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ORIGINAL ARTICLE

Unsteady mixed convection heat transfer along a vertical stretching surface with variable viscosity and viscous dissipation

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KEYWORDS

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Abstract The effect of variable viscosity on laminar mixed convection flow and heat transfer along a semi-infinite unsteady stretching sheet taking into account the effect of viscous dissipation is studied. The flow of the fluid and subsequent heat transfer from the stretching surface is investigated with the aid of suitable transformation variables. Solutions for the velocity and temperature fields are obtained for some representative values of the unsteadiness parameter, variable viscosity parameter, mixed convection parameter and Eckert number. Typical velocity and temperature profiles, the local skin friction coefficient and the local heat transfer rate are presented at selected controlling parameters.

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1. Introduction

Boundary layer flow on a moving continuous surface is an important type of flow occurring in a number of engineering processes. Aerodynamic extrusion of plastic sheets, cooling of metallic sheets in a cooling bath, which would be in the form of an electrolyte, crystal growing, the boundary layer along a liquid film in condensation process and polymer sheet extruded continuously from a die are the practical applications of moving surfaces and also the materials manufactured by extrusion processes and heat treated materials traveling between a feed roll and wind up roll or on a conveyer belt possesses the characteristics of a moving continuous surfaces. After a pioneering work of Sakiadis [1,2] the study of flow and heat transfer characteristics past continuous stretching surfaces has drawn considerable attention, and a good amount of the literature has been generated on this problem (see for instance [3–18]). Although various aspects of this class of boundary layer problems have been tackled, the effect of buoyancy force was ignored in the above-cited reports

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[1–18]. In actual practice, the flow over a continuous material moving through a quiescent fluid is induced by the movement of the solid material and by thermal buoyancy. Therefore, these two mechanisms, surface motion and buoyancy force will determine the momentum and thermal transport processes. The thermal buoyancy force arising due to the heating of a continuously moving surface, under some circumstances, may alter significantly the flow and thermal fields and thereby the heat transfer behavior in the manufacturing process. Lin et al. [19] studied the problem of mixed convection from an isothermal horizontal plate moving in parallel or reversely to a free stream. Ali and Al-Yousef [20–21] investigated the problem of laminar mixed convection adjacent to a moving vertical surface with suction or injection. Karwe and Jaluria [22] showed that the thermal buoyancy effects are more prominent when the plate moves vertically, i.e., aligned with the gravity, than when it is horizontal. Also, an analysis of mixed convection heat transfer from a vertical continuously stretching sheet has been presented by Chen [23]. Ali [24] studied the buoyancy effect on the boundary layer induced by continuous surface stretched with rapidly decreasing velocities. Abo-Eldahab and Abd El-Aziz [25] presented the problem of steady, laminar, hydromagnetic heat transfer by mixed convection over an inclined stretching surface in the presence of space- and temperature-dependent heat generation or absorption effects. Abd El-Aziz [26] investigated the problem of thermal radiation effects on magnetohydrodynamic mixed convection flow of a micropolar fluid past a continuously moving semi-infinite plate for high temperature differences. Abd El-Aziz and Salem [27] studied the effects of coupled heat and mass transfer by natural convection and chemical reaction on the flow of an incompressible, viscous, and electrically conducting fluid past a vertical and linearly stretched permeable sheet with variable surface temperature and mass flux. Salem and Abd El-Aziz [28] investigated the effect of Hall currents and chemical reaction on hydromagnetic flow of a stretching vertical surface with internal heat generation or absorption. In all the above mentioned studies the viscosity of the ambient fluid was assumed to be constant. However, it is known that the fluid viscosity changes with temperature [29], for example the absolute viscosity of water decreases by 240% when the temperature increases from 10 °C to 50 °C. So in order to predict accurately the flow behavior, it is necessary to take into account this variation in viscosity since recent results on the flow due to stretching surface with and without buoyancy force have shown that when this effect is included, the flow characteristics may be substantially changed compared to the constant viscosity case. Pop et al. [30] studied the effect of variable viscosity on flow and heat transfer to a continuous moving flat plate using the similarity solution with no buoyancy force. Mukhopadhyay et al. [31] studied the boundary layer flow over a heated stretching sheet with variable viscosity in the presence of magnetic field. Ali [32] studied the effect of temperature-dependent viscosity on mixed convection heat transfer along a vertical moving surface taking into account the effect of buoyancy force. Abd El-Aziz [33] studied the problem of temperature-dependent viscosity and thermal conductivity effects on combined heat and mass transfer in MHD three-dimensional flow over a stretching surface with Ohmic heating. If the fluid is viscous, considerable heat can be produced even though at relatively low speeds, e.g. in the extrusion of a plastic, and hence the heat transfer results may alter appreciably due to viscous dissipation. Partha et al. [34] presented a similarity solution for mixed convection flow and heat transfer from an exponentially stretching surface by considering viscous dissipation effect in the medium. All of the above investigations have been restricted to steady-state conditions. However, in certain practical problems, the motion of the stretched surface may start impulsively from rest. In these problems the transient or unsteady aspects become of interest. Elbashbeshy and Bazid [35] presented an exact similarity solution for momentum and heat transfer in an unsteady flow whose motion is caused solely by the linear stretching of a horizontal stretching surface. Ali and Magyari [36] presented the problem of unsteady fluid and heat flow induced by a submerged stretching surface while its steady motion is slowed down gradually. Recently, Abd El-Aziz [37–39] extended the problem of Elbashbeshy and Bazid [35] to include various aspects such as thermal radiation, Hall currents and time-dependent chemical reaction. However, the combined effect of buoyancy force, viscous dissipation and variable viscosity on the flow and heat transfer which is important in view point of desired properties of the outcome is not considered in the earlier papers [35–39]. Motivated by all the works mentioned above, it is of interest to extend the work [35] by including such effects on the flow and heat transfer from an unsteady stretching surface. It will be demonstrated that the system of time-dependent governing equations can be reduced to a five-parameter problem by introducing a suitable transformation variables. Accurate numerical solutions are generated by employing shooting method. A comprehensive parametric study is conducted and a representative set of graphical results for the velocity and temperature profiles as well as the skin friction and wall heat transfer coefficients are reported and discussed. The analysis showed that the unsteadiness parameter, buoyancy force, variable viscosity property and viscous dissipation have significant influence on the flow and thermal fields as well as the non-dimensional local skin friction and heat transfer coefficients.

