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Original Article

Applications of the differential operator to a class of meromorphic univalent functions

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Abstract In this paper, we define a new subclass of meromorphic close-to-convex univalent functions defined in the punctured open unit disc by using a differential operator. Some inclusion results, convolution properties and several other properties of this class are studied.

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1. Introduction

Let Σ denote the class of functions of the form

$$f(z) = \frac{1}{z} + \sum_{k=0}^{\infty} a_k z^k, \tag{1.1}$$

which are analytic and univalent in $E^* = \{z : 0 < |z| < 1\} = E\{0\}$. For the functions

$$f(z) = \frac{1}{z} + \sum_{k=0}^{\infty} a_k z^k \text{ and } g(z) = \frac{1}{z} + \sum_{k=0}^{\infty} b_k z^k,\ z \in E^*,$$

analytic in E^*, their Hadamard product or convolution, $f * g$, is the function defined by

$$(f * g)(z) = \frac{1}{z} + \sum_{k=0}^{\infty} a_k b_k z^k, \quad z \in E^*,$$

where $(*)$ stands for convolution sign.

The theory of linear operators plays an important role in geometric function theory. Several differential and integral operators were introduced and studied, see for example [1,3,16,21,22,25,27]. For the recent work on linear operators for

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meromorphic functions, we refer to [4,6,10,11]. In this work we consider the operator defined by El-Ashwah [10] and El-Ashwah and Aouf [11,12]. For λreal, $l > 0$ and $n \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}$, the linear operator $D^n(\lambda,l) : \Sigma \to \Sigma$ was defined by

$$D^n f(z) = \frac{1}{z} + \sum_{k=0}^{\infty}\left[\frac{l+\lambda(k+1)}{l}\right]^n a_k z^k, \quad z \in E^*. \tag{1.2}$$

Clearly $D^0 f(z) = f(z)$ and $D^1(1,1)f(z) = 2f(z)) + zf'(z)$.

It is noted that

$$\lambda z\left(D^n f(z)\right)^{\prime n+1} f(z) - (\lambda + l)D^n f(z), \quad z \in E^*. \tag{1.3}$$

For $\lambda = 1$, the operator $D^n(1,l)f(z)$ was introduced and studied by Cho et al. [7,8]. The case $D^n(\lambda,1)f(z)$ was considered by Al-Oboudi and Al-Zkeri [2]. Further the operators $D^n(1,1)f(z)$ and $D^1(-1,1)f(z)$ were investigated by Uralegaddi and Somanatha [27] and Noor and Ahmad [23] respectively.

For $\alpha,(0 \le \alpha < 1)$, a function $f(z) \in \Sigma$ is said to be meromorphic starlike and convex of order α if it satisfies

$$-\Re e\left\{\frac{zf'(z)}{f(z)}\right\} > \alpha, \quad z \in E,$$

and

$$-\Re e\left\{\frac{\left(zf'(z)\right)'}{f'(z)}\right\} > \alpha, \quad z \in E,$$

respectively. We denote the former class of functions as $\Sigma^*(\alpha)$ and the later one by $\Sigma^k(\alpha)$. These classes have been studied by Pommerenke [24], Clunie [9] and Miller [19,20]. Further a function $f(z) \in \Sigma$ is said to be from the class $\Sigma^c(\alpha)$, if it satisfies

$$-\Re e\left\{z^2 f'(z)\right\} > \alpha, \quad z \in E. \tag{1.4}$$

This class was investigated by Ganigi and Uralegaddi [14], Cho and Owa [5] and Wang and Guo [28].

Definition 1. A function f given by (1.1) is said to belong to the class $\Sigma^g(\alpha)$ of meromorphic close-to-convex functions if there exists a function $g \in \Sigma^*(\alpha)$ such that

$$-\Re e\left\{\frac{zf'(z)}{g(z)}\right\} > 0, \quad z \in E.$$

This class of functions was introduced and studied by Libera and Robertson [17].

