Journal of the Egyptian Mathematical Society (2011) **19**, 33–38

Egyptian Mathematical Society

Journal of the Egyptian Mathematical Society

www.etms-eg.org
www.elsevier.com/locate/joems

REVIEW PAPER

Group theory and unification of forces: Application to 'non-universal' gaugino masses ☆

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Available online 9 November 2011

KEYWORDS

$SO(10)$;
Supersymmetry;
Gaugino mass

Abstract We will give an overview on how to embed the Standard Model (SM), based on $SU(3) \times SU(2) \times U(1)$, within larger groups. We will review the different chains one follows when spontaneously breaking $SO(10)$ down to the SM. Finally, We shall discuss the question of non-universal gaugino masses in supersymmetric $SO(10)$ theories.

1. Introduction

Many systems studied in physics show some form of symmetry. Quantum mechanics, for instance, showed that the elementary systems that matter is made of, such as electrons and protons, are truly identical, not just very similar, so that symmetry in their arrangement is exact, not approximate as in the macroscopic world. Elementary particles were observed to reflect symmetry properties in more esoteric spaces. In all these cases, symmetry can be expressed by certain operations on the systems concerned, and the mathematical language which expresses these properties is that of Group Theory. More specifically, physics uses that part of Group Theory known as the theory of representations, in which matrices acting on the members of a vector space is the central theme. A representation D would be defined to be any mapping of the group G onto a set of linear operators, which would transform the group identity to the identity operator and would map the group multiplication to the natural multiplication in the linear space on which the operators act.

Representations of finite groups proved to be very helpful in, say, the study of crystals and atomic spectra [1]. However, continuous Lie groups and their representations are the ones one studies in order to treat a wide variety of problems in particle physics and unification of forces [2]. We shall give a brief review on the Standard Model (SM), the successful theory describing the world to a very good precision [3]. However, there are some tiny circumstances, like the current experiments in the Large Hadron Collider (LHC), where effects of larger models englobing the SM such as supersymmetry (SUSY) [4] or Grand Unified Theories (GUT) [5], might reveal their effects. For this, we shall also give a brief review of some SUSY GUT models, in particular the phenomenologically interesting model of susy-$SO(10)$, and treat one specific problem in this regard, that of the 'gaugino' masses.

☆ Work presented in the Egypt–France Mathematical Conference, May 3–5, 2010, Cairo.
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Peer review under responsibility of Egyptian Mathematical Society.
doi:10.1016/j.joems.2011.09.012

Production and hosting by Elsevier

2. The Standard Model

The SM group is

$$G_{SM} = U(1) \times SU(2) \times SU(3) \tag{1}$$

where the factor $U(1) \times SU(2)$ corresponds to the electroweak force and represents a unification of electromagnetism and the weak force. In fact, it is the spontaneous symmetry breaking (SSB) which makes the electromagnetic and the weak forces look different, whereas at high energies they are the same. As to the factor $SU(3)$, it corresponds to the strong force, which binds quarks together, and no symmetry breaking here.

As to the particle content of the SM, it is shown in Table 1.

Here, we have written a bunch of $G_{\rm SM} = U(1) \times SU(2) \times SU(3)$ irreducible representations (irreps) as $U \otimes V \otimes W$, where U is a $U(1)$ irrep C_Y, where $Y \in 1/3\mathbb{Z}$ and the underlying vector space is just $\mathbb{C}$ whereas the action is given by: $\alpha.z = \alpha^{3Y}z : \alpha \in U(1), z \in \mathbb{C}$. As to the other factors: V is an $SU(2)$ irrep, either $\mathbb{C}$ or $\mathbb{C}^2$, and W is an $SU(3)$ irrep, either $\mathbb{C}$ or $\mathbb{C}^3$.

Physicists use these irreps to classify the particles as follows. The number Y in C_Y is called the 'hypercharge', and $\mathbb{C}^2 = \langle u, d \rangle$, u and d are called 'isospin up' and 'isospin down', whereas for the 'color space': $\mathbb{C}^3 = \langle r, g, b \rangle$, with r, g and b called 'red', 'green' and 'blue' colors. Finally, The normalization of Y is related to the 'electric charge' Q by: $Q = \frac{Y}{2} + I_3$ where I_3 is the 'diagonal' generator of $SU(2)$. For example, we say the red left-handed up quark is the hypercharge $1/3$, isospin up, red particle and we write $u^r_L = 1 \otimes u \otimes r \in \mathbb{C}_{\frac{1}{3}} \otimes \mathbb{C}^2 \otimes \mathbb{C}^3$, whereas to say that the right-handed electron is the hypercharge -2, isospin singlet which is colorless, we write $e^-_R = 1 \otimes 1 \otimes 1 \in \mathbb{C}_{-2} \otimes \mathbb{C} \otimes \mathbb{C}$.