2. Analysis

Consider the mixed convection, boundary layer flow of a viscous, incompressible fluid along an unsteady stretching sheet, which issues vertically in the upward direction from a slot with velocity

$$u_w = bx/(1 - \alpha t), \tag{1}$$

where b and α are positive constants with dimensions (time)$^{-1}$. Here, b is the initial stretching rate, whereas the effective stretching rate $b/(1 - \alpha t)$ is increasing with time. In the context of polymer extrusion the material properties and in particular the elasticity of the extruded sheet may vary with time even though the sheet is being pulled by a constant force. A schematic representation of the physical model and coordinates system is depicted in Fig. 1. The positive x coordinate is measured along the stretching sheet with the slot as the origin and the positive y coordinate is measured normal to the sheet in the outward direction toward the fluid. The surface temperature T_w of the stretching sheet varies with the distance x from the slot and time t as [40]

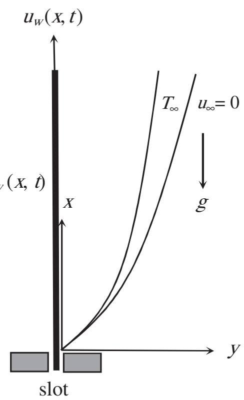

Figure 1 Schematic representation of the physical model and coordinates system.

$$T_w = T_\infty + T_0[bx^2/2v_\infty](1-\alpha t)^{-2}. \tag{2}$$

where T_∞ is the temperature of the ambient fluid, T_0 is a (positive or negative; heating or cooling) reference temperature and v_∞ is the kinematic viscosity of the ambient fluid. It is apt to note here that, the expressions for u_w (x,t) and T_w (x,t) defined in Eqs. (1) and (2) are valid only for time $t < \alpha^{-1}$. The expression (2) for the temperature $T_w(x,t)$ of the sheet represents a situation in which the sheet temperature increases (reduces) if T_0 is positive (negative) from T_∞ at the leading edge in proportion to x^2 and such that the amount of temperature increase (reduction) along the sheet increases with time.

Using Boussinesq approximation for incompressible viscous fluid environment in addition to that, the fluid viscosity is assumed to vary as an inverse linear function of temperature [41]

$$\frac{1}{\mu} = \frac{1}{\mu_\infty}\ [1+\gamma(T-T_\infty)],\ \text{or}\ 1/\mu = a\ (T-T_r), \tag{3}$$

where

$$a = \gamma/\mu_\infty\ \text{and}\ T_r = T_\infty - 1/\gamma, \tag{4}$$

are constants and their values depend on the reference state and the thermal property of the fluid γ. In general, $a > 0$ for liquids and $a < 0$ for gases [42].

The boundary layer equations for mass, momentum and heat transfer in the presence of viscous dissipation take the form

$$\frac{\partial u}{\partial x} + \frac{\partial \upsilon}{\partial y} = 0, \tag{5}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \upsilon\frac{\partial u}{\partial y} = \frac{1}{\rho_\infty}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) + g\beta(T-T_\infty), \tag{6}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + \upsilon\frac{\partial T}{\partial y} = \frac{k}{\rho_\infty c_p}\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho_\infty c_p}\left(\frac{\partial u}{\partial y}\right)^2, \tag{7}$$

along with the boundary conditions

$$\left.\begin{array}{llll} u = u_w, & \upsilon = 0, & T = T_w & \text{at } y = 0 \\ u \to 0, & T \to T_\infty & & \text{as } y \to \infty \end{array}\right\} \tag{8}$$

