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ORIGINAL ARTICLE

Circuit realization, chaos synchronization and estimation of parameters of a hyperchaotic system with unknown parameters

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Abstract In this article, the adaptive chaos synchronization technique is implemented by an electronic circuit and applied to the hyperchaotic system proposed by Chen et al. We consider the more realistic and practical case where all the parameters of the master system are unknowns. We propose and implement an electronic circuit that performs the estimation of the unknown parameters and the updating of the parameters of the slave system automatically, and hence it achieves the synchronization. To the best of our knowledge, this is the first attempt to implement a circuit that estimates the values of the unknown parameters of chaotic system and achieves synchronization. The proposed circuit has a variety of suitable real applications related to chaos encryption and cryptography. The outputs of the implemented circuits and numerical simulation results are shown to view the performance of the synchronized system and the proposed circuit.

MATHEMATICS SUBJECT CLASSIFICATION: 34H10; 34C28; 37M05

1. Introduction

The field of studying dynamical systems has attracted much attention and research activity in recent years. Dynamical systems play an important role in studying various phenomena that undergo spatial and temporal evolution. These phenomena arise from different disciplines including mechanical engineering, electrical engineering, physics, chemistry, biology, and economy [1,2].

Chaos is a very complex nonlinear behavior exists in some dynamical systems and it has some interesting properties such as complicated topological structure, high sensitivity to changes in initial conditions and system parameters. Chaos is characterized by having one positive Lyapunov exponent [3]. The modern applications of chaos and dynamical systems include chaos control, chaos synchronization, electronic circuits, secure communications, image encryption, cryptography, and neuroscience research [4–13].

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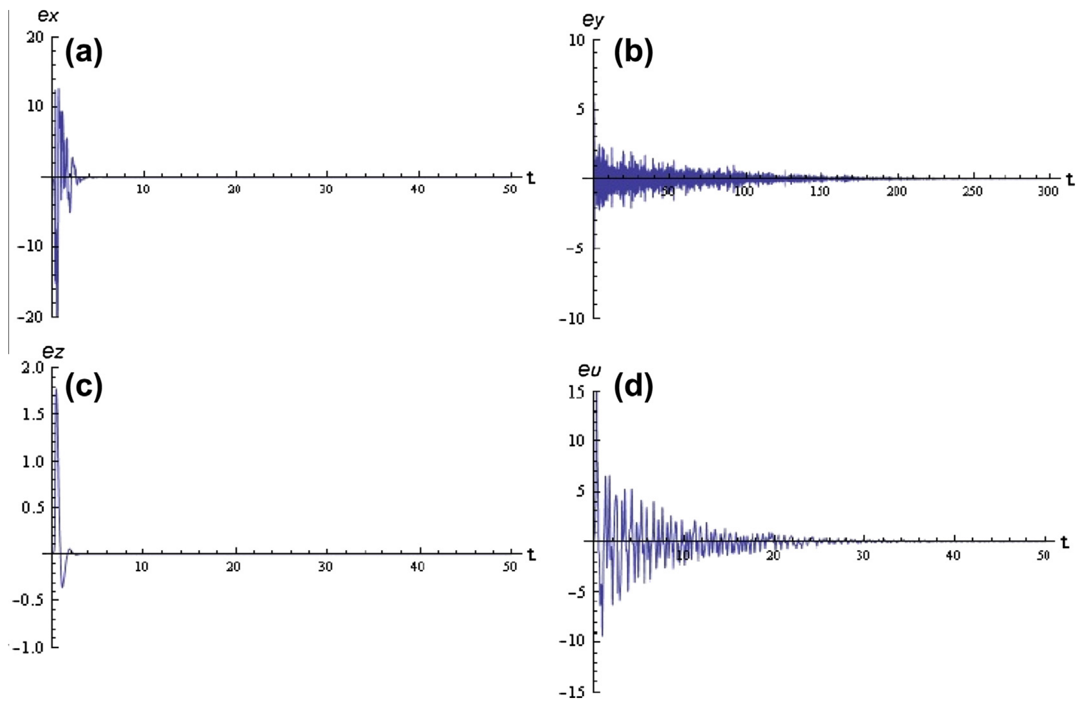

Figure 1 The error dynamics of the state variables (a) x, (b) y, (c) z and (d) u.

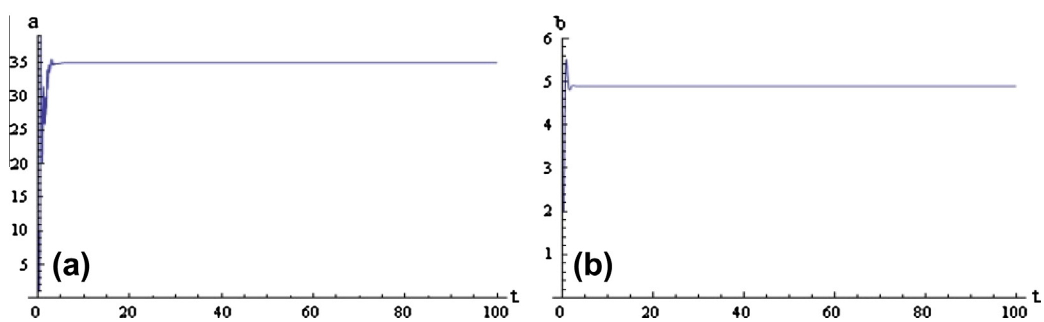

Figure 2 The estimation of the values of parameter a and b of the hyperchaotic system.