Remark 1. In [14] it was shown that if a function $f(z) \in \Sigma^c(\alpha)$, then it is meromorphic close-to-convex of order α.

Let f and g be two analytic functions in E. We say that f is subordinate to g, written $f \prec g$, if there exists a Schwarz function $w(z)$, analytic in E with $w(0) = 0$ and $|w(z)| < 1$ such that $f(z) = g(w(z))$. If g is univalent in E, then $f \prec g$ is equivalent to $f(0) = g(0)$ and $f(E) \subset g(E)$.

A sequence of non-negative numbers $\{c_n\}$ is said to be a convex null sequence if $c_k \to 0$ as $k \to \infty$ and

$$c_0 - c_1 \ge c_1 - c_2 \ge \cdots \ge c_k - c_{k+1} \ge \cdots \ge 0.$$

Now we define the following class of functions by using the operator defined in (1.2).

Definition 2. A function $f(z) \in \Sigma$ is said to be in the class $\Sigma^n(\lambda,\alpha)$, if and only if

$$-\Re e\left\{z^2\left(D^n f(z)\right)'\right\} > \alpha, \quad z \in E, \quad (n \in \mathbb{N}_0).$$

When $n = 0$, we obtain the class $\Sigma^c(\alpha)$ of meromorphic functions, which was studied by Ganigi and Uralegaddi [14], Cho and Owa [5] and Wang and Guo [28].

2. Preliminary results

We need the following results.

Lemma 1 [26]. *If $p(z)$ is analytic in E with $p(0) = 1$ and $\Re e\{p(z)\} > 1/2, z \in E$, then for any analytic function F, in E, the function $P * F$ takes its values in the convex hull of $F(E)$.*

Lemma 2 [13]. *Let $\{c_k\}_{k=0}^{\infty}$ be a convex null sequence. Then the function*

$$p(z) = \frac{c_0}{2} + \sum_{k=1}^{\infty} c_k z^k, \quad z \in E,$$

is analytic and $\Re e\{p(z)\} > 0$ in E.

The following result is due to Hallenbeck and Ruscheweyh.

Lemma 3 [15]. *Let the function $h(z)$ be convex univalent in E with*

$$h(0) = 1, \quad \gamma \neq 0 \quad \text{and} \quad \Re e\gamma > 0, \quad z \in E.$$

Suppose that the function

$$p(z) = 1 + p_1 z + p_2 z^2 + \cdots,$$

is analytic in E and satisfying the following differential subordination

$$p(z) + \frac{zp'(z)}{\gamma} \prec h(z), \quad z \in E,$$

then

$$p(z) \prec q(z) \prec h(z), \quad z \in E,$$

where

$$q(z) = \frac{\gamma}{z^{\gamma}}\int_0^z h(t)t^{\gamma-1}\mathrm{d}t.$$

The function $q(z)$ is convex and is the best dominant.

Lemma 4 [18]. *Let $q(z)$ be a convex function in E and let*

$$h(z) = q(z) + \beta zq'(z),$$

where $\beta > 0$. If $p(z)$ is analytic and satisfies

$$p(z) + \beta zp'(z) \prec h(z), \quad z \in E,$$

then

$$p(z) \prec q(z), \quad z \in E,$$

and this result is sharp.

3. Main results

In this section we shall prove our main results.

Theorem 1. *Let $n \in \mathbb{N}_0, \lambda > 0, 0 \le \alpha < 1$ and let $f(z)$ belong to $\Sigma^{n+1}(\lambda,\alpha)$. Then $f(z)$ belongs to $\Sigma^n(\lambda,\alpha)$. Further*

$$-z^2\left(D^n f(z)\right)' \prec q(z) \prec \frac{1+(2\alpha-1)z}{1+z}, \quad z \in E,$$

where

$$q(z) = \frac{l}{\lambda z^{\frac{l}{\lambda}}} \int_0^z \frac{1+(2\alpha-1)z}{1+z} t^{\frac{l}{\lambda}-1} \mathrm{d}t. \tag{3.1}$$