Now, to define the representation of the SM, we take the direct sum of all the above irreps, defining the reducible representation,

$$F = \mathbb{C}_{-1} \otimes \mathbb{C}^2 \otimes \mathbb{C} \oplus \cdots \oplus \mathbb{C}_{-\frac{2}{3}} \otimes \mathbb{C} \otimes \mathbb{C}^3$$

which we call the 'fermions'. We also have the 'antifermions', F^*, which is just the dual of F. Direct summing these two representations, we get the *Standard Model representation*:

$$V_{SM} = F \oplus F^*$$

3. Grand Unified Theories: GUTs

The SM achieves two properties in that the particles are basis vectors in a representation V of a Lie group $G_{\rm SM}$, and that the classification of particles means the decomposition of the representation into irreps. However, the SM is 'complicated' and needs to be simplified. In fact, the mathematical product of groups: $G_{\rm SM} = U(1) \times SU(2) \times SU(3)$ and $V_{SM} = \mathbb{C}_{-1} \otimes \mathbb{C}^2 \otimes \mathbb{C} \oplus \ldots \oplus \mathbb{C}_{\frac{2}{3}} \otimes \mathbb{C} \otimes \mathbb{C}^{3*}$ is complicated to deal with. Moreover, we need to explain other patterns, such as the fact that we have $\dim V_{\rm SM} = 32 = 2^5$, or the reason of the symmetry between quarks and leptons, and also the asymmetry between left and right.

To remedy these needs, the strategy of GUT is the following. If V is a representation of H and $G_{\rm SM} \subset H$, then V is also a representation of $G_{\rm SM}$, and V may break apart into more $G_{\rm SM}$-irreps than H-irreps. In its turn H might be a subgroup of one larger group G: $H \hookrightarrow G$, and one seeks a 'simple' group G to represent the GUT group.

However, the 'coupling constants' corresponding to the different groups of the SM are energy-dependent [6]:

$$\frac{d\alpha_i}{dq^2} = b_i \alpha_i^2 + \mathcal{O}(\alpha_i^3) \tag{2}$$

where q^2 denotes the energy scale at which we measure α_i and the 'running beta function coefficients' b_1, b_2, b_3 are computed by group theory consideration. So, if unification is correct then there should be a value at which the running coupling constants get the same value

$$\alpha_i(M_X^2) = \alpha_G(M_X^2) \tag{3}$$

M_X is of the order of 10^{15-17} GeV.

Some common examples of 'GUTs' are:

- The Ordinary $SU(5)_{G-G}$, which was due to Georgi and Glashow in the early seventies [7], can be described as: "two isospins + three colors = five things = $\mathbb{C}^5 = (u, d, r, g, b)$". In this model, we have one family of fermions which can be accommodated in an $SU(5)$ reducible representation $\mathbf{5}^* + \mathbf{10}$. Another model based on $SU(5)$ is the 'flipped' $SU(5)' \times U(1)_X$, due to De Rujula, Georgi and Glashow in the early eighties [8], and where $SU(5)'$ is different from $SU(5)_{G-G}$, due to the fact that there are two ways to embed the electric charge generator in $SU(5) \times U(1)$: $SU(3)_C \times SU(2)_L \times U(1)_Z \hookrightarrow SU(5)'$. The weak hypercharge here Y must be a linear combination of Z and X, where Z is defined to be the generator of $SU(5)'$ which commutes with the generators of $SU(3)_C \times SU(2)_L$.
- The other common model is the Pati–Salam model, due to Pati and Salam in the early seventies [9]: $G_{PS} = SU(2) \times SU(2) \times SU(4)$. This model unifies the $\mathbb{C}^3 \oplus \mathbb{C}$ representation of $SU(3)$ into the irrep $\mathbb{C}^4$ of $SU(4)$: $V_{PS} = \mathbb{C}^2 \otimes \mathbb{C} \otimes \mathbb{C}^4 \oplus \mathbb{C} \otimes \mathbb{C}^2 \otimes \mathbb{C}^4 \oplus$ dual. This creates explicit symmetry between quarks and leptons, so that one can see "the lepton number as the fourth color". It also unifies the $\mathbb{C}^2 \oplus \mathbb{C} \oplus \mathbb{C}$ representations of $SU(2)$ into the representation $\mathbb{C}^2 \otimes \mathbb{C} \oplus \mathbb{C} \otimes \mathbb{C}^2$ of $SU(2) \times SU(2)$ which treats left and right more symmetrically.

