ORIGINAL RESEARCH

Estimation for two exponential life time models under joint multiply type-II censoring

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Abstract

Various types of censoring schemes basically type-I and type-II censoring schemes and their modified versions are used in life testing experiments. Most of the tests used in life testing experiments are based on a single sample. A joint censoring scheme is quite useful in conducting comparative life tests of products from different units within the same facility. In this article, we consider two exponential life time models under joint multiply type-II censoring scheme, which is a generalization of usual type-II censoring scheme, implemented on the two samples. We have considered maximum likelihood estimation and Bayesian estimation for estimating the reliability of the product under such a censoring scheme. The results are compared with the results obtained under usual type-II censoring scheme. In Bayes estimation the effect of prior parameters on mean life time and reliability of the product is discussed. We have used the local influence approach for identifying observations that strike a disproportionate effect in the maximum likelihood estimate of the reliability in the model. The life time data set of air-conditioning systems of two Boeing 720 jet airplanes "7914" and "7913" are used to apply the theory developed in the paper.

Keywords: Maximum likelihood estimation, Asymptotic variance, Influence measure, Inverse gamma prior, Bayes estimation

Mathematics Subject Classification: 62F15, 62F30, 62N02, 62N05

Introduction

Different censoring schemes are available in the literature to optimize time, cost and efficiency of the life testing experiment. Among different censoring schemes, type-I and type-II are the two most popular censoring schemes. In a type-I censoring scheme, the experiment is terminated at a prefixed time point. In type-I censoring, number of failures is purely random. So, it may happened that the observed number of failures during the pre-determined fixed time may be very small or sometimes zero which will lead to a biased or inefficient statistical analysis of associated model parameters. To get a certain number of failures, type-II censoring scheme is used. But in type-II censoring scheme, the test termination time is random and it may takes more than the expectation of the experimenter, which increases the cost of the experiment. The work done under such types of censoring schemes is mostly on the basis of single censored sample.

However, in practical market, there are situations in which the experimenter wishes to compare the product produced by different producers under the same facility. From a practical point of view, it is necessary to identify one item in presence of the other relative items. This problem requires sampling of the different production lines, and then the joint censoring scheme is appeared. To this end, the reliability comparative life test is apparently desirable. In order to guarantee the same experimental condition that items are exposed to, a joint censoring model is used to describe the test model.

Recently, two sample joint censoring schemes becoming popular for a life testing experiment mainly to optimize time and cost. Two independent samples of sizes *m* and *n* are selected from these lines and put simultaneously on a life testing experiment. Then, the experimenter follows a joint complete sample scheme or a joint censoring scheme and terminates the life testing experiment when a pre-determined number of failures (say, *r*) occur, which is known as joint type-II censoring.

To determine reliability of a product, life time distribution plays an important role. Selection of a proper life time model for the product is an art. In literature various types of life time models are available like: exponential, Weibull, Rayleigh, gamma, logistic, Pareto, Kumaraswamy etc. The life time model can be selected on the basis of the failure rate of the product. Exponential model is used for the product having constant failure rate. Weibull life time model is a very general life time model which can be used when the product has decreasing, constant or an increasing failure rate.

Exponential life time model is such a life time model which possesses many interesting properties and very much mathematically tractable. Life times of many electrical and electronic items follow exponential distribution. In this paper we have utilized exponential life time model. A lot of work has been done based on exponential life time model under various types of censoring schemes.

In the literature, joint type-II censoring scheme, joint progressive type-II censoring scheme and statistical analysis based upon such schemes have been discussed earlier. Balakrishnan and Rasouli [1] developed likelihood inference for the parameters of two exponential populations under joint type-II censoring. They developed inferential methods based on maximum likelihood estimates and compared their performance with those based on some other approaches such as Bootstrap. Shafay et al. [2] derived the Bayesian inference for the unknown parameters of two exponential populations under joint type II censoring. They developed the estimates with the use of squared-error, linear-exponential and general entropy loss functions.

Balakrishnan and Feng [3] generalized Balakrishnan and Rasouli [1] and Shafay et al. [2] works by considering a jointly type-II censored sample arising from *h* independent exponential populations. Ashour and Abo-Kasem [4, 5] derived Bayesian and non-Bayesian estimators for two generalized exponential populations under joint type-II censored scheme.

Parsi and Bairamov [6] extended the scheme of two joint type-II censoring to a joint progressive type-II censoring scheme and considered expected values of the number of failures for two populations under joint type-II progressive censoring. Rasouli and Balakrishnan [7] provided the exact likelihood inference for two exponential populations under this censoring scheme. They have also derived the exact lower confidence

bounds, exact confidence intervals, and simultaneous confidence regions for the parameters of the model based on classical and Bayesian setup.

Balakrishnan and Su et al. [8] extended the joint progressive censoring model to general K population model and studied exact likelihood inference of the unknown parameters for exponential distributions. Nadi and Gildeh [9] used progressive first-failure censored sample for estimating the lifetime performance index of products for two-parameter exponential distribution.

Mondal and Kundu [10] recently introduced a balanced joint progressive type-II censoring scheme and provided the exact likelihood inference for the two exponential populations. It is observed that it has certain advantages over the joint progressive censoring scheme originally introduced by Rasouli and Balakrishnan [7].

In this paper we suggest a joint multiply type-II censoring scheme for the product having two exponential life time distributions. The rest of the paper is organized as follows: In "Methods" section, we first introduce the multiply type-II censoring and then it is utilized for development of joint multiply type-II censoring model and its likelihood function. Maximum likelihood estimates and their asymptotic standard errors of the mean life time and reliability of the two similar line products are derived. Based on the asymptotic normal distributions of maximum likelihood estimators, asymptotic confidence intervals for mean life times of the products are derived. Then after an influence measure is considered, by which the influence of individual observations on maximum likelihood estimates can be determined. Bayes estimation of mean life times and reliabilities of the products are also discussed. In "Results and discussion" section, the real data set regarding air-conditioning system of a fleet of 13 Boeing 720 jet airplanes considered by Proschan [11] is used to exemplify the results obtained in the earlier sections. Finally overall conclusion is given in "Conclusion" section.