Proof. Let $f(z) \in \Sigma^{n+1}(\lambda,\alpha)$, then from Definition 1, we have

$$-\Re e\left\{z^2\left(D^{n+1} f(z)\right)'\right\} > \alpha, \quad z \in E, \quad (n \in \mathbb{N}_0).$$

Set

$$p(z) = -z^2\left(D^n f(z)\right)'. \tag{3.2}$$

Then $p(z)$ is analytic in E with $p(0) = 1$. Differentiation of (3.2) with the use of (1.3), yields

$$p(z) + \frac{\lambda}{l} z p'^2\left(D^{n+1} f(z)\right)',$$

which can be written as

$$p(z) + \frac{zp'(z)}{\gamma} = -z^2\left(D^{n+1} f(z)\right)' \prec h(z) = \frac{1+(2\alpha-1)z}{1+z},$$

where $\gamma = l/\lambda$. Using Lemma 3, we have

$$p(z) \prec q(z) \prec h(z), \quad z \in E,$$

where $q(z)$ is given in (3.1). Moreover, the function $q(z)$ is convex and is the best dominant. □

Putting $n = 0$, in Theorem 1, we obtain the following result.

Corollary 1. *For $0 \le \alpha < 1$ and $\lambda > 0$. Let $f(z) \in \Sigma$, satisfy the following inequality*

$$\Re e\left\{-z^2\left(\left(1+\frac{2\lambda}{l}\right)f'(z) + \frac{\lambda}{l} z f''(z)\right)\right\} > \alpha,$$

then

$$\Re e\left\{-z^2 f'(z)\right\} > \alpha,$$

that is $f(z) \in \Sigma^c(\alpha)$.

Remark 2. Since $\Sigma^0(\alpha) = \Sigma^c(\alpha)$ is a subclass of meromorphic close-to-convex functions of order α, the univalence of members in $\Sigma^n(\lambda,\alpha)$ is a consequence of Theorem 1.

Theorem 2. *Let $n \in \mathbb{N}_0, \lambda > 0, 0 \le \alpha < 1$. Let $q(z)$ be a convex function with $q(0) = 1$ and let $h(z)$ be a function such that*

$$h(z) = q(z) + zq'(z), \quad z \in E.$$

If $f(z) \in \Sigma^{n+1}(\lambda,\alpha)$ and satisfies the differential subordination

$$-z^2\left(D^{n+1} f(z)\right)' \prec h(z), \quad z \in E,$$

then

$$-z^2\left(D^n f(z)\right)' \prec q(z), \quad z \in E,$$

and this result is sharp.

Proof. Set

$$p(z) = -z^2\left(D^n f(z)\right)', \tag{3.3}$$

then $p(z)$ is analytic and $p(0) = 1$. By differentiating (3.3) and using (1.3), we have

$$p(z) + \frac{\lambda}{l} z p'^2\left(D^{n+1} f(z)\right)' \prec h(z) = q(z) + zq'(z).$$

By using Lemma 4 for $\beta = \lambda/l$, we have

$$p(z) \prec q(z),$$

or

$$-z^2\left(D^n f(z)\right)' \prec q(z), \quad z \in E,$$

and this result is sharp. square

Theorem 3. *Let $f(z) \in \Sigma, \lambda \neq 0$ and $0 < \alpha \le 1/2$. Suppose that for arbitrary r, $(0 < r < 1)$, $f(z)$ satisfies the conditions*

$$\min_{|z|\le r} \Re e\left\{-z^2\left(D^n f(z)\right)'\right\} = \min_{|z|\le r}\left|-z^2\left(D^n f(z)\right)'\right|,$$

and

$$\frac{l}{\lambda}\Re e\left\{\frac{\left(D^{n+1} f(z)\right)'}{\left(D^n f(z)\right)'} - 1\right\} > \alpha - 1, \quad z \in E.$$