Proceeding with the analysis, we introduce the following similarity variable η and the dimensionless variables f and θ:

$$\eta = \sqrt{\frac{b}{v_\infty(1-\alpha t)}}y, \tag{9}$$

$$\psi = \sqrt{v_\infty b/(1-\alpha t)}\cdot x\cdot f(\eta), \tag{10}$$

$$T = T_\infty + T_0[bx^2/2v_\infty]\ (1-\alpha t)^{-2}\ \theta(\eta), \tag{11}$$

where $\psi(x, y, t)$ is the stream function satisfying the continuity Eq. (5) with $u = \partial\psi/\partial y$ and $\upsilon = -\partial\psi/\partial x$. The components of velocity can be readily expressed as:

$$u = u_w f'(\eta), \quad \upsilon = -\sqrt{v_\infty b/(1-\alpha t)}\cdot f(\eta) \tag{12}$$

Making use of Eqs. (9)–(11), Eqs. (6) and (7) reduce to

$$\frac{\theta_r}{\theta_r-\theta}f''' + \frac{\theta_r}{(\theta_r-\theta)^2}f''\theta' - A\left(f' + \frac{\eta}{2}f''\right) + ff'' - (f')^2 + \lambda\theta = 0, \tag{13}$$

$$\frac{1}{\Pr}\theta'' - \frac{A}{2}(4\theta + \eta\theta') + f\theta' - 2f'\theta + \frac{\theta_r}{\theta_r-\theta}Ec(f'')^2 = 0, \tag{14}$$

The transformed boundary conditions are:

$$\left.\begin{array}{llll} f = 0, & f' = 1, & \theta = 1 & \text{at } \eta = 0 \\ f' \to 0, & \theta \to 0 & & \text{as } \eta \to \infty \end{array}\right\} \tag{15}$$

where a prime denotes ordinary differentiation with respect to η, $\theta = (T-T_\infty)/(T_w-T_\infty)$ is the non-dimensional temperature, $\theta_r = (T_r-T_\infty)/(T_w-T_\infty) = -1/(\gamma(T_w-T_\infty))$ is the variable viscosity parameter, $A = \alpha/b$ is the unsteadiness parameter, $Ec = \frac{2bv_\infty}{T_0 c_p} = \frac{u_w^2}{c_p(T_w-T_\infty)}$ is the Eckert number which characterizes the viscous dissipation effect, $\Pr = v_\infty\rho_\infty c_p/k$ is the Prandtl number and $\lambda = \frac{g\beta T_0 x}{2v_\infty b} = \frac{Gr_x}{Re_x^2}$ is the mixed convection parameter with $Gr_x = g\beta(T_w-T_\infty)x^3/v_\infty^2$ is the Grashof number. The case in which $\lambda = 0$ corresponds to the forced convection regime while that in which λ is large corresponds to the free convection regime. For $t = 0$ $(A = 0)$ Eqs. (13) and (14) reduce to those of steady flow and for $t > 0$ $(A \neq 0)$ it applies to unsteady flow. It is worth mentioning that, when $\lambda = Ec = 0$, and $\theta_r \to \infty$, Eqs. (13) and (14) reduce to those of Ali and Magyari [36] when $c = n = 2$ in their equations.For practical applications, the physical quantities of major interest are the local friction coefficient C_{fx}

$$C_{fx} = \frac{2\mu(\partial u/\partial y)_{y=0}}{\rho_\infty u_w^2} = \frac{2\theta_r}{\theta_r - 1}Re_x^{-1/2}f''(0), \tag{16}$$

and the local Nusselt number Nu_x

$$Nu_x = -\frac{x}{T_0}(\partial T/\partial y)_{y=0} = -\frac{Re_x^{3/2}}{2(1-\alpha t)}\theta'(0), \tag{17}$$

where $Re_x = u_w x/v_\infty$ is the local Reynolds number based on the sheet velocity u_w.

3. Numerical solution

The non-linear coupled differential Eqs. (13) and (14) subject to the boundary conditions (15) constitute a two-point boundary value problem has been solved numerically by the shooting method. The differential Eqs. (13) and (14) were first formulated as a set of five first-order simultaneous equations of for five unknowns following the method of superposition [43]. To solve this system we require five initial conditions while we have only two initial conditions $f(0)$ and $f'(0)$ on f and one initial condition $\theta(0)$ on θ. Still there are two initial conditions $f''(0)$ and $\theta'(0)$ which are not prescribed. However the values of $f'(\eta)$ and $\theta(\eta)$ are known at $\eta \to \infty$. Now we employ the numerical shooting technique where these two ending boundary conditions are utilized to produce two known initial conditions at $\eta = 0$. The problem has been solved numerically using fourth-order Runge–Kutta integration scheme.