Table 1 Particle content of the SM.

Name	Symbol	$G_{\rm SM}$ irrep
Left–handed doublets	$\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	$\mathbb{C}_{-1} \otimes \mathbb{C}^2 \otimes \mathbb{C}$
Left–handed quarks	$\begin{pmatrix} \frac{u^r_L}{d^r_L} & \frac{u^g_L}{d^g_L} & \frac{u^b_L}{d^b_L} \end{pmatrix}$	$\mathbb{C}_{\frac{1}{3}} \otimes \mathbb{C}^2 \otimes \mathbb{C}^3$
Right–handed neutrino	ν_R	$\mathbb{C}_0 \otimes \mathbb{C} \otimes \mathbb{C}$
Right–handed electron	e^-_R	$\mathbb{C}_{-2} \otimes \mathbb{C} \otimes \mathbb{C}$
Right–handed up quarks	$u^r_R \quad u^g_R \quad u^b_R$	$\mathbb{C}_{\frac{4}{3}} \otimes \mathbb{C} \otimes \mathbb{C}^3$
Right–handed down quarks	$d^r_R \quad d^g_R \quad d^b_R$	$\mathbb{C}_{-\frac{2}{3}} \otimes \mathbb{C} \otimes \mathbb{C}^3$

Table 2 Particle content in Pati–Salam model

Name	Symbol	$SU(2) \times SU(2) \times SU(4)$ irrep
Left-handed fermions	$\begin{pmatrix} \nu_L & u^r_L & u^g_L & u^b_L \\ e_L & d^r_L & d^g_L & d^b_L \end{pmatrix}$	$\mathbb{C}^2 \otimes \mathbb{C} \otimes \mathbb{C}^4$
Right-handed fermions	$\begin{pmatrix} \nu_R & u^r_R & u^g_R & u^b_R \\ e^-_R & d^r_R & d^g_R & d^b_R \end{pmatrix}$	$\mathbb{C} \otimes \mathbb{C}^2 \otimes \mathbb{C}^4$

We summarize the features of the Pati–Salam model in Table 2.

4. Susy-$SU(10)$

There are many advantages which can be achieved when one builds a GUT based on the group $SU(10)$ [10]. First, the embedding of left–right symmetry can be done in this rank-5 simple group $SU(10)$ with an irrep 16-dim: the 'spinorial representation' whose decomposition under $SU(3)\times SU(2)\times U(1)$ shows the correct quantum numbers to describe one family of fermions where also the right-handed neutrino is present. Second, $SU(10)$ is the minimal left–right symmetric GUT that 'gauges' the B-L symmetry and where the gauge interactions conserve parity, thus making parity a part of a continuous symmetry.

Moreover, the model can be rich enough, since there are several breaking chains one can follow from $G = SO(10)$ down to the SM:

$$G \overset{\Phi_1}{\to} H \overset{\Phi_2}{\to} SM \tag{4}$$

Also, for the 'SSB-Higgs Mechanism' to work for $G_1 \overset{\Phi}{\to} G_2$, the branching rule of Φ under this decomposition should contain a 'singlet component', to take a vacuum expectation value (vev) upon the breaking. For example, the irrep **54** can break $SU(10)$ into G_{422} since we have the branching rule

$$\mathbf{54} \overset{SO(10)\supset G_{422}}{=} (1,1,1) + (\mathbf{3},\mathbf{3},1) + (1,1,\mathbf{20}) + (\mathbf{2},\mathbf{2},\mathbf{6}) \tag{5}$$

Another kind of symmetry can be imposed on the model which is SUSY [4]. The essential motivation to have SUSY in the particle physics model is what is called the 'Hierarchy Problem'. The existence of two scales, the electroweak scale (M_W 100 GeV) and the GUT scale (M_X 10^{15} GeV) so different creates a fundamental problem: How is it possible to keep these two scales incommunicado ? This problem arises whenever 'fundamental scalars' are present. Their mass gets quadratically divergent contribution when 1-loop radiative corrections are taken into account. This happens because there is no symmetry able to keep scalars (virtually) massless in contrast to 'gauge' or 'chiral' symmetries which keep bosons or fermions massless.

