i+1)Q + i)Q^i, \end{aligned} \tag{10}$$

The high order derivatives of functions$Q(\xi)$can be found in Refs. [18] by Kudryashov and Hirota [19].

4.2. Defining the values of unknown parameters

We substitute expressions (10) in Eq. (6). After it we take $Q(\xi)$ from (10) into account. Thus Eq. (6) takes the form

$$P[Q(\xi)],$$

where $P[Q(\xi)]$ is a polynomial of functions $Q(\xi)$. Then we collect all terms with the same powers of functions $Q(\xi)$ and equate these expressions equal to zero. As a result we obtain system of algebraic equations. Solving this system we get the values of unknown parameters.

3. Our method to the 3D- Gross–Pitaevskii equation with periodic potential

To seek exact analytical wave solutions of Eq. (1) we take the similarity transformation [15] by Malfliet,

$$\hbar(x,y,z,t) = \psi(\xi)e^{ik(\alpha x+\gamma y+\lambda z+\beta t)}, \quad \xi = x + y + z - ct \tag{11}$$

We substitute Eq. (11) into Eq. (1) and obtain the following ordinary differential equation

$$\begin{aligned} &3\psi'' + i[k(\alpha+\gamma+\lambda) - c]\psi' - [k^2(\alpha^2+\gamma^2+\lambda^2) \\ &\quad + k\beta + 2U]\psi - g\psi^3 = 0, \end{aligned} \tag{12}$$

The pole of the Eq. (12) is equal to $N = 1$, thus we look for exact solution in the forms

$$U = a_0 + a_1 Q(\xi). \tag{13}$$

Substituting (13) in Eq. (12) and taking (10) into account we obtain the polynomial of functions $Q(\xi)$. Collecting all terms with the same power of functions $Q(\xi)$ and equate this expressions to zero. Then we obtain the system of algebraic equations. By solving this system, we find that values of parameters as follow cases:

Case 1:

$$a_1 = \sqrt{\frac{6}{g}},$$
$$a_0 = \sqrt{-\frac{[k^2(\alpha^2+\gamma^2+\lambda^2)+k\beta+U]}{g}},$$
$$c = 2k(\alpha+\gamma+\lambda) - 2k^2 i(\alpha^2+\gamma^2+\lambda^2) - 2k\beta i - 2iU - 3i. \tag{14}$$

Taking the solution set (14) along with (7) and (13) we have solutions of (12) as follows

$$\psi_1 = \sqrt{-\frac{[k^2(\alpha^2+\gamma^2+\lambda^2)+k\beta+U]}{g}} + \sqrt{\frac{6}{g}}(1+\exp(x+y+z-(2k(\alpha+\gamma+\lambda) - 2k^2 i(\alpha^2+\gamma^2+\lambda^2) - 2k\beta i - 2iU - 3i)t))^{-1},$$

Thus, we have the solitary wave solution of the 3D- Gross–Pitaevskii equation with space and time modulated potential and nonlinearity is in the following form:

$$\hbar_1(x,y,z,t) = \left[\sqrt{-\frac{[k^2(\alpha^2+\gamma^2+\lambda^2)+k\beta+U]}{g}} + \sqrt{\frac{6}{g}}(1+\exp(x+y+z-(2k(\alpha+\gamma+\lambda) - 2k^2 i(\alpha^2+\gamma^2+\lambda^2) - 2k\beta i - 2iU - 3i)t))^{-1}\right] \times \exp(ik(\alpha x+\gamma y+\lambda z+\beta t)),$$

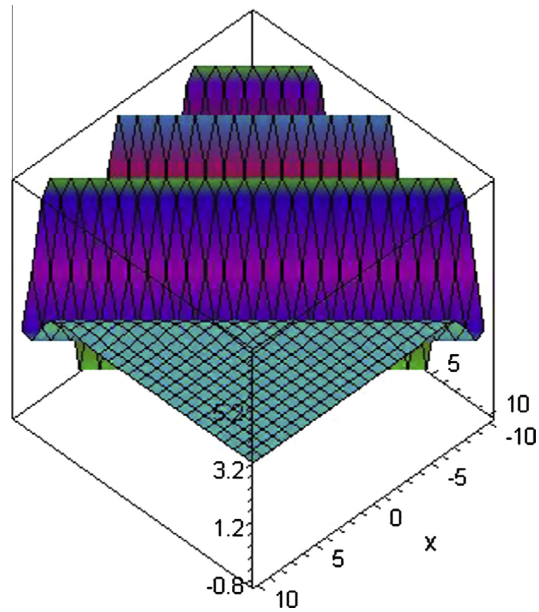

Figure 1 Complex solution corresponding to $\hbar_3(x,y,z,t)$ in two dimensional. For $y = z = 1$ and $-10 \le x \le 10$ and $0 \le t \le 10$.