4. Results and discussion

In order to verify the validity and accuracy of the present analysis, results for the local heat transfer rate were compared with those of Grubka and Bobba [5] and Chen [23] for forced convection flow on a linearly steady stretching surface in the absence of viscous dissipation. The comparison in the above cases is found to be in excellent agreement, as shown in Table 1. The excellent agreement between the present results and previously published data is an encouragement for further study of the effects of other parameters on the flow and heat transfer characteristics of the continuously stretching sheet.

Figs. 2 and 3 show the velocity $f'(\eta)$ and temperature $\theta(\eta)$ profiles for various values of variable viscosity parameter θ_r and mixed convection parameter λ. The presence of the thermal buoyancy effects represented by finite values of λ ($\lambda \neq 0$) has the tendency to induce more flow along the surface. This is reflected in the increases in $f'(\eta)$ as λ increases, for given θ_r, as shown in Fig. 2. Distinctive peak in the velocity profiles which is a characteristic of free convection flows is also observed for large values of the mixed convection parameter λ ($\lambda = 20$) in Fig. 2. On the other hand, the velocity decreases as θ_r increases for both forced ($\lambda = 0$) and mixed ($\lambda = 1$) convection regimes. However, for large values of λ (dominant free convection) the velocity significantly increases with increasing θ_r near the plate where $0 \leqslant \eta \leqslant 1$ but the opposite result is observed far from the plate where $\eta > 1$. In other words, an increase in the values of θ_r causes an increase in the velocity $f'(\eta)$ near the stretching surface whereas it produces lower velocities toward the edge of the boundary layer only in natural convection process. Fig. 3 depicts the graphs for the temperature profiles for the same data values as used in Fig. 2. It is noticed from this figure that the effect of increasing values of λ is to decrease thermal boundary layer thickness. Further, the temperature increases as θ_r increases for both forced ($\lambda = 0$) and mixed ($\lambda = 1$) convection regimes while the situation is completely reversed for free convection regime ($\lambda = 20$). Namely, the temperature of the fluid decreases as θ_r increases for large values of the mixed convection parameter λ.

Figs. 4 and 5 show the velocity $f'(\eta)$ and temperature $\theta(\eta)$ profiles for various values of the variable viscosity parameter θ_r and the unsteadiness parameter A. From these figures it observed that increasing values of A results in decreasing the velocity and temperature keeping other parameters fixed.

Table 1 Comparison of the values of $-\theta'(0)$ for $A = \lambda = Ec = 0$, $\theta_r \to \infty$ and selected values of Pr with previously published data.

Pr	Grubka and Bobba [5]	Chen [23]	Present results
0.01	0.0294	0.02942	0.02948
0.72	1.0885	1.08853	1.08855
1.0	1.3333	1.33334	1.33333
3.0	2.5097	2.50972	2.50972
7.0	–	3.97150	3.97151
10	4.7969	4.79686	4.79687
100	15.712	15.7118	15.7120

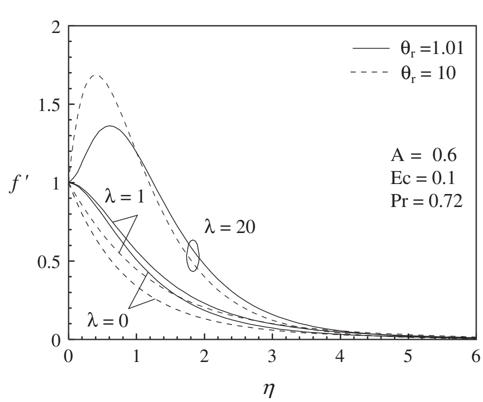

Figure 2 Dimensionless velocity profiles for various values of λ and θ_r.

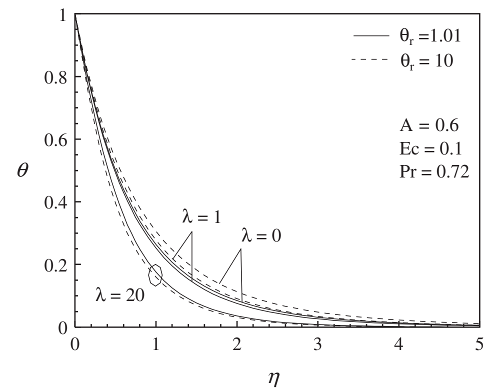

Figure 3 Dimensionless temperature profiles for various values of λ and θ_r.

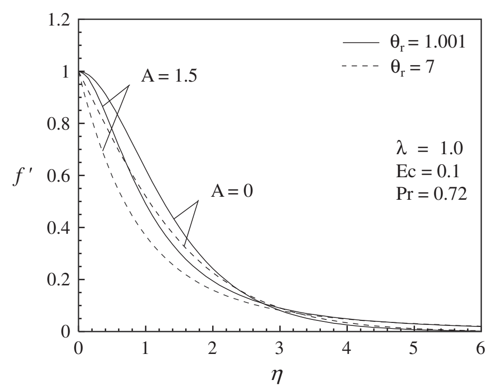

Figure 4 Dimensionless velocity profiles for various values of θ_r and A.