Here, history gives us a precedent lesson: The electron self-energy in classical electromagnetism goes like $e^2/a(a\to 0)$ i.e. it is linearly divergent. In quantum theory, fluctuations of the electromagnetic fields (in the single electron theory) generate a quadratic divergence. If these divergences are not canceled, one would expect that QED should break down at an energy of order m_e/a far below the Planck scale (a severe hierarchy problem). However, the linear and quadratic divergences will cancel exactly if one makes a bold hypothesis consisting of the existence of the positron, i.e. we 'double' the particle spectrum and the divergence problem is solved. One can repeat this lesson for the SM, in that we take the SM particle content and double the particle spectrum. Then, we introduce a new symmetry (SUSY) that relates fermions to bosons: for every fermion (gauge boson) there is a superpartner boson (fermion), called sfermion (gaugino), of equal mass. Now we compute the self-energy of an elementary scalar and find that, since SUSY relates it to the self energy of a fermion which is only logarithmically divergent, the quadratic divergences cancel! However, no superpartners have been seen which implies that susy, if it exists in nature, must be a broken symmetry.

5. Susy $SU(10)$ non-universality

As we saw, GUTs are among the most promising models for physics beyond the SM. Moreover, SUSY is necessary to make the huge hierarchy between the GUT scale and the electroweak scale stable under radiative corrections. The apparent unification of the measured gauge couplings within the Minimal Supersymmetric extension of the Standard Model (MSSM) [4] at scale $M_{\mathrm{GUT}}\sim 2\times 10^{16}$ GeV is considered as an experimental evidence for SUSY GUT.

Universal boundary conditions for gaugino masses, as well as other soft terms, at the high scale (the unification scale or Plank scale) are adopted in the mSUGRA or CMSSM [11]. If the discrepancy between the SM and the experimental determinations of (g-2) [12] is confirmed at the 3-σ level, this could be interpreted as strong evidence against the CMSSM. However, these universal boundary conditions adopted in the mSUGRA are simple assumptions about the nature of high-scale physics and may remove some interesting degrees of freedom.

Non-universal gaugino masses may arise in supergravity models in which a non-minimal gauge field kinetic term is induced by the SUSY-breaking vev of a chiral superfield that is charged under the GUT group [13]

$$\mathcal{L} \supset \int d^2\theta f_{ab}(\Phi) W^a W^b + h.c. \supset \frac{< F_\Phi >_{ab}}{M} \lambda^a \lambda^b$$

where $f_{AB} = f_0(\Phi_s)\delta_{AB} + \sum_n f_n(\Phi_s)\frac{\Phi^n_{AB}}{M} + \cdots$ with M is a mass parameter, Φ_s and Φ^n are the singlet and non-singlet chiral superfields, respectively, the $\lambda^{a,b}$ are the gaugino fields and the F_Φ is the auxiliary field component of Φ.

In conventional models of supergravity breaking, the assumption that only the singlet field F_{Φ_s} gets a vev is made so that one obtains universal gauge masses. However, in principle, the chiral superfield Φ which communicates SUSY breaking to the gaugino fields can lie in any representation found in the symmetric product of two adjoints. As we said before, there can be more than one breaking chain from a GUT group G down to the SM group if G is a large symmetry group, like $SU(10)$. Indeed, for $SU(10)$, we have the decomposition:

$$(\mathbf{45}\times\mathbf{45})_{\mathrm{symmetric}} = \mathbf{1} + \mathbf{54} + \mathbf{210} + \mathbf{770}$$

where only **1** yields universal masses.