Then, we have

$$f(z) \in \Sigma^n(\lambda,\alpha).$$

Proof. Let

$$p_1(z) = -z^2\left(D^n f(z)\right)', \tag{3.4}$$

then $p_1(z)$ is analytic and $p_1(0) = 1$. From (1.3), it follows that

$$\frac{p_1(z)}{z} = -\left\{\frac{l}{\lambda}\left(D^{n+1} f(z)\right) - \left(1+\frac{l}{\lambda}\right)\left(D^n f(z)\right)\right\},$$

which on differentiation, yields

$$\frac{zp_1'(z)}{p_1(z)} = 1 + \left\{\frac{l}{\lambda}\frac{\left(D^{n+1} f(z)\right)'}{\left(D^n f(z)\right)'} - \left(1+\frac{l}{\lambda}\right)\right\}.$$

Now by the hypothesis of theorem and using a result by Wang and Guo [[28], Lemma 2.2], we have

$$\Re e p_1(z) > \alpha, \quad z \in E.$$

This completes the proof. square

For $n = 0$, we have the following result.

Corollary 2. *Let $f(z) \in \Sigma$ and $0 < \alpha \leq 1/2$. Suppose that for arbitrary r, $(0 < r < 1)$, $f(z)$ satisfies the conditions*

$$\min_{|z| \leq r} \Re e\left(-z^2 f'(z)\right) = \min_{|z| \leq r} \left|-z^2 f'(z)\right|,$$

and

$$\Re e\left\{1 + \frac{\left(zf'(z)\right)'}{f'(z)}\right\} > \alpha - 1, \quad z \in E.$$

Then, we have

$$f(z) \in \Sigma^c(\alpha).$$

Theorem 4. *Let $f(z) \in \Sigma$ and $1/2 < \alpha < 1$. Suppose that for arbitrary r, $(0 < r < 1)$, $f(z)$ satisfies the conditions*

$$\min_{|z| \leq r} \Re e\left\{-z^2 \left(D^n f(z)\right)'\right\} = \min_{|z| \leq r} \left|-z^2 \left(D^n f(z)\right)'\right|,$$

and

$$\frac{l}{\lambda} \Re e\left\{\frac{\left(D^{n+1} f(z)\right)'}{\left(D^n f(z)\right)'} - 1\right\} > \frac{\alpha}{2} - 1, \quad z \in E.$$

Then, we have

$$f(z) \in \Sigma^n(\lambda, \alpha).$$

Proof. Let

$$p_1(z) = -z^2 \left(D^n f(z)\right)', \tag{3.5}$$

then $p_1(z)$ is analytic and $p_1(0) = 1$. Proceeding in a similar way as in the proof of previous theorem, we have

$$\frac{z p_1'(z)}{p_1(z)} = 1 + \left\{\frac{l}{\lambda} \frac{\left(D^{n+1} f(z)\right)'}{\left(D^n f(z)\right)'} - \left(1 + \frac{l}{\lambda}\right)\right\}.$$

Now by the hypothesis of theorem and using a result by Wang and Guo [[28], Lemma 2.4], we have

$$\Re e\{p_1(z)\} > \alpha, \quad z \in E.$$

This completes the proof. square

Corollary 3. *Let $f(z) \in \Sigma$ and $1/2 < \alpha < 1$. Suppose that for arbitrary r, $(0 < r < 1)$, $f(z)$ satisfies the conditions*

$$\min_{|z| \leq r} \Re e\left\{-z^2 f'(z)\right\} = \min_{|z| \leq r} \left|-z^2 f'(z)\right|,$$

and

$$\Re e\left\{1 + \frac{\left(zf'(z)\right)'}{f'(z)}\right\} > \frac{\alpha}{2} - 1, \quad z \in E.$$