Methods

Multiply type-II censoring

A generalization of type-II censoring scheme is known as multiply type-II censoring scheme. The following two types of multiply type-II censoring schemes are available:

- Under this scheme, from n items on the test we observe only the r_1-th, r_2-th,$\cdots$,r_k-th failure times X_{r_1}, X_{r_2}, ...,X_{r_k}, where $1 \leq r_1 < r_2 < \cdots < r_k \leq n$, and the rest of the data are not available.
- In life testing experiments the test is terminated either at predetermined time observed (type-I censoring) or at a predetermined number of failures observed (type-II censoring). Such censoring schemes may be from left or right. Sometimes left and right censoring appears together, this is known as doubly censoring. Furthermore, if mid censoring arises amongst the doubly censoring in the type II censoring scheme, the scheme is also known as multiply type-II censoring.
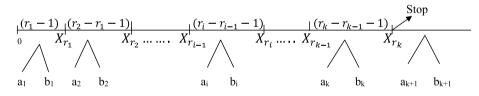
Applications of multiply type-II censoring scheme are available in literature. Some of the references related to the multiply type-II censoring scheme described in (a) are

Balakrishnan [12], Balakrishnan et al. [13], Shah and Patel [14], Kim and Han [15], and in (b) are Upadhyay et al. [16], Kong [17], Shah and Patel [18], Kang et al. [19], Patel and Patel [20] and Shafay and Sultan [21].

Joint multiply type-II censoring

Let there are two lines of similar products and our aim is to study the relative merits of these two products. A sample of size m is drawn from one product line, called type-A and another sample of size n is drawn from the other product line, called type-B.

Suppose that, $Y_1, Y_2, \ldots, Y_m$ the lifetimes of m specimens of product type-A, are independent identically distributed random variables from distribution function $F(y)$ and density function $f(y)$, and $W_1, W_2, \ldots, W_n$, the lifetimes of n specimens of product type-B, are also independent identically distributed random variables from distribution function $G(w)$ and density function $g(w)$. Further, suppose $X_1 < X_2 < \cdots < X_N$ denote the order statistics of the $N = m + n$ random variables $\{Y_1, Y_2, \ldots, Y_m;\ W_1, W_2, \ldots, W_n\}$. Here we assume that the probability distributions of Y and W are continuous, a unique ordering is always possible, since theoretical ties do not exist. Even though, if two observations from Y and W are equal, we can break the tie using random arrangement of these observations.

Suppose that only k ordered failure times $X_{r_1}, X_{r_2}, \ldots, X_{r_k}$ are observed out of $X_1, X_2, \ldots, X_N$ ordered failure times. Here experimenter pre-fixes the values of $r_1, r_2, \ldots, r_k$ before conducting the life testing experiment. In this scheme, initial $r_1 - 1$ failures, some intermediate failures and last $N - r_k$ failures are not observed, since our aim is to save time and cost of the experiment. The joint multiply type-II censoring scheme can be visualized graphically as follows.

where a_i = number of failures of type-A with $X_{r_{i-1}} < Y < X_{r_i}$, $i = 1, 2, \ldots, k$; $r_0 = 0$, $X_0 = 0$; b_i = number of failures of type-B with $X_{r_{i-1}} < W < X_{r_i}$, $i = 1, 2, \ldots, k$; a_{k+1} = number of failures of type-A with $X_{r_k} < Y = m - m_k - \sum_{i=1}^{k} a_i$; b_{k+1} = number of failures of type-B with $X_{r_k} < W = n - n_k - \sum_{i=1}^{k} b_i = R_{k+1} - a_{k+1}$; Let m_k = number of failures of type-A out of the k observed failures $= \sum_{i=1}^{k} Z_i$; n_k = number of failures of type-B out of the k observed failures $= \sum_{i=1}^{k} (1 - Z_i) = k - m_k$; Note that m_k and n_k both simultaneously cannot be zero. If any one of them is zero the problem reduces to estimation based on single sample only.

Denote $R_1 = r_1 - 1$, $R_i = r_i - r_{i-1} - 1$, $i = 1, 2, \ldots, k$ and $R_{k+1} = a_{k+1} + b_{k+1}$

Then, under the joint multiply type-II censoring scheme, the observable data consist of (Z, X), where

$X = \{X_{r_1}, X_{r_2}, \ldots, X_{r_k}\}$, with $1 < r_1 < r_2 < \cdots < r_k < N$, and

$Z = \{Z_1, Z_2, \ldots, Z_k\}$ with $Z_i = 1$ or 0 according to Z_i is from Y or W failure.

Then the likelihood function of (Z, X) will be given by

$$L = C\{F(x_{r_1})\}^{a_1}\{G(x_{r_1})\}^{b_1}\prod_{i=1}^{k}\{f(x_{r_i})\}^{z_i}\prod_{i=1}^{k}\{g(x_{r_i})\}^{1-z_i}\prod_{i=2}^{k}[F(x_{r_i}) - F(x_{r_{i-1}})]^{a_i}$$

$$\times \prod_{i=2}^{k}[G(x_{r_i}) - (x_{r_{i-1}})]^{b_i}\{\overline{F}(X_{r_k})\}^{a_{k+1}}\{\overline{G}(X_{r_k})\}^{b_{k+1}} \tag{1}$$

where $C = \frac{m!n!}{\prod_{i=1}^{k+1}a_i!\prod_{i=1}^{k+1}b_i!}$.

Consider exponential life time models for life time Y and W as

$$F(y) = 1 - e^{-y/\theta_1} \text{ and } G(w) = 1 - e^{-w/\theta_2}, \; y > 0, \; w > 0; \; \theta_i > 0, \; i = 1, 2. \tag{2}$$

Assuming $T_i = x_{r_i} - x_{r_{i-1}}$, $i = 2,...,k$ and substituting Eq. (2) in Eq. (1) the likelihood function becomes

$$L = L(\theta|X) = \frac{C}{\theta_1^{m_k}\theta_2^{n_k}}\left(1 - e^{-\frac{x_{r_1}}{\theta_1}}\right)^{a_1}\exp\left(-\frac{u_k}{\theta_1}\right)\prod_{i=2}^{k}\left[1 - e^{-T_{\frac{i}{\theta_1}}}\right]^{a_i}\left(1 - e^{-\frac{x_{r_1}}{\theta_2}}\right)^{b_1}\exp\left(-\frac{v_k}{\theta_2}\right)$$

$$\times \prod_{i=2}^{k}\left[1 - e^{-T_{\frac{i}{\theta_2}}}\right]^{b_i} \tag{3}$$

where

$$\theta = (\theta_1, \theta_2), \; u_k = \sum_{i=2}^{k}a_i x_{r_{i-1}} + \sum_{i=1}^{k}z_i x_{r_i} + a_{k+1}x_{r_k}$$

and

$$v_k = \sum_{i=2}^{k}b_i x_{r_{i-1}} + \sum_{i=1}^{k}(1 - z_i)x_{r_i} + b_{k+1}x_{r_k}$$