Here we make two basic assumptions: First, we omit the 'possible' situation of a linear combination of the above irreps (i.e. Dominant contribution to the gaugino masses coming from one of the non-singlet F-components). Second, we assume that $SU(10)$ gauge symmetry group is broken down at GUT scale M_{GUT} into an intermediate group H which, in turn, breaks down to the SM at some intermediate scale M_{HB}

$$SO(10) \overset{@\mathbf{M_{GUT}}}{\to} H \overset{@\mathbf{M_{HB}}}{\to} \mathrm{SM} \equiv SU(3)\times SU(2)\times U(1).$$

In this regard, the successful couplings unification in the MSSM favors a single GUT scale, in that the M_{HB} should not be too far from M_{GUT}, and previous studies [14–17] assume no intermediate scales between M_{GUT} and M_{EW} for simplicity.

Nonetheless, recent studies [18] show that in GUTs with large number of fields, renormalization effects significantly modify the scale at which quantum gravity becomes strong and this in turn can modify the boundary conditions for coupling unification. Any one of three options – threshold corrections due to the mass spectrum near the unification scale, gravity induced non-renormalizable operators near the Plank scale, or presence of additional light Higgs multiplets – can permit unification with the intermediate scale lower [19]. Current work [20] investigates the intermediate scale dependence of non-universal gaugino masses in supersymmetric $SU(10)$.

5.1. Calculation details for one specific chain

Let us take the chain $SO(10) \xrightarrow{\mathbf{54}} H = G_{422} \xrightarrow{\mathbf{16}} SM \equiv SU(3) \times SU(2) \times U(1)$. The **54** irrep can be represented as a traceless and symmetric 10×10 matrix which takes the vev:

$$< 54 >= v\ Diag(2,2,2,2,2,2,-3,-3,-3,-3)$$

with the indices $1,\ldots 6$ corresponding to $SO(6) \simeq SU(4)_C$ while those of $7,\ldots\ 0$ (0 means 10) correspond to $SO(4) \simeq SU(2)_L \times\ SU(2)_R$. We then use a **16** irrep to break $SU(4)$ into the SM having the branching rule:

$$\mathbf{16} \overset{SO(10)\supset SM}{=} (\mathbf{3},\mathbf{2})_{1/3} + (\mathbf{3},\mathbf{1})_{2/3} + (\bar{\mathbf{3}},\mathbf{1})_{-4/3} + (\mathbf{1},\mathbf{2})_{-1} + (\mathbf{1},\mathbf{1})_{2} + (\mathbf{1},\mathbf{1})_{0},$$

When the neutral component $(\mathbf{1,1})_0$ of **16** develops a vev then G_{422} will be broken to SM. The gauge supermultiplets **45** of $SU(10)$ would also be decomposed under the two breakings:

$$A(45) = A(15,1,1) + A(1,3,1) + A(1,1,3) + A(6,2,2)$$

Under SM we have

$$A(15,1,1) = A(8,1)_0 + A(3,1)_{4/3} + A(\bar{3},1)_{-4/3} + A(1,1)_0,$$
$$A(1,1,3) = A(1,3)_0,$$
$$A(1,3,1) = A(1,1)_2 + A(1,1)_{-2} + A(1,1)_0.$$

One needs to identify the weak hypercharge Y generator as a linear combination of the $(\mathbf{1,1})_0$ parts of the generators **15** of $SU(4)$ and **3** of $SU(2)_R$.

For this we write

$$SU4_c \times SU2_R \xrightarrow{(\mathbf{4,2,1})} SU3_c \times U1_{B-L} \times SU2_R \rightarrow SU3_c \times U1_Y$$
$$\mathbf{16} \overset{SO(10)\supset G_{422}}{=} (\mathbf{4},\mathbf{2},1) + (\bar{\mathbf{4}},\mathbf{2},1),$$

where

$$\mathcal{D}_\mu \Phi = \partial_\mu \Phi - i g_4 \frac{T^b A^b}{2} \phi^a - i g_2 \frac{\boldsymbol{\tau}.B}{2} \phi^s$$

with $T^b\ (\boldsymbol{\tau})$, $b = 1,\ldots,15$; $a = 1,\ldots,4$; $s = 1,2$, are the generalized Gellman (Pauli) matrices, and $\langle \phi^a \rangle = v\delta^{a4}, \langle \phi^s \rangle = v\delta^{s1.}$

Concentrating on the mixing of $U(1) \subset SU(4)_C$ and the other $U(1)' \subset SU(2)_R$, the corresponding A^{15}, B^3 components will mix and we obtain the neutral gauge boson mass terms:

$$\langle D_\mu \Phi \rangle \langle D_\mu \Phi \rangle^+ = \frac{v^2}{4} \left(\sqrt{\frac{3}{2}} g_4 A^{15} - g_R B^3 \right)^2$$