A hyperchaotic system is a system that is characterized by having more than one positive Lyapunov exponents for its chaotic attractor. Hyperchaos was firstly introduced by Rossler [14] then several hyperchaotic systems are introduced and studied extensively (see for example [3,15–18], and references therein). The hyperchaotic systems have higher unpredictability and more randomness than simple chaotic systems. So, hyperchaos is preferred in many applications including secure communications, chaos based image encryption, and cryptography.

In [15], the following fourth order hyperchaotic system was introduced

$$\begin{aligned} \dot{x} &= a(y - x) + hyz, \\ \dot{y} &= cx - lxz + y + u, \\ \dot{z} &= xy - bz, \\ \dot{u} &= -ky, \end{aligned} \tag{1}$$

where x, y, z, and u are the state variables of the system and the constants a, b, c, l, h, and k are the parameters of the system to be tuned. This system has only one equilibrium point that is $(0, 0, 0, 0)$. As this hyperchaotic system has larger positive Lyapunov exponents than the already known hyperchaotic systems [15], and hence higher complexity and unpredictability, the presented circuit can be considered as a possible ideal choice for chaos generation in telecommunication systems. Also, the hyperchaotic behavior exists within a large range of the six parameters of the hyperchaotic system which gives the system the advantage of possessing a large domain of secret keys in real applications of chaotic based image encryption.

Chaos control and synchronization of system (1) are studied in [16,17]. In [16], hybrid projective synchronization is applied to the hyperchaotic system (1) whereas in [17], adaptive chaos synchronization is presented between two identical drive and response system with uncertain parameters. In this work, we investigate chaos synchronization in the practical and more realistic case that occurs when all the parameters of the drive system are unknown and the values of the parameters of the response system are not determined yet. During the process of synchronization, the values of the parameters of the drive

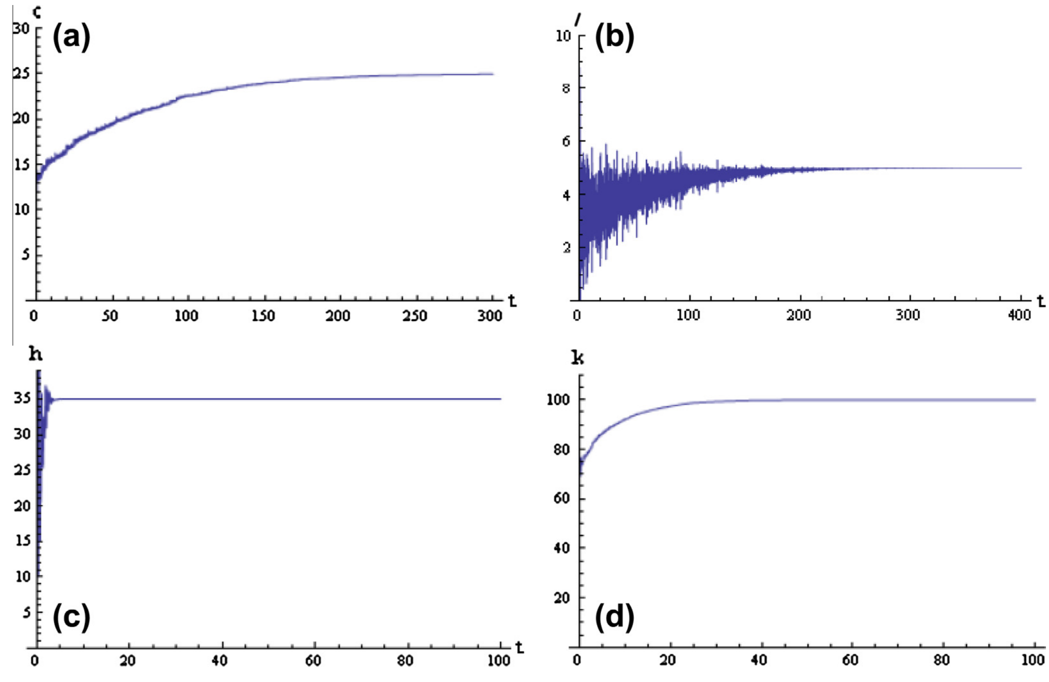

Figure 3 The estimation of the values of parameter c, l, h and k of the hyperchaotic system.