Maximum likelihood estimate (MLE) and asymptotic standard error

The MLEs $\hat{\theta}_1$ and $\hat{\theta}_2$ of the parameters θ_1 and θ_2 are obtained by maximizing Eq. (3). To maximize the likelihood function in Eq. (3) we derive the likelihood equations

$$\frac{\partial \log L}{\partial \theta_1} = 0 \text{ and } \frac{\partial \log L}{\partial \theta_2} = 0.$$

Solving the above equations we get MLEs of mean life times θ_1 and θ_2. The maximum likelihood method does not admit explicit solutions, but we have two equations as

$$m_k\theta_1 = u_k - \frac{a_1 x_{r_1}e^{-(x_{r_1}/\theta_1)}}{1 - e^{-(x_{r_1}/\theta_1)}} - \sum_{i=2}^{k}\left\{\frac{a_i T_i e^{-(T_i/\theta_1)}}{1 - e^{-(T_i/\theta_1)}}\right\} \tag{4}$$

and

$$n_k \theta_2 = v_k - \frac{b_1 x_{r_1} e^{-(x_{r_1}/\theta_2)}}{1 - e^{-(x_{r_1}/\theta_2)}} - \sum_{i=2}^{k} \left\{ \frac{b_i T_i e^{-(T_i/\theta_2)}}{1 - e^{-(T_i/\theta_2)}} \right\} \tag{5}$$

Using any method of iteration, the Eqs. (4) and (5) can be solved for θ_1 and θ_2. The solutions of the two equations give us the MLEs $\hat{\theta}_1$ and $\hat{\theta}_2$ of the parameters θ_1 and θ_2 respectively.

Let $I(\theta_1, \theta_2) = I_{i,j}(\theta_i, \theta_j)\, i, j = 1, 2$ denote the Fisher information matrix of the parameters θ_1 and θ_2, whereand consequently the observed Fisher information matrix is given by

$$I_{i,j}(\theta_i, \theta_j) = -E\left(\frac{\partial^2 logL}{\partial \theta_i \partial \theta_j} \right)$$

$$\hat{I}(\theta_1, \theta_2) = \begin{pmatrix} -\frac{\partial^2 \log L}{\partial \theta_1^2} & 0 \\ 0 & -\frac{\partial^2 \log L}{\partial \theta_2^2} \end{pmatrix}_{(\theta_1, \theta_2) = (\hat{\theta}_1, \hat{\theta}_2)} \tag{6}$$

where

$$\frac{\partial^2 \log L}{\partial \theta_1^2} = \frac{2}{\theta_1^3}\left\{ \frac{a_1 x_{r_1} e^{-(x_{r_1}/\theta_1)}}{1 - e^{-(x_{r_1}/\theta_1)}} \right\} - \frac{a_1 x_{r_1}^2}{\theta_1^4} \frac{e^{-(x_{r_1}/\theta_1)}}{\left(1 - e^{-(x_{r_1}/\theta_1)}\right)^2} - \frac{2u_k}{\theta_1^3} + \frac{m_k}{\theta_1^2} + \frac{2}{\theta_1^3}\sum_{i=2}^{k} \frac{a_i T_i e^{-(T_i/\theta_1)}}{1 - e^{-(T_i/\theta_1)}}$$

$$- \frac{1}{\theta_1^4}\sum_{i=2}^{k} \frac{a_i T_i^2 e^{-(T_i/\theta_1)}}{\left(1 - e^{-(T_i/\theta_1)}\right)^2} \tag{7}$$

and

$$\frac{\partial^2 \log L}{\partial \theta_2^2} = \frac{2}{\theta_2^3}\left\{ \frac{b_1 x_{r_1} e^{-(x_{r_1}/\theta_2)}}{1 - e^{-(x_{r_1}/\theta_2)}} \right\} - \frac{b_1 x_{r_1}^2}{\theta_2^4} \frac{e^{-(x_{r_1}/\theta_2)}}{\left(1 - e^{-(x_{r_1}/\theta_2)}\right)^2} - \frac{2v_k}{\theta_2^3} + \frac{n_k}{\theta_2^2} + \frac{2}{\theta_2^3}\sum_{i=2}^{k} \frac{b_i T_i e^{-(T_i/\theta_2)}}{1 - e^{-(T_i/\theta_2)}}$$

$$- \frac{1}{\theta_2^4}\sum_{i=2}^{k} \frac{b_i T_i^2 e^{-(T_i/\theta_2)}}{\left(1 - e^{-(T_i/\theta_2)}\right)^2} \tag{8}$$

Hence asymptotic standard errors (ASEs) of MLEs are obtained by

$$\text{ASE}(\hat{\theta}_1) = \sqrt{\frac{-1}{E\left(\frac{\partial^2 \log L}{\partial \theta_1^2}\right)}} \text{ and } \text{ASE}(\hat{\theta}_2) = \sqrt{\frac{-1}{E\left(\frac{\partial^2 \log L}{\partial \theta_2^2}\right)}} \tag{9}$$

Then, by using the asymptotic normality of the MLEs, we can express the asymptotic $(1 - \alpha)100\%$ confidence intervals for θ_1 and θ_2 as

$$\hat{\theta}_1 \pm Z_{\alpha/2}\text{ASE}(\hat{\theta}_1) \text{ and } \hat{\theta}_2 \pm Z_{\alpha/2}\text{ASE}(\hat{\theta}_2) \tag{10}$$

where $Z_{\alpha/2}$ denotes the upper $\alpha/2$ percentage point of the standard normal distribution.

MLEs of Reliabilities at time t of the product of type-A and type-B are respectively given by

$$\widehat{R_A}(t) = e^{-t/\hat{\theta}_1} \text{ and } \widehat{R_B}(t) = e^{-t/\hat{\theta}_2} \tag{11}$$

The ASE of MLE of reliability at time t for the product A is calculated as

$$\mathrm{ASE}_A(t) = \sqrt{\frac{\mathrm{d}R_A(t)}{\mathrm{d}\theta_1} V(\hat{\theta}_1)}\Bigg|_{\theta=\hat{\theta}_1} \tag{12}$$

Similarly the asymptotic standard error of MLE of reliability at time t for the product B can be calculated.

Influence measure

In this section we have considered the influence of individual observations on maximum likelihood estimates. We have used the method considered by Poon and Tang [22].