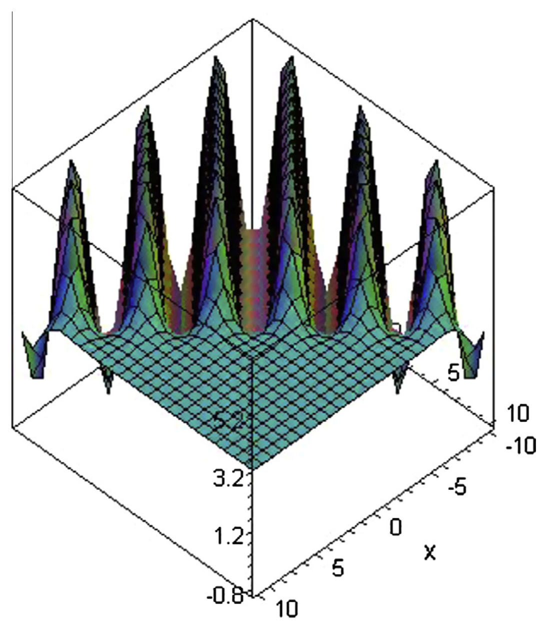

Figure 2 Complex solution corresponding to $\hbar_4(x,y,z,t)$ in two dimensional. For $y = z = 1$ and $-10 \le x \le 10$ and $0 \le t \le 10$.

Case 2:

$$a_1 = \frac{1}{3}\frac{-9+2ik\,\alpha+2ik\,\gamma+2ik\,\lambda - i(2k(\alpha+\gamma+\lambda) - 2k^2 i(\alpha^2+\gamma^2+\lambda^2) - 2k\beta i - 2iU - 3i)}{\sqrt{-\frac{k^2\alpha^2+k^2\gamma^2+k^2\lambda^2+k\beta+U}{g}}},$$
$$a_0 = \sqrt{-\frac{[k^2(\alpha^2+\gamma^2+\lambda^2)+k\beta+U]}{g}},$$
$$c = 2k(\alpha+\gamma+\lambda) - 2k^2 i(\alpha^2+\gamma^2+\lambda^2) - 2k\beta i - 2iU - 3i. \tag{15}$$

Using the conditions (15) and (7) in (13), we obtain the solution of (1)

$$\hbar_2(x,y,z,t) = \left[\sqrt{-\frac{[k^2(\alpha^2+\gamma^2+\lambda^2)+k\beta+U]}{g}} + \frac{1}{3}\frac{-9+2ik\ \alpha+2ik\ \gamma+2ik\ \lambda-i(2k(\alpha+\gamma+\lambda)-2k^2i(\alpha^2+\gamma^2+\lambda^2)-2k\beta i-2iU-3i)}{\sqrt{-\frac{k^2\alpha^2+k^2\gamma^2+k^2\lambda^2+k\beta+U}{g}}} \times (1+\exp(x+y+z-(2k(\alpha+\gamma+\lambda)-2k^2i(\alpha^2+\gamma^2+\lambda^2)-2k\beta i-2iU-3i)t))^{-1}\right] \times \exp(ik(\alpha x+\gamma y+\lambda z+\beta t)),$$

Case 3:

$$a_1 = -\sqrt{\frac{6}{g}},\quad a_0 = -\sqrt{-\frac{[k^2(\alpha^2+\gamma^2+\lambda^2)+k\beta+U]}{g}},\quad c = 2k(\alpha+\gamma+\lambda)-2k^2i(\alpha^2+\gamma^2+\lambda^2)-2k\beta i-2iU-3i. \tag{16}$$

Now, taking the solution set (16) along with (7) into account, Eq. (13) becomes

$$\hbar_3(x,y,z,t) = \left[-\sqrt{-\frac{[k^2(\alpha^2+\gamma^2+\lambda^2)+k\beta+U]}{g}} - \sqrt{\frac{6}{g}}(1+\exp(x+y+z-(2k(\alpha+\gamma+\lambda) -2k^2i(\alpha^2+\gamma^2+\lambda^2)-2k\beta i-2iU-3i)t))^{-1}\right] \times \exp(ik(\alpha x+\gamma y+\lambda z+\beta t)),$$

Case 4:

$$a_1 = \frac{1}{3}\frac{-9+2ik\ \alpha+2ik\ \gamma+2ik\ \lambda-i(2k(\alpha+\gamma+\lambda)-2k^2i(\alpha^2+\gamma^2+\lambda^2)-2k\beta i-2iU-3i)}{\sqrt{-\frac{k^2\alpha^2+k^2\gamma^2+k^2\lambda^2+k\beta+U}{g}}},\quad a_0 = -\sqrt{-\frac{[k^2(\alpha^2+\gamma^2+\lambda^2)+k\beta+U]}{g}},\quad c = 2k(\alpha+\gamma+\lambda)-2k^2i(\alpha^2+\gamma^2+\lambda^2)-2k\beta i-2iU-3i. \tag{17}$$