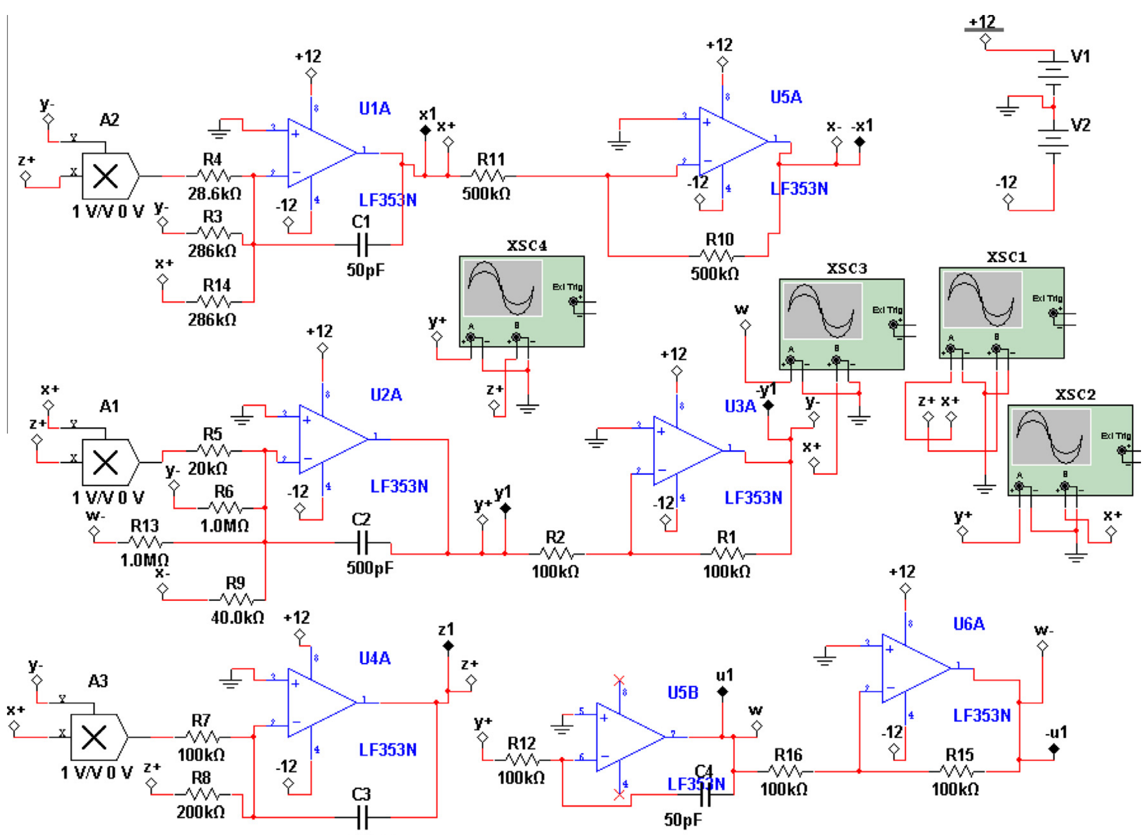

Figure 4 Circuit implementation of the master system.

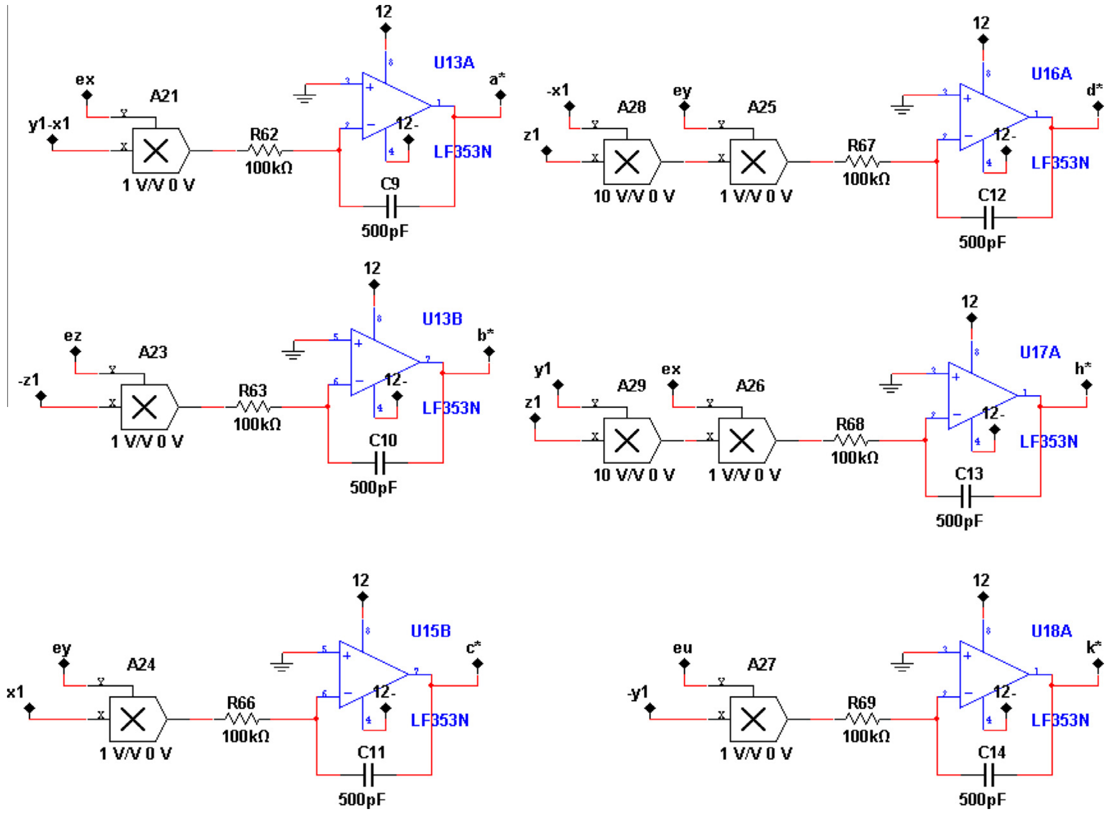

Figure 5 Circuit schematic of the updating laws circuit.