Let $L(\theta)$ be the likelihood function of θ given in Eq. (1). Define the case weight perturbation $w = (w_1, w_2, \ldots, w_N)'$ and corresponding perturbed log-likelihood function will be

$$\log L(\theta|w) = \sum_{i=1}^{a_1} w_i \log\left(F\left(x_{r_1},\theta_1\right)\right) + \sum_{i=a_1+1}^{r_1-1} w_i \log\left(G\left(x_{r_1},\theta_2\right)\right) + \sum_{i=1}^{k} w_{r_i} z_i \log\left(f\left(x_{r_1},\theta_1\right)\right)$$
$$+ \sum_{i=1}^{k} w_{r_i}(1-z_i) \log\left(g\left(x_{r_1},\theta_2\right)\right) + \sum_{j=2}^{k}\sum_{i=r_{j-1}+1}^{a_j+r_{j-1}} w_i \log\left[F\left(x_{r_j},\theta_1\right) - F\left(x_{r_{j-1}},\theta_1\right)\right]$$
$$+ \sum_{j=2}^{k}\sum_{i=a_j+r_{j-1}+1}^{r_j-1} w_i \log\left[G\left(x_{r_j},\theta_2\right) - G\left(x_{r_{j-1}},\theta_2\right)\right]$$
$$+ \sum_{i=r_k+1}^{a_{k+1}+r_k} w_i \log\left[1 - F\left(x_{r_k},\theta_1\right)\right] + \sum_{i=a_{k+1}+r_k+1}^{N} w_i \log\left[1 - G\left(x_{r_k},\theta_2\right)\right] \tag{13}$$

Using Eq. (2) in Eq. (13) it can be further simplified as

$$\log L(\theta|w) = \sum_{i=1}^{a_1} w_i \log\left(1 - \exp\left(-\frac{x_{r_1}}{\theta_1}\right)\right) + \sum_{i=a_1+1}^{r_1-1} w_i \log\left(1 - \exp\left(-\frac{x_{r_1}}{\theta_2}\right)\right)$$
$$- \left(\frac{1}{\theta_1}\right)\left\{\sum_{i=1}^{k} w_{r_i} z_i x_{r_i} + \sum_{j=2}^{k}\sum_{i=r_{j-1}+1}^{a_j+r_{j-1}} w_i x_{r_{j-1}} + \sum_{i=r_k+1}^{a_{k+1}+r_k} w_i x_{r_k}\right\}$$
$$- \left(\frac{1}{\theta_2}\right)\left\{\sum_{i=1}^{k} w_{r_i}(1-z_i) x_{r_i} + \sum_{j=2}^{k}\sum_{i=a_j+r_{j-1}+1}^{r_j-1} w_i x_{r_{j-1}} + \sum_{i=a_{k+1}+r_k+1}^{N} w_i x_{r_k}\right\}$$
$$- \sum_{i=1}^{k} w_{r_i} z_i \log\theta_1 + \sum_{j=2}^{k}\sum_{i=r_{j-1}+1}^{a_j+r_{j-1}} w_i \log\left(1 - \exp\left(-\frac{T_j}{\theta_1}\right)\right)$$
$$- \sum_{i=1}^{k} w_{r_i}(1-z_i) \log\theta_2 + \sum_{j=2}^{k}\sum_{i=a_j+r_{j-1}+1}^{r_j-1} w_i \log\left(1 - \exp\left(-\frac{T_j}{\theta_2}\right)\right) \tag{14}$$

If $w = w_0 = (1, 1, \ldots, 1)'$, from Eq. (14) we see that $\log L(\theta|w_0) = \log L(\theta)$. If i-th observation is deleted i.e. $w_i=0$ and if such deletion leads to very different MLE of

$\theta = (\theta_1, \theta_2)$, then it leads to influence of i-th observation. In a similar manner if a small perturbation of w_i from $w_i = 1$ leads to a very different MLE of $\theta = (\theta_1, \theta_2)$, it will be evidence of influence of i-th observation. Thus, if $\log L(\theta|w)$ becomes maximum under $\hat{\theta}_w$, then the change of $\hat{\theta}_w$, as a function of w reveals the information about how influential an individual observation is. Cook [23] proposed the displacement $D(w)$ as

$$D(w) = \log L\left(\hat{\theta}|w_0\right) - \log L\left(\hat{\theta}_w|w_0\right) \tag{15}$$

The directions giving large change of $D(w)$ at w_0 are of interest. Cook [23] proposed the following straightforward computation method of such a direction as described below:

$$\text{Let } H = \left. \frac{\partial^2 \log L(\theta)}{\partial \theta_1 \partial \theta_2} \right|(\theta_1, \theta_2) = \left(\hat{\theta}_1, \hat{\theta}_2\right) \tag{16}$$

which is a 2×2 matrix.

Define

$$A = \left(\frac{\partial^2 \log L(\theta|w)}{\partial(\theta_1, \theta_2)\partial w} \right)_{(\theta_1, \theta_2)=(\hat{\theta}_1, \hat{\theta}_2), w=w_0} \tag{17}$$

which is a $2 \times N$ matrix, $N = m + n$ is a total sample size.

The $(1, j)$th element of the matrix A is given by

$$\left. \frac{\partial^2 \log L(\theta|w)}{\partial \theta_1 \partial wj} \right| \theta_1 = \hat{\theta}_1, w = w_{0j} \tag{18}$$

and $(2, j)$th element as

$$\left. \frac{\partial^2 \log L(\theta|w)}{\partial \theta_2 \partial wj} \right| \theta_2 = \hat{\theta}_2, w = w_{0j} \tag{19}$$

Using the Eqs. (7) and (8) the matrix H can be computed at MLE of the parameters.

Then matrix Λ is defined as

$$\Lambda = -A'(H)^{-1}A \tag{20}$$

Poon and Poon [24] introduced the basic perturbation direction (pd$_i$) with the help of diagonal elements and trace value of the matrix Λ.

$$\text{pd}_i = \frac{\Lambda_{ii}}{\sqrt{\text{trace}\left(\Lambda^2\right)}}, \quad i = 1, 2, \ldots N \tag{21}$$

and suggest that the observations with large pd$_i$ values as influential. From Eqs. (16) to (17), influence measures can be computed with given density function $f(x_i, \theta)$. To identify the large values of pd$_i$, Poon and Poon [24] introduced the reference constant (c) as

$$c = \frac{2\text{trace}(\Lambda)}{N\sqrt{\text{trace}\left(\Lambda^2\right)}} \tag{22}$$

which can be used to identify the observations having large pd$_i$ values.

