Journal of the Egyptian Mathematical Society (2016) 24, 479–486

Egyptian Mathematical Society

Journal of the Egyptian Mathematical Society

www.etms-eg.org
www.elsevier.com/locate/joems

Original Article

Dual solutions in hydromagnetic stagnation point flow and heat transfer towards a stretching/shrinking sheet with non-uniform heat source/sink and variable surface heat flux

Mohamed Abd El-Aziz [a,b,*]

[a] *Helwan University, Faculty of Science, Mathematics Department, P.O. Box, 11795 Cairo, Egypt*
[b] *King Khalid University, Faculty of Science, Mathematics Department, Abha 9004, Saudi Arabia*

Received 30 July 2014; revised 27 June 2015; accepted 20 September 2015
Available online 18 November 2015

Keywords

Dual solutions;
Stagnation point flow;
Shrinking/stretching sheet;
Magnetic field;
Heat source/sink

Abstract The steady stagnation-point flow and heat transfer of a viscous, incompressible and heat generating/absorbing fluid over a shrinking sheet in the presence of a non-uniform heat source/sink is considered. The system of partial differential equations was transformed to a system of ordinary differential equations, which was solved numerically. Numerical results were obtained for the skin friction coefficient, the surface temperature as well as the velocity and temperature profiles for some values of the governing parameters. The study reveals that the range of velocity ratio parameter for which the solution exists increases as the magnetic field increase.

Math Subject Classification: 76N20-76R05-80A20

1. Introduction

Investigation on boundary-layer flow and heat transfer in a quiescent fluid driven by a continuous stretching sheet have been extensively investigated during the past decades owing to its importance in industrial and engineering applications. Examples are heat treatment of materials manufactured in an extrusion process and a casting process of materials. Cooling of stretching sheets is needed to assure the best quality of the material and requires dedicated control of the temperature and, therefore, knowledge of flow and heat transfer in such systems. Motivated by the process of polymer extrusion, in which the

* Corresponding author at: Helwan University Faculty of Science, Mathematics Department -Helwan University P.O.Box, 11795 Helwan-Cairo Egypt. Tel: +20223715874.
E-mail address: m_abdelaziz999@yahoo.com
Peer review under responsibility of Egyptian Mathematical Society.

Production and hosting by Elsevier

S1110-256X(15)00079-6
http://dx.doi.org/10.1016/j.joems.2015.09.004

Nomenclature

a, b, c	prescribed constants
A^*	space-dependent heat source/sink parameter
B^*	temperature-dependent heat source/sink parameter
B_0	magnetic induction
C_{fx}	local friction coefficient
c_p	specific heat at constant pressure
f	dimensionless stream velocity
k	thermal conductivity
M	magnetic parameter
Nu_x	local Nusselt number
Pr	Prandtl number
q'''	non-uniform heat source/sink
q_w	surface heat flux
Re_x	local Reynolds number
T	fluid temperature
T_∞	ambient temperature
u	velocity of the fluid in the x-direction
u_e	velocity of the ambient fluid
u_w	velocity of the stretching/shrinking sheet
v	velocity of the fluid in the y-direction
x, y	axial and normal coordinates

Greek symbols

η	similarity variable
θ	dimensionless temperature
λ	velocity ratio parameter
λ_c	critical value of velocity ratio parameter
μ	dynamic viscosity
ψ	stream function
ν	kinematic viscosity
ρ	fluid density
σ	electric conductivity