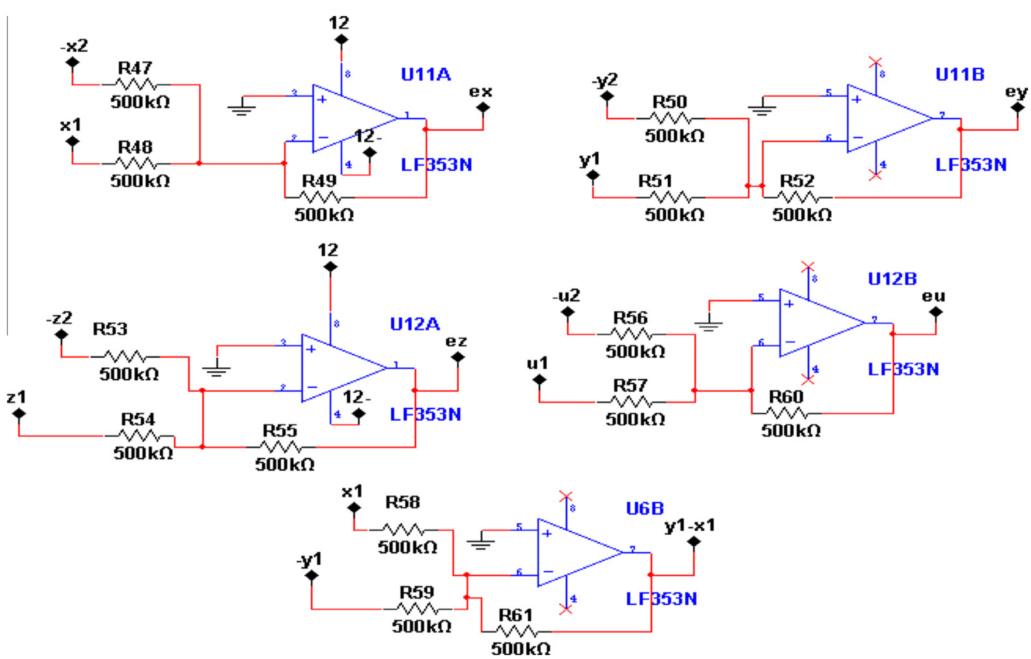

Figure 6 Circuit implementation of error dynamics of the state variables.

system are estimated by the updating laws that we deduce, and hence, the parameters of the response system are determined according to the estimations of the parameters of the drive system.

The aim of this paper is to present a circuit realization of this chaos synchronization scheme. We implement the master (drive) circuit and the slave (response) circuit, then the adaptive control and estimator circuit are proposed. The last circuit

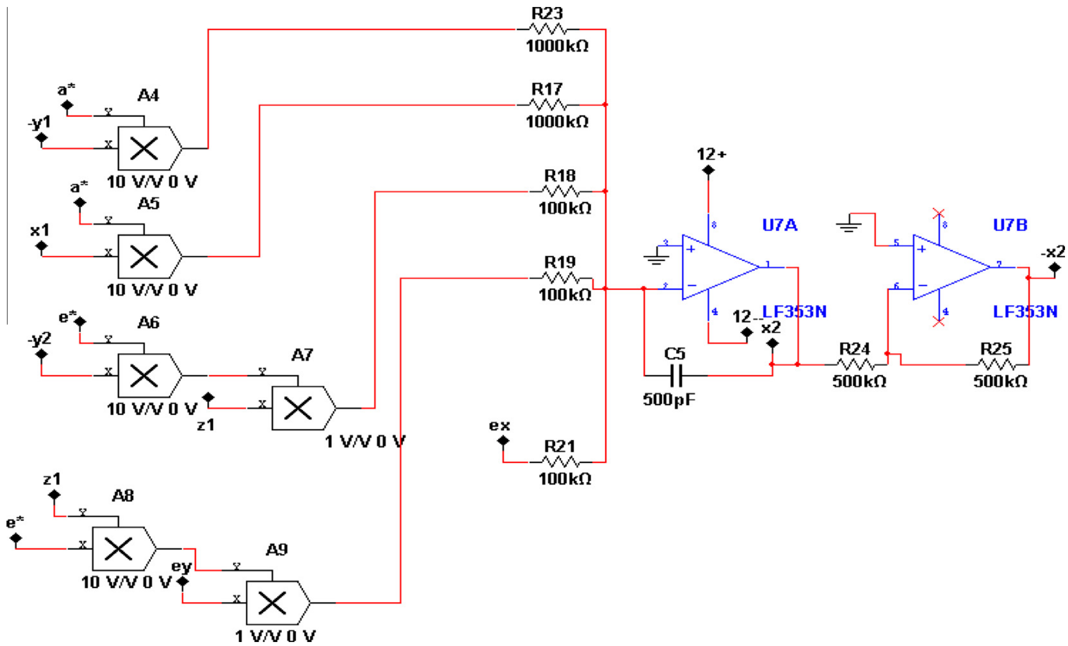

Figure 7 Circuit implementation of the state variable x in the slave system.