Bayes estimation

In this section we consider the Bayes estimates of mean life time and reliability of the products of type-A and type-B. To obtain Bayes estimate of the parameter θ of the distribution, we should decide prior distribution of θ. Prior distribution can be determined by analyst's pre-data understanding/knowledge/belief about θ. Usually the parametric form of the prior is chosen such that the posterior distribution of θ be of the same form i.e. belongs to the same family of the prior distributions. Use of such a prior is mostly for mathematical and computational convenience in practice. Usually the Bayes estimate of the parameter θ fall somewhere between the prior and likelihood estimates. Thus Bayes estimate of the parameter depends on the initial beliefs about the parameter θ.

Unfortunately, when both the parameters θ_1 and θ_2 are unknown then there does not exist any natural conjugate priors. In this article, similarly as in Kundu and Pradhan [25], we use the inverse gamma priors for the parameters θ_1 and θ_2.

The inverse gamma priors $\text{IG}(c_1, d_1)$ and $\text{IG}(c_2, d_2)$ for θ_1 and θ_2 respectively defined as

$$\pi_1(\theta_1) = \frac{e^{-d_1/\theta_1} d_1^{c_1}}{\theta_1^{c_1+1} \Gamma c_1} = \text{IG}(c_1, d_1) \text{ and } \pi_2(\theta_2) = \frac{e^{-d_2/\theta_2} d_2^{c_2}}{\theta_2^{c_2+1} \Gamma c_2} = \text{IG}(c_2, d_2), \qquad (23)$$

$$\theta_i > 0, \ a_i > 0, \ b_i > 0, \ i = 1, 2.$$

On the basis of the likelihood function in Eq. (3) and above independent inverse gamma prior distributions, the joint posterior density function of θ_1 and θ_2 can be constructed as

$$h(\theta_1, \theta_2 | \underline{x}) = L \pi_1(\theta_1) \pi_2(\theta_2)$$

$$\propto \frac{e^{-\frac{(u_k+d_1)}{\theta_1}} (u_k+d_1)^{m_k+c_1}}{\theta_1^{m_k+c_1+1} \Gamma(m_k+c_1)} \frac{e^{-\frac{(v_k+d_2)}{\theta_2}} (v_k+d_2)^{n_k+c_2}}{\theta_2^{n_k+c_2+1} \Gamma(n_k+c_2)} \frac{\left(1 - e^{-\left(\frac{x_{r_1}}{\theta_1}\right)}\right)^{a_1} \left(1 - e^{-\left(\frac{x_{r_1}}{\theta_2}\right)}\right)^{b_1}}{(u_k+d_1)^{m_k+c_1} (v_k+d_2)^{n_k+c_2}} \qquad (24)$$

$$\times \prod_{i=2}^{k} \left[1 - e^{-T_{\frac{i}{\theta_1}}}\right]^{a_i} \prod_{i=2}^{k} \left[1 - e^{-T_{\frac{i}{\theta_2}}}\right]^{b_i}$$

$$\propto \text{IG}(m_k + c_1, u_k + d_1) \text{IG}(n_k + c_2, v_k + d_2) h_3(\theta_1, \theta_2 | \underline{x}) \qquad (25)$$

where

$$h_3(\theta_1, \theta_2 | \underline{x}) = \frac{\left(1 - e^{-\left(\frac{x_{r_1}}{\theta_1}\right)}\right)^{a_1} \left(1 - e^{-\left(\frac{x_{r_1}}{\theta_2}\right)}\right)^{b_1}}{(u_k+d_1)^{m_k+c_1} (v_k+d_2)^{n_k+c_2}} \prod_{i=2}^{k} \left[1 - e^{-T_{\frac{i}{\theta_1}}}\right]^{a_i} \prod_{i=2}^{k} \left[1 - e^{-T_{\frac{i}{\theta_2}}}\right]^{b_i} \qquad (26)$$

From the expression of the posterior distribution given in (14) it is quite difficult to obtain Bayes estimates of the parameters in closed form, so we use approximation method to evaluate them. There are several approximation methods to obtain Bayes estimates of the parameters. Here, we use importance sampling method proposed by Kundu and Pradhan [25]. The importance sampling method can be used to derive estimates of parameters.

Using importance sampling approach Bayes estimates of θ_1 and θ_2 can be obtained as follows:

Step 1: Generate θ_1 from IG$(m_k + c_1, u_k + d_1)\sim$Inverse gamma$(m_k + c_1, u_k + d_1)$

Step 2: Generate θ_2 from IG$(n_k + c_2, v_k + d_2)\sim$ Inverse gamma$(n_k + c_2, v_k + d_2)$

Step 3: Repeat Steps 1 and 2 N times to obtain $(\theta_{11}, \theta_{21}), \ldots, (\theta_{1N}, \theta_{2N})$

Step 4: The Bayes estimate $\hat{\varepsilon}_B$ of $\varepsilon(\theta_1, \theta_2)$, any function of θ_1 and θ_2 under the squared-error loss function can then be approximated as

$$\hat{\varepsilon}_B = \frac{\sum_{i=1}^N \varepsilon(\theta_{1i}, \theta_{2i})h_3(\theta_{1i}, \theta_{i2}|\underline{x})}{\sum_{i=1}^N h_3(\theta_{1i}, \theta_{2i}|\underline{x})} \tag{27}$$

The Bayes estimate of θ_1 is obtained by considering $\varepsilon(\theta_1, \theta_2) = \theta_1$ in the above computation.

Similarly, the Bayes estimate of θ_2 can be computed. The Bayes estimates of the reliabilities at time t of the two types of products can be obtained by replacing the function $\varepsilon(\theta_1, \theta_2)$ by an expression of reliability function given in (11). Some further applications of this method can also be found in Kundu and Pradhan [25] and Rastogi and Tripathi [26].

Results and discussion

In this section we perform the analysis of a real data set to illustrate how the proposed methods work in practice. We have used the following data set proposed by Proschan [11]. Proschan [11] presented data on intervals between failures (in hours) of the air conditioning system of a fleet of 13 Boeing 720 jet airplanes. After analyzing the data, he observed that the failure distribution of the air-conditioning system for each of the planes was well approximated by exponential distributions. These data sets were used by Rasouli and Balakrishnan [7] and Mondal and Kundu [27].

For the purpose of illustration, we chose here the planes "7914" and "7913," and we assume the failure times of air-conditioning systems in the two planes follow exponential life time model with means $\theta_1 > 0$ and $\theta_2 > 0$ respectively. The samples of sizes $m = 24$ and $n = 27$ for plane 7914 and 7913 are considered respectively. The corresponding failure time data are presented in Table 1.

Here we call the air-conditioning systems of plane 7914 as product of type-A and of plane 7913 as product of type-B. As an example, we have constructed two jointly multiply type-II censored samples with $k = 9$ and $k = 6$ from the above data to exemplify the results obtained in the earlier sections, which are shown in the Tables 2 and 3 respectively.