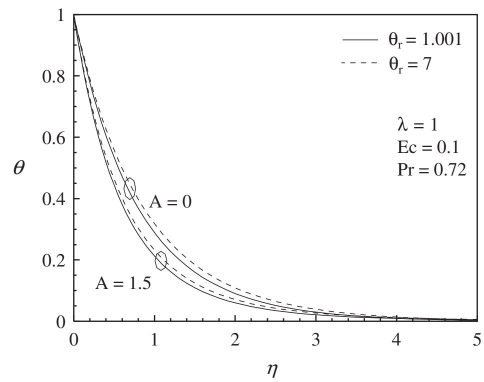

Figure 5 Dimensionless temperature profiles for various values of θ_r and A.

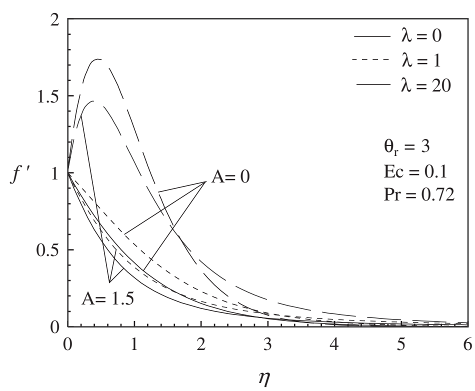

Figure 6 Dimensionless velocity profiles for various values of λ and A.

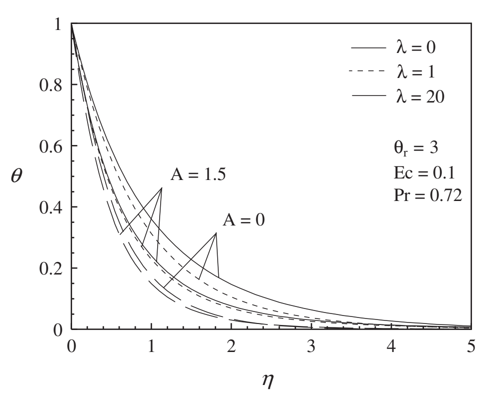

Figure 7 Dimensionless temperature profiles for various values of λ and A.

Fig. 4 reveals that the effect of θ_r on the velocity $f'(\eta)$ in the unsteady flow ($A = 1.5$) is more pronounced than the steady flow ($A = 0$). However, the effect of θ_r on the temperature θ (η) profiles in the steady flow is more pronounced than the unsteady flow as shown in Fig. 5.

Figs. 6 and 7 present typical steady ($A = 0$) and unsteady ($A = 1.5$) state velocity $f'(\eta)$ and temperature θ (η) profiles for various values of the mixed convection parameter λ. From these figures it is observed that the effect of λ on the velocity and temperature in the steady flow is more prominent than

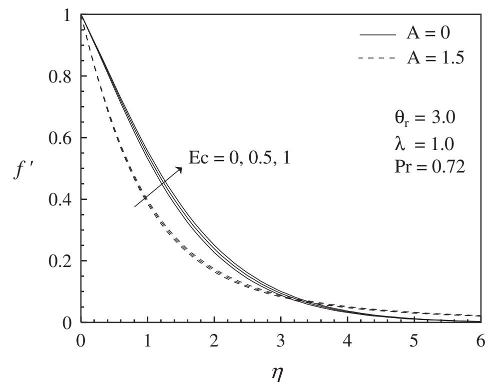

Figure 8 Dimensionless velocity profiles for various values of Ec and A.

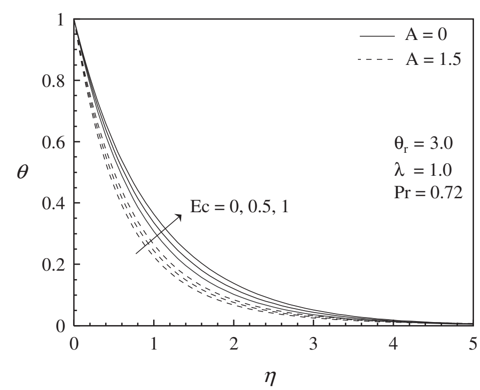

Figure 9 Dimensionless temperature profiles for various values of Ec and A.

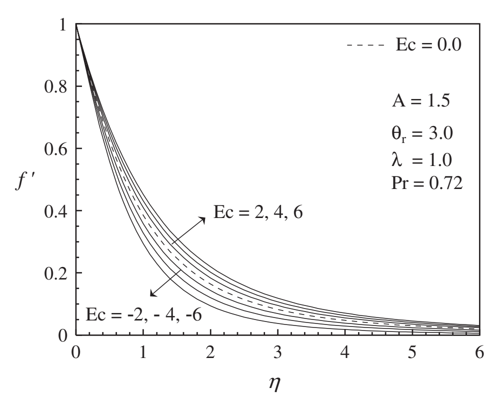

Figure 10 Dimensionless velocity profiles for various values of Ec.

the unsteady flow. Also the effect on the flow and thermal fields become more so as the strength of the mixed convection parameter λ increases.