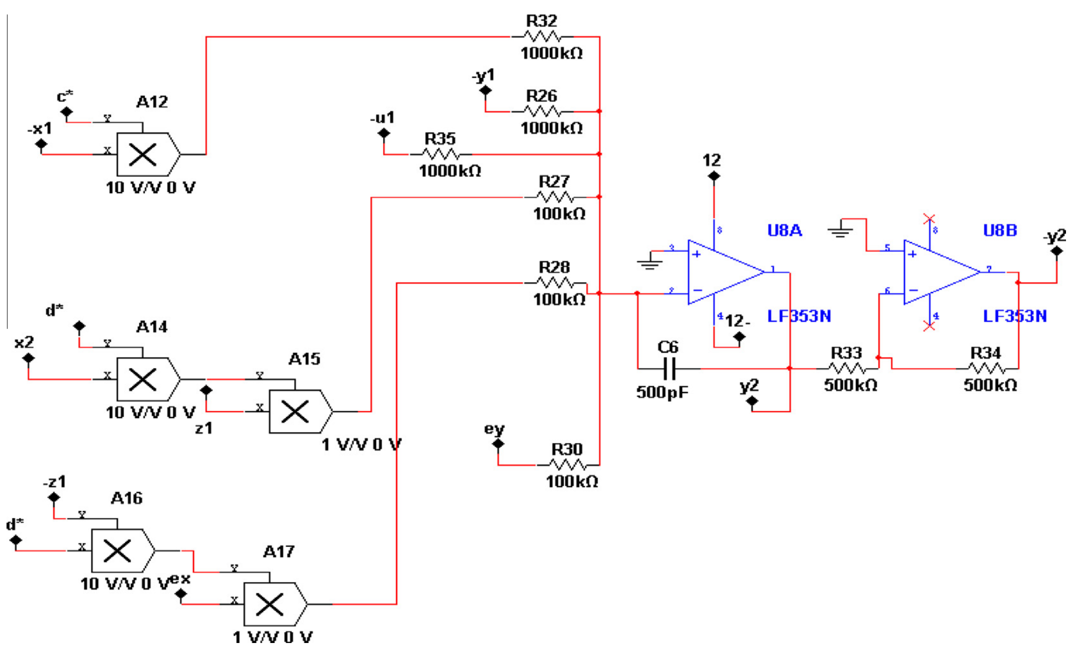

Figure 8 Circuit implementation of the state variable y in the slave system.

estimates the values of the unknown parameters of the system and updates the values of parameters of the slave circuit automatically. The proposed circuit can be used in many modern applications, for example, secure communications systems, chaos based cryptography systems, and image and real time video encryption.

This paper is organized as follows: chaos synchronization is studied in Section 2. Section 3 contains the circuit implementations of the synchronized system. Finally, the conclusion of the work is discussed in Section 4.

2. Chaos synchronization

In this section, the master system corresponding to the hyperchaotic system (1) is assumed to have fully unknown parameters. It can be written in the following form:

$$\begin{aligned} \dot{x}_1 &= a(y_1 - x_1) + hy_1z_1, \\ \dot{y}_1 &= cx_1 - lx_1z_1 + y_1 + u_1, \\ \dot{z}_1 &= x_1y_1 - bz_1, \\ \dot{u}_1 &= -ky_1, \end{aligned} \tag{2}$$