Using the Eqs. (4), (5) and (9) the MLEs and their asymptotic standard errors are calculated and presented in the Table 4. The results are obtained in case of joint multiply type-II as well as for usual joint type-II censored samples.

Table 1 Failure times of air-conditioning systems in two airplanes

Plane	Failure times
7914	3, 5, 5, 13, 14, 15, 22, 22, 23, 30, 36, 39, 44, 46, 50, 72, 79, 88, 97, 102, 139, 188, 197, 210
7913	1, 4, 11, 16, 18, 18, 18, 24, 31, 39, 46, 51, 54, 63, 68, 77, 80, 82, 97, 106, 111, 141, 142, 163, 191, 206, 216

Table 2 Joint multiply type-II censoring scheme with $k=9$

i	1	2	3	4	5	6	7	8	9	
r_i	3	8	14	22	27	31	37	40	43	
X_{r_i}	4	14	22	39	51	72	97	106	141	
Z	0	1	1	0	0	1	1	0	0	
a_i	1	3	1	5	3	0	2	1	1	3
b_i	1	1	4	2	1	3	3	1	1	5
R_i	2	4	5	7	4	3	5	2	2	8
T_i	–	10	8	17	12	21	25	9	35	

Table 3 Joint multiply type-II censoring scheme with $k=6$

i	1	2	3	4	5	6	
r_i	3	8	14	22	27	31	
X_{r_i}	4	14	22	39	51	72	
Z	0	1	1	0	0	1	
a_i	1	3	1	5	3	0	8
b_i	1	1	4	2	1	3	12
R_i	2	4	5	7	4	3	20
T_i	–	10	8	17	12	21	

Table 4 MLE and ASE

Censoring scheme	*k*	MLE		ASE	
		$\hat{\theta}_1$	$\hat{\theta}_2$	ASE$(\hat{\theta}_1)$	ASE$(\hat{\theta}_2)$
Joint multiply type-II	9	59.51328	86.02658	12.08575	9.25042
	6	63.47124	88.61522	12.96159	22.89991
Joint type-II	9	54.16666	125.3333	22.11366	72.36123
	6	81.33333	93.33333	46.95782	53.88602

MLEs of reliabilities at time $t=100$ for both the types of products along with their asymptotic standard errors are calculated using the Eqs. (11) and (12) and shown in the Table 5.

Estimation of θ_1 is quite stable over different choice of k, the estimation of θ_2 seems to be quite sensitive to the joint multiply type II censoring scheme employed when k is small. This effect also observed in reliability of the products.

When two populations are not identical and consider smaller samples from them, then ordinary joint type-II censoring scheme resulting in poor inference for mean of both the populations. From Table 4, we observed that asymptotic standard errors for each of the product A and product B are smaller in case of joint multiply type-II censoring scheme than that of under joint type-II censoring scheme. Which reveals the advantage of joint multiply type-II censoring over the joint type-II censoring.

Now we calculate influence measure for the joint multiply type-II censored sample with $k=6$, with the data given in Table 3.

Table 5 Reliability and ASE

Censoring scheme	k	Reliability at $t = 100$		ASE	
		type-A	type-B	type-A	type-B
Joint multiply type-II	9	0.18632	0.31273	0.00526	0.00422
	6	0.20690	0.32352	0.00513	0.00412
Joint type-II	9	0.15784	0.45029	0.00538	0.00287
	6	0.29244	0.34252	0.00442	0.00393

In our example, we have total 51 products including both the types A and type B, so the order of matrix A will be 2×51.

The 51 elements $a_{1w_j}, j = 1, 2, \ldots, 51$ of the first row of matrix A can be calculated using Eq. (18) as

$$a_{1w_j} = \frac{-\frac{x_{r_1}}{\theta_1^2} e^{-\frac{x_{r_1}}{\theta_1}}}{1 - e^{-\frac{x_{r_1}}{\theta_1}}}, \quad \text{for } j = 1$$

$$a_{1w_{r_j}} = \frac{Z_j x_{r_j}}{\theta_1^2} - \frac{Z_j}{\theta_1}, \quad \text{for } j = 1, 2, \ldots, 6$$

$$a_{1w_j} = \frac{-\frac{T_2}{\theta_1^2} e^{-\frac{T_2}{\theta_1}}}{1 - e^{-\frac{T_2}{\theta_1}}} + \frac{x_3}{\theta_1^2}, \text{ for } j = 4, 5, 6$$

$$a_{1w_j} = \frac{-\frac{T_3}{\theta_1^2} e^{-\frac{T_3}{\theta_1}}}{1 - e^{-\frac{T_3}{\theta_1}}} + \frac{x_8}{\theta_1^2}, \quad \text{for } j = 9$$

$$a_{1w_j} = \frac{-\frac{T_4}{\theta_1^2} e^{-\frac{T_4}{\theta_1}}}{1 - e^{-\frac{T_4}{\theta_1}}} + \frac{x_{14}}{\theta_1^2}, \quad \text{for } j = 15 \text{ to } 19$$

$$a_{1w_j} = \frac{-\frac{T_5}{\theta_1^2} e^{-\frac{T_5}{\theta_1}}}{1 - e^{-\frac{T_5}{\theta_1}}} + \frac{x_{22}}{\theta_1^2}, \quad \text{for } 23 \text{ to } 25$$

$$a_{1w_j} = \frac{-\frac{T_6}{\theta_1^2} e^{-\frac{T_6}{\theta_1}}}{1 - e^{-\frac{T_6}{\theta_1}}} + \frac{x_{27}}{\theta_1^2}, \quad \text{for } 28$$

$$a_{1w_j} = \frac{x_{31}}{\theta_1^2}, \quad \text{for } j = 32 \text{ to } 39$$

and

$a_{1w_j} = 0,$ for all remaining j.

Similarly the 51 elements a_{2w_j}, $j = 1, 2, \ldots, 51$ of the second row of the matrix A are calculated using Eq. (19) by replacing θ_1 by θ_2 in the expressions of the elements of the first row of matrix A given above. Replacing the MLEs $\hat{\theta}_1$ and $\hat{\theta}_2$ in place of θ_1 and θ_2, and substituting the values in vector w_0 the numeric values of the elements of the matrix A can be obtained.