The effect of Eckert number Ec and the unsteadiness parameter A on the velocity $f'(\eta)$ and temperature θ (η) profiles is shown Figs. 8 and 9. The fluid temperature is observed to be increased in the medium as Ec increases because of the internal heat generated due to viscous dissipation ($Ec \neq 0$) in the medium as shown in Fig. 9. This has the direct effect in increasing

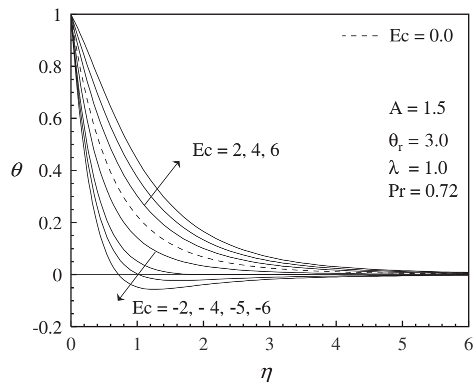

Figure 11 Dimensionless temperature profiles for various values of *Ec*.

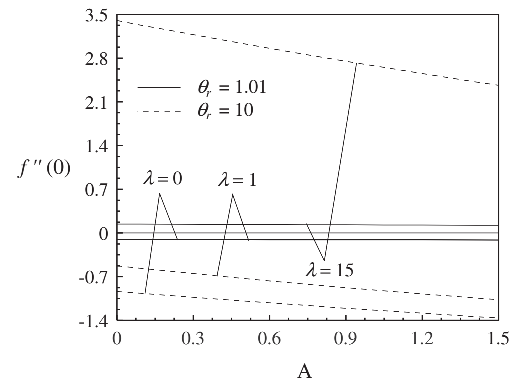

Figure 12 Local skin friction coefficient f'' (0) versus A for various values of λ and θ_r.

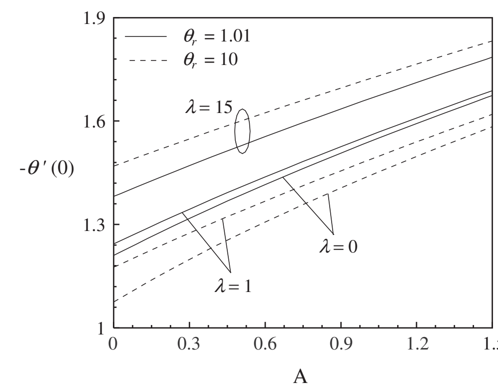

Figure 13 Local heat transfer rate $-\theta'$ (0) versus A for various values of λ and θ_r.

the fluid velocity due to the increases in the values of the Eckert number *Ec* as depicted in Fig. 8. Furthermore, Figs. 8 and 9 reveal that the velocity and temperature profiles in the unsteady flow exhibit similar fashions to those in the steady flow. Also viscous dissipation demonstrates a more pronounced influence on the velocity and temperature distribution in the case of steady flow.

Figs. 10 and 11 illustrate the effect of viscous dissipation on the velocity and temperature profiles for $A = 1.5$, $\theta_r = 0.3$, $\lambda = 1$ and $\Pr = 0.72$. According to the definition of Eckert number, a positive *Ec* corresponds to fluid heating (heat is being supplied across the walls into the fluid) case ($T_w > T_\infty$) so that the fluid is being heated whereas a negative *Ec* means that the fluid is being cooled. From Fig. 10 it is seen that the velocity $f'(\eta)$ of the fluid heating case ($Ec > 0$) is higher than that of the case without viscous dissipation ($Ec = 0$), whereas the opposite trend is observed for the fluid cooling case ($\mathrm{Ec} < 0$). Fig. 11 shows that the dimensionless temperature increases when the fluid is being heated ($Ec > 0$) but decrease when the fluid is being cooled ($Ec < 0$). For $Ec < 0$ the dimensionless fluid temperature $\theta(\eta)$ decreases monotonically with η, from unity at the wall toward its free stream value. It is noted from the definition of θ that this behavior implies the monotonous decrease in the actual fluid temperature in the horizontal direction from the sheet temperature T_w to the free stream temperature. The increase in the fluid temperature due to viscous heating is observed to be more pronounced for a higher value of *Ec*, as expected. On the other hand, for $Ec < 0$ (i.e. $T_w <$ - T_∞) the dimensionless fluid temperature θ decreases with η rapidly at first, arriving at a negative minimum value, for $Ec < -4$ and then increases more gradually to its free stream value. Correspondingly, the actual fluid temperature in the horizontal direction increases at first from the surface temperature T_w to a maximum value and then decrease to its free stream value. It should be noted that for the fluid cooling case ($Ec < 0$) a negative θ indicates the excess of the free stream temperature T_∞ over the actual fluid temperature T because of the viscous dissipation effect.