extrudate emerges from a narrow slit, Crane [1] was the first to give a similarity solution in a closed analytical form for the two-dimensional flow caused by a stretching plate. Subsequently; various aspects of the flow and heat transfer over a stretching surface have been examined by several investigators [2–7]. It is known that the properties of the final product depend greatly on the rate of cooling involved in manufacturing processes. It would be beneficial to have a controlled cooling system for these processes. An electrically conducting and heat generating/absorbing fluids seems to be a good candidate for some industrial applications such as in polymer technology and metallurgy because the flow can be regulated by external means through a magnetic field. The applied magnetic field may play an important role in controlling momentum and heat transfers in the boundary layer flow of different fluids over a stretching sheet. The use of magnetic fields has been also used in the process of purification of molten metals from non-metallic inclusions. Many works have been reported on flow and heat transfer of electrically conducting fluids over a stretched surface in the presence of magnetic field (see for instance, Chakrabarti and Gupta [8], Andersson [9], Chiam [10], Mahmoud [11], Abo-Eldahab and Abd El-Aziz [12,13], Abd El-Aziz and Salem [14] and Abd El-Aziz [15,16]). In recent times, the problem of flow and heat transfer over a shrinking sheet is relatively a new consideration in the laminar boundary layer flow. The surface velocity on the boundary towards a fixed point is known as a shrinking phenomenon. Shrinking sheet is a surface which decreases in size to a certain area due to an imposed suction or external heat. The flow induced by shrinking sheet exhibits quite distinct physical phenomena from the forward stretching flow. A search on the literature about this flow showed a few publications on the subject since it is quite a new type of flow. A steady boundary layer flow over a shrinking sheet is not possible as the vorticity generated in this case is not confined within the boundary layer. To maintain boundary layer structure the flow needs a certain amount of external suction at the porous sheet. Wang [17] first brought in the concept of the flow developed due to shrinking sheet while studying the behavior of liquid film on an unsteady stretching sheet. The existence and uniqueness of the similarity solution of the equation for the flow due to a shrinking sheet with suction were established by Miklavčič and Wang [18]. The flow induced by a shrinking sheet with constant or power-law velocity distribution was investigated recently by Fang [19] and Fang et al. [20].Wang [21] studied the stagnation flow towards a shrinking sheet and found that solutions do not exist for larger shrinking rates and may be non-unique in the two-dimensional case. The flow over an unsteady shrinking sheet was studied by Fang et al. [22] and the solution is an exact solution of the unsteady Navier–Stokes equations. Yacob et al. [23] investigated the heat transfer characteristics occurring during the melting process due to a stretching/shrinking sheet in a micropolar fluid. Bhattacharyya [24] analyzed the effects of partial slip on steady boundary layer stagnation-point flow of an incompressible fluid and heat transfer towards a shrinking sheet. The problem of a steady mixed convection flow on a moving plate in nanofluids has been investigated numerically by Subhashini and Sumathi [25]. Roşca and Pop [26] analyzed the problem of unsteady viscous flow over a curved stretching/shrinking surface with mass suction. The study of the boundary layer magnetohydrodynamic (MHD) flow towards a shrinking sheet has gained considerable attention of many researchers because of its frequent occurrence in industrial technology, geothermal applications, and high temperature plasmas applicable to nuclear fusion energy conversion, liquid metal fluids and MHD power generation systems. Shrinking problem can also be applied to study the capillary effects in smaller pores, the shrink-well behavior and the hydraulic properties of agricultural clay soils since associated changes in hydraulic and mechanical properties of such soils will seriously hamper predictions of the flow and transport processes which are essential for agricultural development and environmental management strategies. Also, shrinking film is one of the common applications of shrinking problems in industries. The shrinking film is very useful in packaging of bulk products since it can be unwrapped easily with adequate heat. Muhaimina et al. [27] studied the effect of chemical reaction, heat and mass transfer on nonlinear boundary layer past a porous shrinking sheet in the presence of suction. A series solution of three-dimensional MHD and rotating flow over a porous shrinking sheet was obtained by Hayat et al. [28]

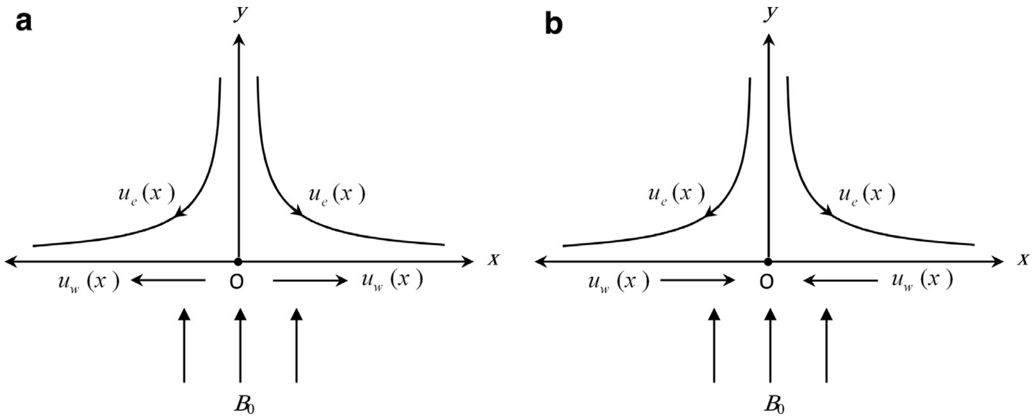

Fig. 1 A sketch of the physical model.