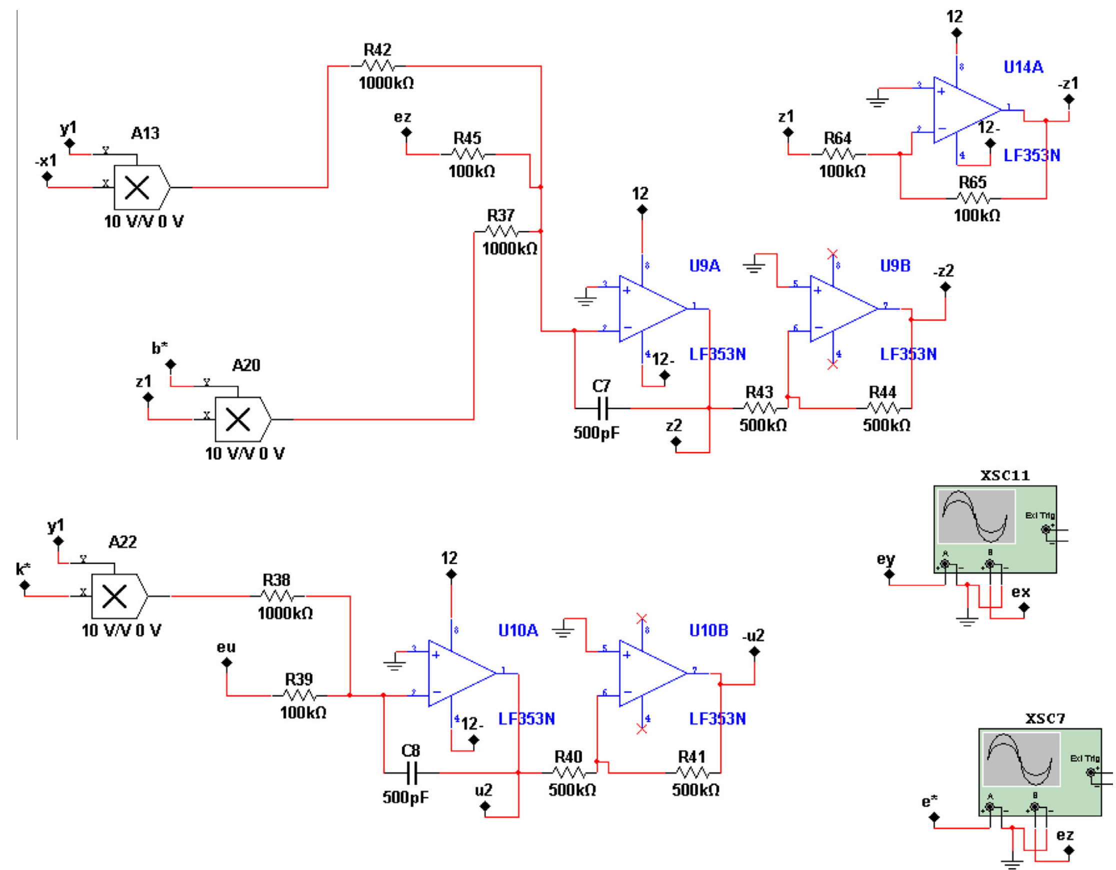

Figure 9 Circuit implementation of the state variables z and u in the slave system.

where a, b, c, l, h, and k are the unknown parameters to be estimated.

On the other hand, the parameters of the slave system considered have any arbitrary initial values when starting the process of chaos synchronization. Then, the values of these parameters are modified according to the estimations of the values of parameters of the master system (2). The slave system has the following form:

$$\begin{aligned}
\dot{x}_2 &= \hat{a}(y_2 - x_2) + \hat{h}y_2z_2 + v_1,\\
\dot{y}_2 &= \hat{c}x_2 - \hat{l}x_2z_2 + y_2 + u_2 + v_2,\\
\dot{z}_2 &= x_2y_2 - \hat{b}z_2 + v_3,\\
\dot{u}_2 &= -\hat{k}y_2 + v_4,
\end{aligned} \tag{3}$$

where $\hat{a}, \hat{b}, \hat{c}, \hat{l}, \hat{h}$, and $\hat{k}$ are the estimated parameters and v_1, v_2, v_3, and v_4 are the control functions to be determined.

The state errors between the slave system and the master system are defined as follows

$$e_x = x_2 - x_1, \quad e_y = y_2 - y_1, \quad e_z = z_2 - z_1, \quad \text{and } e_u = u_2 - u_1, \tag{4}$$

whereas the parameters errors are defined by

$$\begin{aligned}
&\tilde{a} = \hat{a} - a, \quad \tilde{b} = \hat{b} - b, \quad \tilde{c} = \hat{c} - c, \quad \tilde{l} = \hat{l} - l,\\
&\tilde{h} = \hat{h} - h, \quad \text{and } \tilde{k} = \hat{k} - k.
\end{aligned} \tag{5}$$

The control functions are chosen as follows

$$\begin{aligned}
v_1 &= \hat{a}(e_x - e_y) - \hat{h}(y_2e_z - z_1e_y) - m_1e_x,\\
v_2 &= -\hat{c}e_x + \hat{l}(x_2e_z + z_1e_x) - e_y - e_u - m_2e_y,\\
v_3 &= x_1y_{1_}x_2y_2 + \hat{b}e_z - m_3e_z,\\
v_4 &= \hat{k}e_y - m_4e_u,
\end{aligned} \tag{6}$$

where $m_1,\ m_2,\ m_3$, and m_4 are positive feedback gains.