Typical variations in the local skin friction coefficient in terms of the wall velocity gradient $f''(0)$ and the local heat transfer rate $-\theta'$ (0) are illustrated as a function of the unsteadiness parameter A in Figs. 12 and 13 for various values of λ and θ_r. From Fig. 12 it is interesting to note that the effect of mixed convection parameter λ on the local skin friction coefficient for large values of θ_r ($\theta_r = 10$, where the viscosity approaches its constant value) is much larger than that for lower values of θ_r ($\theta_r = 1.01$, where the viscosity variation with temperature is very large). In other words, the effect of λ on f'' (0) is insignificant when $\theta_r \to 1$ as compared to that when ($\theta_r = 10$). Also Fig. 12 reveals that the effect of θ_r is to decrease the skin friction coefficient f'' (0) for both forced ($\lambda = 0$) and mixed ($\lambda = 1$) convection regimes but the opposite trend occurs for free convection regime ($\lambda = 15$). These behaviors are consistent with the results of the velocity profiles shown in Fig. 2. Furthermore, the maximum effect of the variable viscosity and mixed convection parameters on the local skin friction coefficient occurs when the flow is steady ($A = 0$) as shown in Fig. 12. In addition, for given θ_r and λ, the skin friction coefficient f'' (0) decreases with increasing the unsteadiness parameter A. From Figs. 12 and 13 it is clear that the local skin friction coefficient f'' (0) and the local heat transfer rate $-\theta'$ (0) are both increased with increasing λ for given θ_r and A. This is due to the fact that a positive λ induces a favorable pressure gradient that enhances the flow and heat transfer in the boundary layer. Further, the effect of λ on the $-\theta'$ (0) becomes more significant for higher values of the variable viscosity parameter as one can see from Fig. 12 by comparing the curves with $\theta_r = 1.01$ and $\theta_r = 10$ for given A. Also, it is noted from Fig. 13 that an increase in the value of θ_r produces a decrease in the local heat transfer rate $-\theta'$ (0) for both forced ($\lambda = 0$) and mixed ($\lambda = 1$) convection flow but the opposite result obtained for free convection regime ($\lambda = 15$). These behaviors agree quite with the results of the

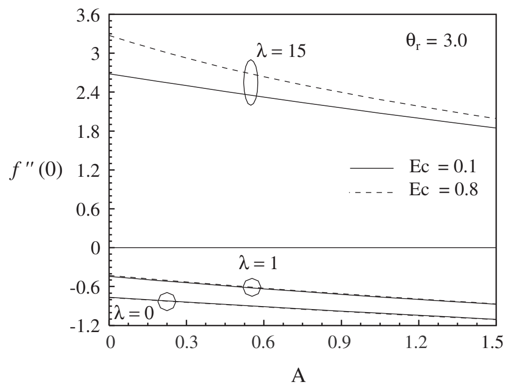

Figure 14 Local skin friction coefficient f'' (0) versus A for various values of λ and Ec.

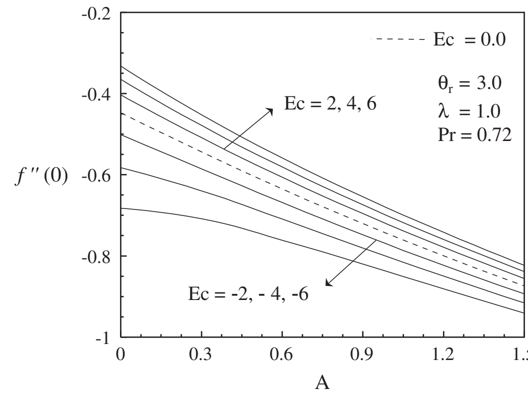

Figure 16 Local skin friction coefficient f'' (0) versus A for various values of Ec.

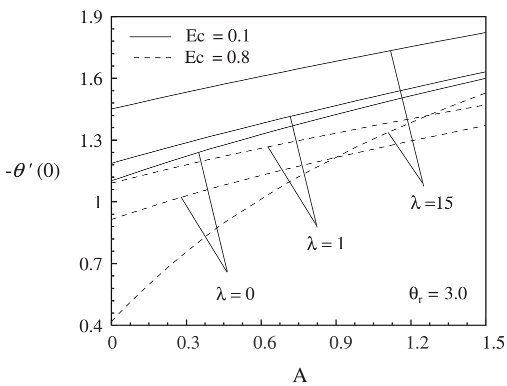

Figure 15 Local heat transfer rate $-\theta'$ (0) versus A for various values of λ and Ec.