This quadratic form in B^3, A^{15} has a zero eigenvalue whose corresponding eigenstate is identified as the massless $U(1)_Y$

Table 3 Gaugino mass ratios at intermediate scale M_{HB} in the different cases. To each ratio correspond four columns, the first of which gives the general formula whereas the other three give the result when M_{HB} is taking a specific value. Bracketed values denote the gaugino mass ratios when $M_{HB} = M_{GUT}$ evaluated at the same specific energy scale (10^3 or 10^8 GeV) as the case of $M_{HB} \neq M_{GUT}$. The following numerical values are taken: $M_{GUT} = 10^{16}$, $\alpha = 0.1$. Mass scales are evaluated in GeV. The parameter m is equal to $\frac{M_1}{M_2}$.

Irrep	H	M_1/M_3				M_2/M_3			
$M_{HB}=$			M_{GUT}	10^8	10^3		M_{GUT}	10^8	10^3
54	G_{422}	$\frac{-5R(2_R,4)}{6+4R(2_R,4)}$	−1/2	0.88 (3.21)	2.27 (1.96)	$-\frac{3}{2}R(2_L,4)$	−3/2	0.93 (3.13)	1.45 (1.58)
	$SU2 \times SO7$	1	1	1 (−6.42)	1(−3.92)	$-\frac{7}{3}R(2_L,4)$	−7/3	−0.36 (4.88)	−0.31 (2.45)
210	G_{422}	$m = \frac{-3}{3+2R(2_R,4)}$	$m = -\frac{3}{5}$	$m = -1.70\ (=-1.84)$	$m = -2.82(=-2.24)$	∞	∞	∞	∞
	H_{51}	$\frac{-95R(1_X,5)}{24+R(1_X,5)}$	−19/5	2.21 (24.38)	2.67 (14.90)	1	1	1 (−2.09)	1 (−1.05)
770	G_{422}	$\frac{19R(2_R,4)}{6+4R(2_R,4)}$	19/10	−3.34 (−12.19)	−8.63(−7.45)	$\frac{5}{2}R(2_L,4)$	5/2	−1.55 (−5.22)	−2.42(−2.63)
	$SU2 \times SO7$	1	1	1 (−6.42)	1 (−3.92)	$7R(2_L,4)$	7	1.09(−14.63)	0.93(−7.36)
	H_{51}	$\frac{385R(1_X,5)}{24+R(1_X,5)}$	77/5	−8.95 (−98.80)	−10.85 (−60.40)	1	1	1 (−2.09)	1 (−1.05)

gauge boson E, whereas the orthogonal combination F is a massive vector boson:

$$F = \cos\theta A^{15} - \sin\theta B^3$$
$$E = \sin\theta A^{15} + \cos\theta B^3$$
$$\text{with : } \cos\theta = \frac{\sqrt{\frac{3}{2}}g_4}{c}, \quad \sin\theta = \frac{g_R}{c} : c^2 = g_R^2 + \frac{3}{2}g_4^2$$

It is convenient to define [5] the 4×4 (2×2) matrices

$$\mathbf{A} = \frac{T^b A^b}{\sqrt{2}} \text{ with } A_b^a \equiv (\mathbf{A})_{ab}, \quad \mathbf{B} = \frac{\tau^r B^r}{\sqrt{2}} \text{ with}$$
$$B_s^r \equiv (\mathbf{B})_{rs} \Rightarrow A_4^4 = -\frac{\sqrt{3}}{2}A^{15}, \quad B_1^1 = \frac{B^3}{\sqrt{2}}$$

and denote the gaugino fields of the $SU(4)_C$ ($SU(2)_{L,R}$) group by λ_b^a ($\lambda_{sL,R}^r$), with $a,b = 1,\ldots,4$ and $\lambda_a^a = 0$ ($r,s = 1,2$ with $\lambda_r^r = 0$), i.e. λ_b^a lie in the same supermultiplet as A_b^a.

We thus can determine the E-coefficient of the expressions of A_4^4 and B_1^1, and by SUSY we have:

$$\lambda_4^4 = -\frac{\sqrt{3}}{2}(\sin\theta\lambda + \cos\theta\widetilde{\lambda})$$
$$\lambda_{1R}^1 = \frac{1}{\sqrt{2}}(\cos\theta - \sin\theta\widetilde{\lambda})$$

where λ is the gaugino field lying in the same supermultiplet as the $U(1)_Y$ gauge field E, whereas $\tilde{}$ is the superpartner of the massive vector boson F.