Then, the error dynamical system between the master system (2) and the slave system (3) is expressed by

$$\begin{aligned}
\dot{e}_x &= \tilde{a}(y_1 - x_1) + \tilde{h}y_1z_1 - m_1e_x,\\
\dot{e}_y &= \tilde{c}x_1 - \tilde{l}x_1z_1 - m_2e_y,\\
\dot{e}_z &= -\tilde{b}z_1 - m_3e_z,\\
\dot{e}_u &= -\tilde{k}y_1 - m_4e_u.
\end{aligned} \tag{7}$$

The next step is to derive the updating laws. We consider the Lyapunov function V defined by

$$\begin{aligned}
V(e_x, e_y, e_z, e_u, \tilde{a}, \tilde{b}, \tilde{c}, \tilde{d}, \tilde{h}, \tilde{k}) = &\frac{1}{2}\Big(e_x^2 + e_y^2 + e_z^2 + e_u^2 + \tilde{a}^2 + \tilde{b}^2\\
&+ \tilde{c}^2 + \tilde{l}^2 + \tilde{h}^2 + \tilde{k}^2\Big).
\end{aligned} \tag{8}$$

Differentiation of V along the trajectories of (7) gives

$$\begin{aligned}
\frac{dV}{dt} = &\tilde{a}(y_1 - x_1)e_x + \tilde{h}y_1z_1e_x - m_1e_x^2 + \tilde{c}x_1e_y - \tilde{d}x_1z_1e_y\\
&- m_2e_y^2 - \tilde{b}z_1e_z - m_3e_z^2 - \tilde{k}y_1e_u - m_4e_u^2 + \tilde{a}\frac{d\tilde{a}}{dt} + \tilde{b}\frac{d\tilde{b}}{dt}\\
&+ \tilde{c}\frac{d\tilde{c}}{dt} + \tilde{l}\frac{d\tilde{l}}{dt} + \tilde{h}\frac{d\tilde{h}}{dt} + \tilde{k}\frac{d\tilde{k}}{dt}.
\end{aligned} \tag{9}$$

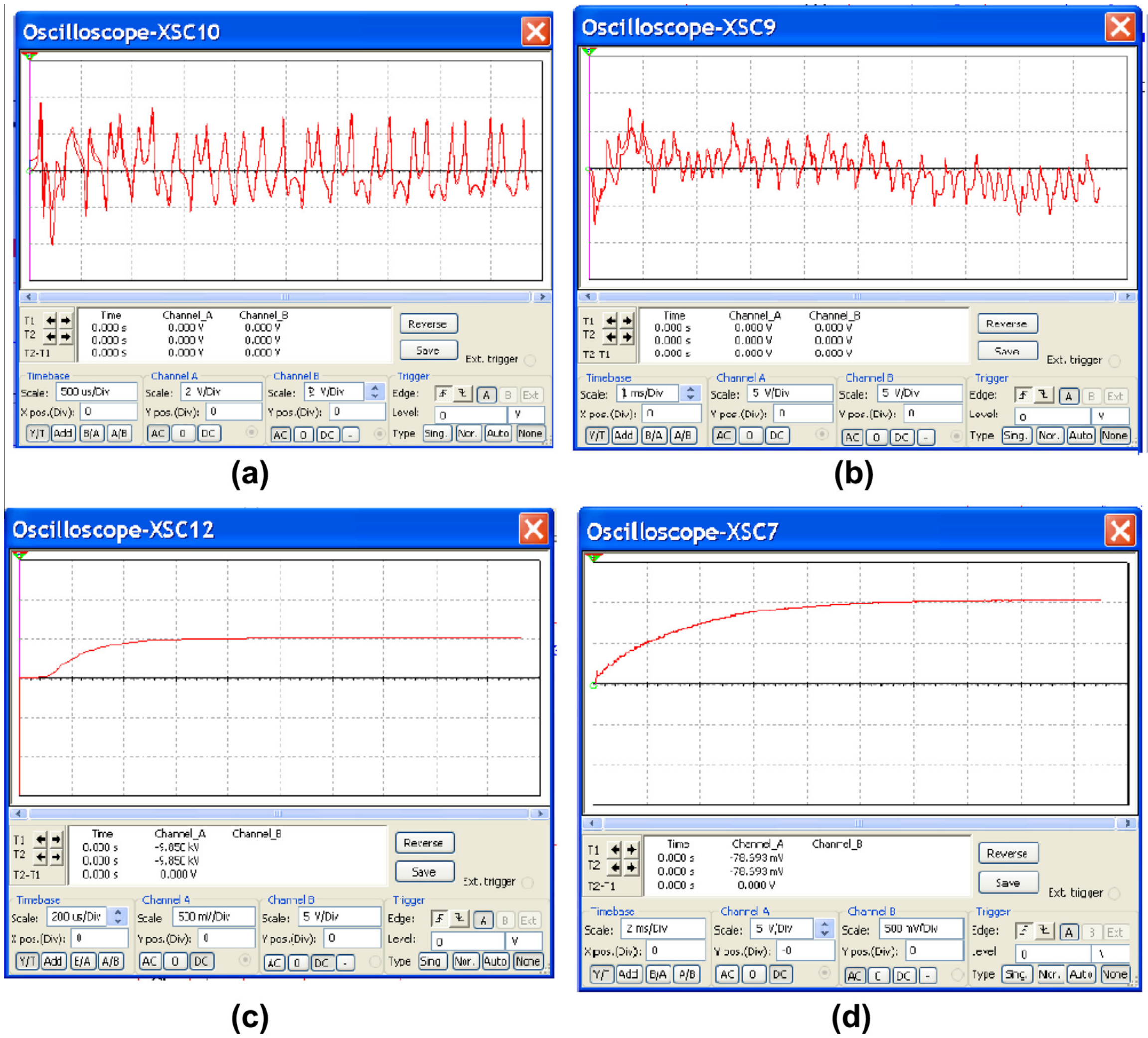

Figure 10 The synchronization of state variables (a) x and (b) u of the master and the slave systems. The estimation of unknown parameters b and k are illustrated in Fig. (c) and (d), respectively.