The matrix H^{-1} can be obtained from Eq. (16) at MLEs as

$$H^{-1} = \begin{bmatrix} -168.002810 & 0 \\ 0 & -524.405878 \end{bmatrix}$$

Hence the elements of matrix Λ can be calculated using Eq. (20). The basic perturbation direction values (pd_i) from Eq. (21) comes out as given in Table 6.
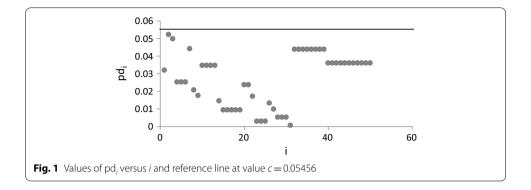
From Eq. (22) the reference constant c is calculated and comes out 0.05456.

To identify the observations which have greater effect on MLEs, we find out the pd_i values. The plot of pd_i versus i, with reference line at constant c is given in Fig. 1. From the plot we observe that no observations have pd_i values more than the reference constant c. Hence none of the observations influence MLEs in a disproportionate way.

Bayes estimates of mean life times and reliabilities of the products are presented in Tables 7 and 8 respectively based on the "Methods" section in case of $k = 9$ and 6 for different choice of prior parameters c_1, d_1, c_2 and d_2. There are many ways of determining the values of prior parameters. The values for different prior parameters can be chosen by utilizing the prior information from the past data. Here we have adopted the method used by Sultana et al. [28]. Suppose that N numbers of past data are available from the given population. Let $\hat{\theta}_{1_j}$ and $\hat{\theta}_{2_j}$, $j = 1, 2, \ldots N$ denote the corresponding MLEs of unknown parameters θ_1 and θ_2. The selection of prior parameters can be made by equating mean and variance of $\hat{\theta}_{1_j}$ and $\hat{\theta}_{2_j}$ with the corresponding mean and variance of the prior distributions.

From the above table, we observe that increase in the value of $c_1(c_2)$ Bayes estimate of $\theta_1(\theta_2)$ are sensitive and decreases in both the cases of censoring schemes, but Bayes estimate of $\theta_1(\theta_2)$ remains almost stable under changes in the value of $d_1(d_2)$. Under the

Table 6 Values of pd_i

i	pd_i	i	pd_i	i	pd_i	i	pd_i	i	pd_i
1	0.032053	11	0.034785	21	0.023740	31	0.000617	41	0.036101
2	0.052264	12	0.034785	22	0.017143	32	0.043944	42	0.036101
3	0.049860	13	0.034785	23	0.002952	33	0.043944	43	0.036101
4	0.025273	14	0.014579	24	0.002952	34	0.043944	44	0.036101
5	0.025273	15	0.009428	25	0.002952	35	0.043944	45	0.036101
6	0.025273	16	0.009428	26	0.013330	36	0.043944	46	0.036101
7	0.044246	17	0.009428	27	0.009853	37	0.043944	47	0.036101
8	0.020746	18	0.009428	28	0.005278	38	0.043944	48	0.036101
9	0.017592	19	0.009428	29	0.005278	39	0.043944	49	0.036101
10	0.034785	20	0.023740	30	0.005278	40	0.036101	50	0.036101
								51	0.036101

Fig. 1 Values of pd_i versus i and reference line at value $c = 0.05456$

Table 7 Bayes estimates of mean life times

c_1	d_1	c_2	d_2	k = 9		k = 6	
				$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\theta}_1$	$\hat{\theta}_2$
18	999	40	2861	73.7492	80.9237	65.1143	74.6591
15	999	40	2861	83.4231	81.0132	73.9114	74.6439
21	999	40	2861	66.1775	80.8727	58.2935	74.7057
18	1010	40	2861	74.1121	80.9243	65.4957	74.6591
18	990	40	2861	73.4522	80.9232	64.8022	74.6592
18	999	35	2861	73.7936	89.3611	65.0836	82.8931
18	999	45	2861	73.7049	74.0086	65.1433	67.9528
18	999	40	2840	73.7488	80.5592	65.1145	74.2678
18	999	40	2900	73.7497	81.6007	65.1139	75.3859
26	1503	88	7526	69.3600	86.0126	63.4718	83.6187
23	1503	88	7526	75.8186	86.0316	69.4822	83.5422
29	1503	88	7526	63.9656	86.0114	58.4701	83.7024
26	1450	88	7526	68.0226	86.0117	62.0660	83.6213
26	1400	88	7526	66.7609	86.0109	60.7398	83.6240
26	1525	88	7526	69.9152	86.0130	64.0554	83.6176
26	1503	91	7526	69.3591	83.6080	63.4917	81.1899
26	1503	85	7526	69.3611	88.5629	63.4513	86.1991
26	1503	88	7520	69.3600	85.9574	63.4718	83.5609
26	1503	88	7530	69.3600	86.0494	63.4718	83.6572

reduction of number of failures observed, $k = 9$ to $k = 6$, in joint multiply type-II censoring scheme Bayes estimate of both the parameters decreases. The same concussions can be seen for reliability of the product also.

Conclusion

We have developed a maximum likelihood estimation of reliability for the products are being produced by two lines under the same facility based on multiply type-II joint censoring scheme. Bayesian estimation is considered using the importance sampling method. The effect of prior parameters on Bayes estimates is examined. The influence measure to identify observations which have disproportionately influence on MLEs of the parameters is also considered.

Table 8 Bayes estimates of reliability at time $t = 100$

c_1	d_1	c_2	d_2	$k = 9$		$k = 6$	
				$\hat{R}_1(t)$	$\hat{R}_2(t)$	$\hat{R}_1(t)$	$\hat{R}_2(t)$
18	999	40	2861	0.25770	0.29062	0.21529	0.26200
15	999	40	2861	0.30158	0.29102	0.25847	0.26193
21	999	40	2861	0.22067	0.29040	0.17988	0.26222
18	1010	40	2861	0.25942	0.29062	0.21723	0.26200
18	990	40	2861	0.25629	0.29062	0.21371	0.26200
18	999	35	2861	0.25791	0.32659	0.21514	0.29928
18	999	45	2861	0.25749	0.25893	0.21544	0.22956
18	999	40	2840	0.25770	0.28900	0.21529	0.26015
18	999	40	2900	0.25771	0.29362	0.21529	0.26540
26	1503	88	7526	0.23651	0.31267	0.20690	0.30243
23	1503	88	7526	0.26742	0.31275	0.23711	0.30210
29	1503	88	7526	0.20944	0.31266	0.18082	0.30279
26	1450	88	7526	0.22990	0.31266	0.19965	0.30244
26	1400	88	7526	0.22360	0.31266	0.19275	0.30245
26	1525	88	7526	0.23924	0.31267	0.20989	0.30243
26	1503	91	7526	0.23651	0.30238	0.20701	0.29180
26	1503	85	7526	0.23652	0.32331	0.20680	0.31345
26	1503	88	7520	0.23651	0.31243	0.20690	0.30218
26	1503	88	7530	0.23651	0.31282	0.20690	0.30260

To exemplify the results obtained in the papers, two multiply type-II joint censored samples with different termination times $k = 9$ and $k = 6$ are considered from the real life example of failure times of air-conditioning systems in two airplanes 7914 and 7913.