The final step consists of writing the part of the Lagrangian containing the gaugino mass term:

$$\mathcal{L}_{mass} = M_4(\lambda_b^a)^2 + M_{2R}(\lambda_s^r)_R^2 + M_{2L}(\lambda_s^r)_L^2 \tag{6}$$

The first stage of breaking from $G = SO(10)$ to $H = G_{422}$ gives:

$$\underset{M}{SO10} \rightarrow \underset{-\frac{3}{2}M=M_{2L}}{SU2_L} \times \underset{-\frac{3}{2}M=M_{2R}}{SU2_R} \times \underset{M=M_4}{SU4}$$

In $SU4$ indices ($a,b = 1,\ldots,4$, $\alpha,*\beta = 1,\ldots,3$), we have:

$$M_4\lambda_b^a\lambda_a^b = M_4\lambda_\beta^\alpha\lambda_\alpha^\beta + M_4(\lambda_4^4)^2 + \cdots$$

One needs to single out the '$SU(3)$-traceless' gaugino field $\hat{\lambda}_\beta^\alpha = \lambda_\beta^\alpha - \frac{1}{3}\delta_\beta^\alpha\lambda_\gamma^\gamma$ and thus we have $M_4\lambda_b^a\lambda_a^b = M_4\hat{\lambda}_\beta^\alpha\hat{\lambda}_\alpha^\beta + \frac{4}{3}M_4(\lambda_4^4)^2 + \cdots$. Similarly, in $SU2_R$ indices, we have: $M_2\lambda_s^r\lambda_r^s = 2M_2(\lambda_1^1)^2 + \ldots$.

Thus, one can write finally:

$$\mathcal{L}_{mass} = M_4(\hat{\lambda}_\beta^\alpha)^2 + \frac{4}{3}M_4(\frac{3}{2}\frac{g_2^2}{c^2})\lambda^2 + 2M_{2R}(\frac{3}{2}\frac{g_4^2}{c^2})\lambda^2 + M_{2L}(\lambda_s^r)^2$$

and we get at $M_{\rm HB}$ the gaugino masses in the ratio:

$$\frac{M_2(t)}{M_3(t)} = \frac{r_2(t,t_0)}{r_4(t,t_0)}(-\frac{3}{2}), \quad \frac{M_1(t)}{M_3(t)} = \frac{(g_2^2(t) - \frac{9}{4}g_4^2(t))}{(g_2^2(t) + \frac{3}{2}g_4^2(t))}$$

where $r_i(t,t_0) = \frac{\alpha_i(t)}{\alpha_i(t_0)}$, $t = ln\frac{M_{\rm GUT}^2}{Q^2}$ with $Q^2 = M_{\rm HB}^2$ and $t_0 = 0$ corresponding to $Q^2 = M_{\rm GUT}^2$. When ($M_{\rm HB} = M_{\rm GUT}$) we get M_a ($a = 1,2,3$) in the ratio $-\frac{1}{2} : -\frac{3}{2} : 1$.

5.2. Results

We summarize in Table 3 the results of the gaugino masses corresponding to the several breaking chains from $SU(10)$ down to the SM:

Numerically, and using the renormalization group equations for the running of the coupling constants, we find a significant change in the case where the two breaking scales are distant apart from where they are equal. Although some model complexifications might affect the coupling constants evolution, and consequently the values of the derived gaugino mass ratios, however the above conclusion concerning the significant influence of the existence of multi-stages in the breaking chain would remain unchanged. The derived mass ratios would be reflected in the electroweak energy scale measurements due to take place in the near future experiments, like the LHC, with interesting phenomenological consequences.

6. Summary and conclusion

Especially with the advent of the LHC, many ideas of 'new physics' can be tested, one of which is SUSY GUTs. The concept of 'Symmetry' has provided us with a very strong tool guiding us to understand the physical world, and the mathematics of symmetry is given by Group Theory. GUT, based on continuous symmetry groups, allow to interpret many experimental data, particularly that they have many new signatures (like non-universality of gaugino masses, of which we presented the detailed results for the GUT group being equal to $SU(10)$) which can be tested in the near-future experiments.

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