Then, we have

$$f(z) \in \Sigma^c(\alpha).$$

Theorem 5. *If $f(z) \in \Sigma^0(\alpha) = \Sigma^c(\alpha)$, then*

$$z\left\{f(z) * \frac{m(1-2z)}{\left\{z(1-z)^2\right\}}\right\} - 1 \neq 0, \quad \theta \in [0, 2\pi) \quad \text{and} \quad z \in E,$$

where

$$m = \frac{\left(1 + e^{i\theta}\right)\left(1 + (2\alpha - 1)e^{-i\theta}\right)}{1 + (2\alpha - 1)^2 + (2\alpha - 1)\cos\theta}. \tag{3.6}$$

Proof. If $f(z) \in \Sigma^0(\alpha) = \Sigma^c(\alpha)$, then by definition, we have

$$-\Re e\left(z^2 f'(z)\right) > \alpha,$$

or using subordination we can write

$$-z^2 f'(z) \prec \frac{1 + (2\alpha - 1)z}{1 + z}.$$

Now according to the definition of subordination, there exists a function $w(z)$ analytic in E, with $w(0) = 0$ and $|w(z)| < 1$, $z \in E$, such that

$$-z^2 f'(z) = \frac{1 + (2\alpha - 1)w(z)}{1 + w(z)}, \quad z \in E.$$

Or we can write

$$-z^2 f'(z) \frac{\left(1 + e^{i\theta}\right)}{1 + (2\alpha - 1)e^{i\theta}} - 1 \neq 0, \quad z \in E \quad \text{and} \quad \theta \in [0, 2\pi). \tag{3.7}$$

Since

$$-zf'(z) = f(z) * \frac{1 - 2z}{z(1-z)^2},$$

then (3.7) can be written as

$$z\left[f(z) * \frac{m(1-2z)}{z(1-z)^2}\right] - 1 \neq 0, \quad \theta \in [0, 2\pi) \quad \text{and} \quad z \in E,$$

where m is given by (3.6), which is the desired convolution condition. This completes the proof. □

Theorem 6. *The class $\Sigma^n(\lambda, \alpha)$, is a convex set.*

Proof. Let the functions

$$f_1(z) = \frac{1}{z} + \sum_{k=0}^{\infty} a_{k_1} z^k,$$

and

$$f_2(z) = \frac{1}{z} + \sum_{k=0}^{\infty} a_{k_2} z^k,$$

be in the class $\Sigma^n(\lambda, \alpha)$. For $t \in (0, 1)$, it is enough to show that the function

$$h(z) = (1-t)f_1(z) + tf_2(z),$$

is in the class $\Sigma^n(\lambda, \alpha)$. Since

$$h(z) = \frac{1}{z} + \sum_{k=0}^{\infty} [(1-t)a_{k_1} + ta_{k_2}]z^k,$$

then

$$-z^2\left(D^n h(z)\right)' = 1 + \sum_{k=0}^{\infty} -k(1-t)\left[\frac{l+\lambda(1+k)}{l}\right]^n a_{k_1} z^{k+1} + \sum_{k=0}^{\infty} -kt\left[\frac{l+\lambda(1+k)}{l}\right]^n a_{k_2} z^{k+1},$$

from which we can write

$$-\Re e\left\{z^2\left(D^n h(z)\right)'\right\} = (1-t)\Re e\left\{1 + \sum_{k=0}^{\infty} -k\left[\frac{l+\lambda(1+k)}{l}\right]^n a_{k_1} z^{k+1}\right\} + t\Re e\left\{1 + \sum_{k=0}^{\infty} -k\left[\frac{l+\lambda(1+k)}{l}\right]^n a_{k_2} z^{k+1}\right\}. \quad (3.8)$$

Since $f_1(z)$ and $f_2(z)$ belongs to $\Sigma^n(\lambda, \alpha)$, this implies that

$$\Re e\left\{1 + \sum_{k=0}^{\infty} -k\left[\frac{l+\lambda(1+k)}{l}\right]^n a_{k_i} z^{k+1}\right\} > \alpha, \quad (i = 1, 2). \quad (3.9)$$

From (3.8) and (3.9), we have

$$\Re e\left\{-z^2\left(D^n h(z)\right)'\right\} > \alpha.$$

This completes the proof. □

Theorem 7. *Let $f(z) \in \Sigma^n(\lambda, \alpha)$ and $g(z) \in \Sigma$ such that*

$$\Re e\{zg(z)\} > \frac{1}{2}.$$

Then $(f * g)(z) \in \Sigma^n(\lambda, \alpha)$.