The derivatives of the errors of parameters with respect to time are chosen as

$$\frac{d\tilde{a}}{dt} = (x_1 - y_1)e_x, \quad \frac{d\tilde{b}}{dt} = z_1 e_z, \quad \frac{d\tilde{c}}{dt} = -x_1 e_y,$$
$$\frac{d\tilde{l}}{dt} = x_1 z_1 e_y, \quad \frac{d\tilde{h}}{dt} = -y_1 z_1 e_x, \quad \text{and} \quad \frac{d\tilde{k}}{dt} = y_1 e_u. \tag{10}$$

Substituting from (10) in (9) gives

$$\frac{dV}{dt} = -\left(m_1 e_x^2 + m_2 e_y^2 + m_3 e_z^2 + m_4 e_u^2\right), \tag{11}$$

which is a negative definite function. Then, the error dynamics is globally exponentially stable about the origin.

From (5), we have $\frac{d\tilde{a}}{dt} = \frac{d\hat{a}}{dt}, \frac{d\tilde{b}}{dt} = \frac{d\hat{b}}{dt}, \frac{d\tilde{c}}{dt} = \frac{d\hat{c}}{dt}, \frac{d\tilde{l}}{dt} = \frac{d\hat{l}}{dt}, \frac{d\tilde{h}}{dt} = \frac{d\hat{h}}{dt}$, and $\frac{d\tilde{k}}{dt} = \frac{d\hat{k}}{dt}$. Hence, the following updating laws for the unknown parameters of the master system are obtained:

$$\frac{d\hat{a}}{dt} = (x_1 - y_1)e_x, \quad \frac{d\hat{b}}{dt} = z_1 e_z, \quad \frac{d\hat{c}}{dt} = -x_1 e_y,$$
$$\frac{d\hat{l}}{dt} = x_1 z_1 e_y, \quad \frac{d\hat{h}}{dt} = -y_1 z_1 e_x, \quad \frac{d\hat{k}}{dt} = y_1 e_u. \tag{12}$$

Numerical simulations are carried out for the following values of parameters of the master system (2), which assumed to be unknown, $a = 35, b = 4.9, c = 25, l = 5, h = 35$, and $k = 100$ [15]. The initial values selected for the parameters of the slave system are given as follows: $\hat{a}(0) = 10, \hat{b}(0) = 2, \hat{c}(0) = 12, \hat{l}(0) = 3, \hat{h}(0) = 10$, and $\hat{k}(0) = 69$.

Fig. 1 shows the error between the state variables of the master system (2) and the slave system (3). Figs. 2 and 3 shows the values of the estimated parameters of the slave system (3) versus the time t.

3. Circuit implementation of the hyperchaotic system and process of synchronization

The design and implementation of the proposed circuit are carried out in the following steps:

3.1. Design and implementation of master system circuit

The master system (2), with the values of parameters used in numerical simulations presented in Section 2, is implemented in Fig. 4 which shows the circuit diagram of the master system and the values of circuit elements that are used. As the possible outputs of the operational amplifiers (OP-AMPs) used are smaller than the actual values of state variables of the master system, we scale the values of state variables in the circuit implementation by a factor of 0.1.

3.2. Design and implementation of updating laws circuit

The updating laws circuit, shown in Fig. 5, solves system (12) to estimate the values of the unknown parameters of the master system (2) and provide the values of the estimated parameters to the state variables of slave system circuit.

3.3. Design and implementation of synchronization error circuit

The synchronization error circuit that computes the difference between the state variables of the master and the slave systems is shown in Fig. 6.

3.4. Design and implementation of slave system circuit

Figs. 7–9 show the circuit diagram of the state variables circuit which produces the state variables of the slave system (3). The values of the state variables of the slave system are also scaled by a factor 0.1 as in previous circuit.

Fig. 10 illustrates the synchronization achieved between the master system (2) and the slave system (3) and shows examples of the obtained estimations of the unknown parameters of master system.

4. Conclusion

In this paper, the chaos synchronization of a recently hyperchaotic system is presented. We studied the more practical case of fully unknown parameters of the master system. The circuits of the master system and the slave system of the full synchronized system are implemented. Also we propose and present the circuit implementation for a control and updating unit which achieves chaos synchronization, estimations of the unknown parameters, and updating the parameters of the slave system automatically according to the estimation of the unknown parameters. The circuit simulations show good results and performance for the synchronized system and the proposed circuits. Furthermore, future work can include various improvements that can be added to this work. For examples, the cases of the presence of noise effect on transmitted signals or the presence of unknown nonlinearity of the master system may be investigated analytically, then the appropriate chaos synchronization scheme can be implemented as in this paper.

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