We observe from the outcomes that maximum likelihood estimation of average failure time for plane 7914 is quite stable over different choice of k, and the estimation of average failure time for plane 7913 seems to be quite sensitive to the multiply type II censoring scheme employed when k is small.

We have seen that for small sample i.e. $k = 6$, no observations have siginifcant disproportionate infuenece on the MLEs. We also observe that increase in the value of shape parameter of prior distribution, Bayes estimate of mean failure time are sensitive and decreases in both the cases of censoring schemes, but remain almost stable under the changes in the value of scale parameter.

Declarations

Competing interests
The author declares that she has no competing interests.

References
1. Balakrishnan, N., Rasouli, A.: Exact likelihood inference for two exponential populations under joint type-II censoring. Comput. Stat. Data Anal. **52**(5), 2725–2738 (2008)
2. Balakrishnan, N., Feng, S.: Exact likelihood inference for *k* exponential populations under joint type-II censoring. Commun. Stat.-Simul. Comput. **44**, 591–613 (2014)
3. Shafay, A.R., Balakrishnan, N., Abdel-Aty, Y.: Bayesian inference based on a jointly type-II censored sample from two exponential populations. Commun. Stat.-Simul. Comput. **43**, 1–14 (2013)
4. Ashour, S.K., Abo-Kasem, O.E.: Bayesian and non-Bayesian estimation for two generalized exponential populations under joint type II censored scheme. Pak. J. Stat. Oper. Res. **10**(1), 57–72 (2014)
5. Ashour, S.K., Abo-Kasem, O.E.: Parameter estimation for two Weibull populations under joint type II censored scheme. Int. J. Eng. Appl. Sci. **4**(5), 31–36 (2014)
6. Parsi, S., Bairamov, I.: Expected values of the number of failures for two populations under joint type-II progressive censoring. Comput. Stat. Data Anal. **53**, 3560–3570 (2009)
7. Rasouli, A., Balakrishnan, N.: Exact likelihood inference for two exponential populations under joint progressive type-II censoring. Commun. Stat.-Theory Methods **39**(12), 2172–2191 (2010)
8. Balakrishnan, N., Su, F., Liu, K.Y.: Exact likelihood inference for k exponential populations under joint progressive type-II censoring. Commun. Stat.-Simul. Comput. **44**(4), 902–923 (2015)
9. Nadi, A.A., Gildeh, B.S.: Estimating the lifetime performance index of products for two-parameter exponential distribution with the progressive first-failure censored sample. Int. J. Qual. Res. **10**(2), 389–406 (2016)
10. Mondal, S., Kundu, D.: Inference on Weibull Parameters Under a Balanced Two Sample Type-II Progressive Censoring Scheme. https://arxiv.org/abs/1801.00434, stat. AP. (2018)
11. Proschan, F.: Theoretical explanation of observed decreasing failure rate. Technometrics **5**(3), 375–383 (1963)
12. Balakrishnan, N.: On the maximum likelihood estimation of the location and scale parameters of exponential distribution based on multiply type-II censored samples. J. Appl. Stat. **17**, 55–61 (1990)
13. Balakrishnan, N., Gupta, S.S., Panchapakesan, S.: Estimation of the location and scale parameters of the extreme value distribution based on multiply type-II censored samples. Commun. Stat. Theory Methods **24**(8), 2105–2125 (1995)
14. Shah, J.B., Patel, M.N.: Bayes estimation under asymmetric loss functions multiply type II censored Rayleigh data. J. Indian Stat. Assoc. **46**(1), 67–78 (2008)
15. Kim, C., Han, K.: Estimation of the scale parameter of the Rayleigh distribution with multiply type-II censored sample. J. Stat. Comput. Simul. **79**(8), 965–976 (2009)
16. Upadhyay, S.K., Singh, U., Shastri, V.: Estimation of exponential parameters under multiply type II censoring. Commun. Stat.-Simul. Comput. **25**(3), 801–815 (1996)
17. Kong, F.: Parameter estimation under multiply type-II censoring. Handb. Stat. **17**, 315–335 (1998)
18. Shah, J.B., Patel, M.N.: Bayes estimation of a two-parameter geometric distribution under multiply type II censoring. Int. J. Qual. Stat. Reliab. (2011). https://doi.org/10.1155/2011/618347
19. Kang, S.B., Cho, Y.S., Han, J.T., Kim, J.: An estimation of the entropy for a double exponential distribution based on multiply type-II censored samples. Entropy **14**, 161–173 (2012)
20. Patel, D.T., Patel, M.N.: A Bayesian approach to optimal warranty length for exponential life time distributed product with multiply type-II censoring scheme. Int. J. Math. Stat. **18**(3), 64–81 (2017)
21. Shafay, A., Sultan, K.: Bayesian inference based on multiply type-II censored samples of sequential order statistics from pareto distribution. J. Test. Eval. (2020). https://doi.org/10.1520/JTE20170699
22. Poon, W.Y., Tang, M.L.: Influence measure in maximum likelihood estimate for models of lifetime data. J. Appl. Stat. **28**(6), 737–742 (2001)
23. Cook, R.D.: Assessment of local influence (with discussion). J. R. Stat. Soc. B. **48**, 133–169 (1986)
24. Poon, W.Y., Poon, Y.S.: Conformal normal curvature and assessment of local influence. J. R. Stat. Soc. B. **61**, 51–61 (1999)
25. Kundu, D., Pradhan, B.: Estimating the parameters of the generalized exponential distribution in presence of hybrid censoring. Commun. Stat. Theor. Methods **38**, 2030–2041 (2009)
26. Rastogi, M.K., Tripathi, Y.M.: Inference on unknown parameters of a Burr distribution under hybrid censoring. Stat. Pap. **54**, 619–643 (2013)
27. Mondal, S., Kundu, D.: A new two sample type-II progressive censoring scheme. Commun. Stat. Theor. Methods **48**(10), 2602–2618 (2019)
28. Sultana, F., Tripathi, Y.M., Rastogi, M.K., Wu, S.J.: Parameter estimation for the kumaraswamy distribution based on hybrid censoring. Am. J. Math. Manag. Sci. **37**(3), 243–261 (2018)

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