Proof. Let

$$h(z) = (f * g)(z).$$

Using convolution properties, we have

$$-z^2\left(D^n h(z)\right)'^2 \left(D^n f(z)\right)' * zg(z), \quad z \in E. \quad (3.10)$$

Since $f(z) \in \Sigma^n(\lambda, \alpha)$ and

$$\Re e\{zg(z)\} > \frac{1}{2},$$

then it follows from Lemma 1

$$(f * g)(z) \in \Sigma^n(\lambda, \alpha), \quad z \in E.$$

This completes the proof. □

Theorem 8. *For $\lambda > 1$ and let $f(z)$ and $g(z)$ belong to $\Sigma^n(\lambda, \alpha)$, with*

$$f(z) = \frac{1}{z} + \sum_{k=1}^{\infty} a_k z^k,$$

and

$$g(z) = \frac{1}{z} + \sum_{k=1}^{\infty} b_k z^k, \quad z \in E^*.$$

Then $(f * g)(z) \in \Sigma^n(\lambda, \beta)$, where

$$\alpha \le \beta = \frac{4\alpha - \lambda(2\alpha+1) - 1}{2(1-\lambda)}.$$

Proof. Since $g(z) \in \Sigma^n(\lambda, \alpha)$, we have

$$\Re e\left\{1 + \sum_{k=1}^{\infty} -k\left[\frac{l+\lambda(1+k)}{l}\right]^n b_k z^{k+1}\right\} > \alpha, \quad z \in E. \quad (3.11)$$

For $1 \le \lambda \le 2$. Let $c_0 = 1$ and

$$c_k = \frac{\lambda - 1}{k}\left[\frac{l}{l+\lambda(1+k)}\right]^n, \quad k \ge 1.$$

Then $\{c_k\}_{k=0}^{\infty}$ is a convex null sequence. Therefore by Lemma 2, we have

$$\Re e\left\{1 + \sum_{k=1}^{\infty} \frac{\lambda-1}{k}\left[\frac{l}{l+\lambda(1+k)}\right]^n b_k z^{k+1}\right\} > \frac{1}{2}, \quad z \in E. \quad (3.12)$$

Now taking the convolution of (3.11) and (3.12) and applying Lemma 1, to have

$$\Re e\left\{1 + \sum_{k=1}^{\infty} (1-\lambda) b_k z^{k+1}\right\} > \alpha, \quad z \in E.$$

Or

$$\Re e\{zg(z)\} = \Re e\left\{1 + \sum_{k=1}^{\infty} b_k z^{k+1}\right\} > \frac{\alpha - \lambda}{(1-\lambda)}.$$

Thus

$$\Re e\left\{zg(z) - \frac{2\alpha - \lambda - 1}{2(1-\lambda)}\right\} > \frac{1}{2}.$$

Since $f(z) \in \sum^n(\lambda, \alpha)$, applying Lemma 1, we obtain

$$\Re e\left\{-z^2\left(D^n f(z)\right)' * \left(zg(z) - \frac{2\alpha - \lambda - 1}{2(1-\lambda)}\right)\right\} > \alpha,$$

or we can write

$$\Re e\left\{-z^2\left(D^n f(z)\right)' * zg(z)\right\} > \frac{4\alpha - \lambda(2\alpha + 1) - 1}{2(1-\lambda)}.$$

Hence the result follows from (3.10). □

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