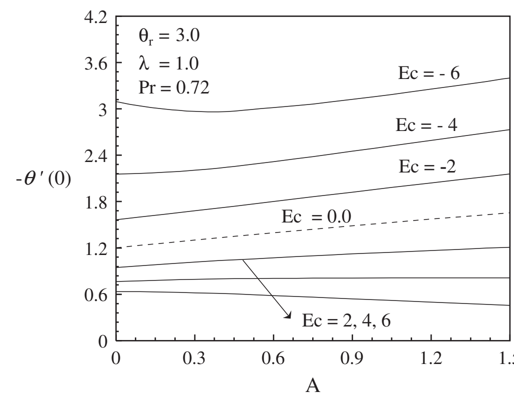

Figure 17 Local heat transfer rate $-\theta'$ (0) versus A for various values of Ec.

temperature profiles shown in Fig. 3. Moreover, from Fig. 13 it is obvious that for given θ_r and λ the heat transfer rate is greatly increased with an increase in the unsteadiness parameter A.

The local skin fiction coefficient f'' (0) and the local heat transfer rate $-\theta'$ (0) are plotted against the unsteadiness parameter A at selected values of λ and Ec in Figs. 14 and 15, respectively. For both forced ($\lambda = 0$) and mixed ($\lambda = 1$) convection regimes, viscous dissipation exhibits a negligible effect on the skin friction coefficient f'' (0) between the fluid and the elastic sheet for all values of A as shown in Fig. 14. Even, for example, for $\lambda = 1$, $A = 0$ and $A = 1.5$ the slopes of different velocity profiles in Fig. 8 at $\eta = 0$ are scarcely discernible. On the other hand, Fig. 14 shows that the local skin fiction coefficient f'' (0) is markedly increased with an increase in the value of Ec for large values of λ ($\lambda = 15$) where the free convection flow is predominant and this increase is more pronounced at smaller values of A. It is clear from Fig. 15 that, increasing the value of Ec is found to reduce the local heat transfer rate $-\theta'$ (0) for all values of λ and A. This is evident from the fact that the presence, as well as an increase the viscous dissipation ($Ec \neq 0$), the fluid is being heated and hence results in a decrease in the wall temperature gradient. Also, the reduction in the local heat transfer rate is found to be more pronounced for a larger mixed convection parameter λ and smaller unsteadiness parameter A. Further, from Fig. 15 it is observed that the local heat transfer rate $-\theta'$ (0) is increased owing to increasing the value of mixed convection parameter λ for small values of the Eckert number ($Ec = 0.1$) and all values of the steadiness parameter A whereas for higher values of the Eckert number ($Ec = 0.8$) this is true only for $A > 1.3$ due to the rapid growth in $-\theta'$ (0) with increasing the unsteadiness parameter A.

Variation in the local skin friction coefficient f'' (0) and the local heat transfer rate $-\theta'$ (0) are illustrated as a function of the unsteadiness parameter A in Figs. 16 and 17, respectively. Fig. 16 reveals that for a given A the local skin friction coefficient f'' (0) increases with viscous dissipation in fluid heating case ($Ec > 0$) and decreases in fluid cooling case ($Ec < 0$). As compared to the case of no viscous dissipation ($Ec = 0$), Fig. 16 demonstrates that for given value of A the heat transfer is enhanced for the fluid cooling case but it is reduced for the fluid heating case.

5. Conclusions

Unsteady laminar mixed convection boundary layer flow and heat transfer along a stretching vertical sheet is studied in the light of variation in fluid viscosity due to temperature differences. The influence of buoyancy along with viscous dissipation on the convective transport in the boundary layer region is also considered. In this study emphasis is given on how velocity field, skin friction, temperature distribution and heat transfer changes due to variation in the governing parameters such as variable viscosity parameter, mixed convection parameter, Eckert number and unsteadiness parameter. Using a suitable transformation, the governing time-dependent boundary layer equations for momentum and thermal energy are reduced to a set of non-linear coupled ordinary differential equations which are then solved numerically by using fourth-order Runge–Kutta scheme with shooting method. As a summary, we conclude that:

1. For all values of the unsteadiness parameter, increasing the variable viscosity parameter may increase or decrease the local skin friction coefficient and local heat transfer rate, depending on the competition between the impacts of the mixed convection parameter and variable viscosity parameter; in a manner that the local skin friction coefficient and the local heat transfer rate are both reduced as the variable viscosity parameter increases for both forced and mixed convection regimes (small values of the mixed convection parameter), while the opposite trend is observed for pure free convection regime (large values of the mixed convection parameter).
2. For all values of the unsteadiness parameter, the effect of increasing values of the mixed convection parameter is to increase the skin friction coefficient greatly for large values of the variable viscosity parameter but only slight effect on the skin friction coefficient for small values of variable viscosity parameter is observed.
3. The effect of increasing values of the mixed convection parameter is to increase the local heat transfer rate for all values of the variable viscosity and unsteadiness parameters.
4. When the viscous dissipation effect is included ($Ec \neq 0$), a significant reduction in the heat transfer rate occurs for all values of the mixed convection and unsteadiness parameters.
5. Due to viscous dissipation effect a considerable increase in the skin friction coefficient occurs for large values of the mixed convection parameter, but only insensible effects are observed for small values of the mixed convection parameter. This is true for all values of the unsteadiness parameter.

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