As above the solution of the 3D-GPE under the condition (17) along with (7) and (13) is

$$\hbar_4(x,y,z,t) = \left[\sqrt{-\frac{[k^2(\alpha^2+\gamma^2+\lambda^2)+k\beta+U]}{g}} + \frac{1}{3}\frac{-9+2ik\ \alpha+2ik\ \gamma+2ik\ \lambda-i(2k(\alpha+\gamma+\lambda)-2k^2i(\alpha^2+\gamma^2+\lambda^2)-2k\beta i-2iU-3i)}{\sqrt{-\frac{k^2\alpha^2+k^2\gamma^2+k^2\lambda^2+k\beta+U}{g}}} \times (1+\exp(x+y+z-(2k(\alpha+\gamma+\lambda)-2k^2i(\alpha^2+\gamma^2+\lambda^2)-2k\beta i-2iU-3i)t))^{-1}\right] \times \exp(ik(\alpha x+\gamma y+\lambda z+\beta t)),$$

4. Conclusion

Complex wave behavior showed in Figs. 1 and 2 when parameters given special values. In this article we constructed the new exact solitary wave solutions for the 3D-GP equation by means of Kudryashov approach. This equation is a general version of the dissipative Gross-Pitaevskii equation. These results show that the four-wave type of ansatz approach is effective and simple method for analyzing three-wave solutions and two-wave solutions of higher dimensional nonlinear evolution equations.

Acknowledgments

With the special thanks from the Editor and the referee(s) for their careful reading and for their comments which greatly improved the paper. I really appreciate your clear and accurate guidance as well as your helpful comments on my work.

References

[1] Ma. Manjun, Zhe Huang, Bright soliton solution of a Gross–Pitaevskii equation, Appl. Math. Lett. 26 (2013) 718–724.

[2] S.A. El-Wakil, M.A. Abdou, New exact travelling wave solutions using modified extended tanh-function method, Chaos Solitons Fract. 31 (4) (2007) 840–852.

[3] E. Fan, Extended tanh-function method and its applications to nonlinear equations, Phys. Lett. A 277 (4–5) (2000) 212–218.

[4] A.M. Wazwaz, The tanh-function method: solitons and periodic solutions for the Dodd-Bullough-Mikhailov and the Tzitzeica-Dodd-Bullough equations, Chaos Solitons Fract. 25 (1) (2005) 55–63.

[5] A. Biswas, Optical solitons with time-dependent dispersion, nonlinearity and attenuation in a Kerr-law media, Int. J. Theor. Phys. 48 (2009) 256–260.

[6] A. Biswas, 1-Soliton solution of the B(m, n) equation with generalized evolution, Commun. Nonlinear Sci. Numer. Simul. 14 (2009) 3226–3229.

[7] A. Biswas, Solitary wave solution for KdV equation with power-law nonlinearity and time-dependent coefficients, Nonlinear Dyn. 58 (2009) 345–348.

[8] A. Biswas, 1-Soliton solution of the K(m, n) equation with generalized evolution, Phys. Lett. A 372 (2008) 4601–4602.

[9] A. Biswas, Solitary wave solution for the generalized Kawahara equation, Appl. Math. Lett. 22 (2009) 208–210.

[10] W.X. Ma, Travelling wave solutions to a seventh order generalized KdV equation, Phys. Lett. A 180 (1993) 221–224.

[11] W. Malfliet, Solitary wave solutions of nonlinear wave equations, Am. J. Phys. 60 (7) (1992) 650–654.

[12] W.X. Ma, T.W. Huang, Y. Zhang, A multiple exp-function method for nonlinear differential equations and its application, Phys. Scr. 82 (2010) 065003.

[13] N.A. Kudryashov, Exact solitary waves of the Fisher equation, Phys. Lett. A 342 (1–2) (2005) 99–106.

[14] N.A. Kudryashov, Simplest equation method to look for exact solutions of nonlinear differential equations, Chaos Soliton Fract. 24 (5) (2005) 1217–1231.

[15] W. Malfliet, Solitary wave solutions of nonlinear wave equations, Am. J. Phys. 60 (7) (1992) 650–654.

[16] W.X. Ma, T.W. Huang, Y. Zhang, A multiple exp-function method for nonlinear differential equations and its application, Phys. Scr. 82 (2010) 065003.

[17] N.A. Kudryashov, Exact solitary waves of the Fisher equation, Phys. Lett. A 342 (1–2) (2005) 99–106.

[18] N.A. Kudryashov, Simplest equation method to look for exact solutions of nonlinear differential equations, Chaos Soliton Fract. 24 (5) (2005) 1217–1231.

[19] R. Hirota, Exact solution of the Korteweg-de Vries equation for multiple collision of solitons, Phys. Rev. Lett. 27 (1971) 1192–1194.