using HAM method. Fang and Zhang [29] obtained a closed-form analytical solution for steady MHD flow over a porous shrinking sheet subjected to applied suction. Recently, Noor et al. [30] obtained a series solution of MHD viscous flow due to a shrinking sheet by applying Adomian decomposition method (ADM). Recently, the boundary layer stagnation-point flow of Carreau fluid model toward a Shrinking sheet with MHD is investigated by Akbara et al. [31]. Very recently, Abbas et al. [32] studied the effect of homogeneous-heterogeneous reactions on an electrically conducting viscous fluid near the stagnation-point past a permeable stretching/shrinking sheet with uniform suction and generalized slip condition. However, the effects of internal heat generation/absorption on the flow and heat transfer towards a shrinking sheet were not analyzed in the previous investigations [17–32]. When there is an appreciable difference between the surface and the ambient fluid, one need to consider the temperature dependent heat source or sink which may exert strong influence on the heat transfer characteristics. Also, the study of heat generation or absorption in moving fluids is important in view of several physical problems such fluids undergoing exothermic or endothermic chemical reactions dealing with chemical reactions and those concerned with dissociating fluids. Heat generation effects may alter the temperature distribution; consequently, the particle deposition rate in nuclear reactors, electronic chips and semiconductor wafers. Although exact modeling of internal heat generation or absorption is quite difficult, some simple mathematical models can express its average behavior for most physical situations. Heat generation or absorption has been assumed to be constant, space-dependent or temperature-dependent (see for instance, Crepeau and Clarksean [33], Abo-Eldahab and Abd El-Aziz [12,13] and Salem and Abd El-Aziz [34]).

Motivated by the above investigations and applications, we intend to investigate the influence of a uniform magnetic field and non-uniform internal heat source/sink on the behavior of the stagnation-point flow and thermal transport of an electrically conducting heat generating/absorbing fluid over a shrinking sheet. The governing partial differential equations are converted into ordinary differential equations by similarity transformation, before being solved numerically using the shooting method. Effects of different governing parameters on the momentum and heat transfer characteristics under prescribed surface heat flux (PST) boundary conditions are explored and discussed in detail.

2. Mathematical formulation

We consider the steady two-dimensional stagnation-point flow of an incompressible, electrically conducting heat generating/absorbing fluid towards a horizontal linearly stretching/shrinking sheet in its own plane with a velocity proportional to the distance from the stagnation-point. The x-axis being along the stretching/shrinking sheet and the y-axis is normal to it. A uniform magnetic field of strength B_0 is assumed to be applied in the positive y-direction (see Fig. 1). The magnetic Reynolds number of the flow is taken to be small enough so that the induced magnetic field is negligible. It is assumed that the velocity of the flow outside the boundary layer is $u_e(x) = a\,x$ and the velocity of the stretching/shrinking sheet is $u_w(x) = c\,x$, where a is a positive constant, while c is a positive (stretching sheet) or a negative (shrinking sheet) constant. It is also assumed that the ambient fluid temperature is maintained at a uniform temperature T_∞ and the sheet is subjected to a heat flux of the form $q_w(x) = b\,x$, where b is constant. In addition, there is no applied electric field and the Hall effect, Joule heating and viscous dissipation are all neglected in this work. With the usual boundary layer approximations the problem is governed by the following equations

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e\frac{du_e}{dx} + \nu\frac{\partial^2 u}{\partial y^2} + \frac{\sigma B_0^2}{\rho}(u - u_e) \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} + \frac{1}{\rho c_p}q''' \tag{3}$$

The appropriate boundary conditions for the velocity components and temperature are given by

$$\begin{aligned} &u = u_w(x) = c\,x, \quad v = 0, \quad k\left(\frac{\partial T}{\partial y}\right) = -q_w(x) = -bx \text{ at } y = 0 \\ &u \to u_e(x) = a\,x, \quad T \to T_\infty \quad \text{as } y \to \infty \end{aligned} \tag{4}$$

The non-uniform heat source/sink q''' for the prescribed surface heat flux (PST) case is modeled as follows

$$q''' = \frac{k u_e(x)}{\nu x}\left[A^*\left(\frac{q_w}{k}\right)\left(\frac{\nu x}{u_e}\right)^{1/2}\exp\left(-y\sqrt{\frac{u_e}{\nu x}}\right) + B^*(T - T_\infty)\right], \tag{5}$$

where A^* and B^*, are the coefficients of space and temperature-dependent heat source/sink, respectively. It is to be noted that the case $A^* > 0$ and $B^* > 0$ corresponds to internal heat source and that $A^* < 0$ and $B^* < 0$ corresponds to internal heat sink.

To obtain similarity solutions for the system of Eqs. (1)–(3), we introduce the following similarity variables

$$\eta = \sqrt{\frac{u_e}{\nu x}}\, y, \qquad f(\eta) = \frac{\psi}{\sqrt{\nu\, u_e x}}, \qquad T = T_\infty + \frac{q_w}{k}\sqrt{\frac{\nu\, x}{u_e}}\,\theta(\eta) \tag{6}$$

where ψ is the stream function defined in the usual way as $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$, which automatically satisfies the continuity Eq. (1). The velocity components are readily obtained as:

$$u = a\,x\,f'(\eta), \;\; v = -\sqrt{\nu\, a}\,f(\eta) \tag{7}$$

Substituting Eq. (6) into Eqs. (2) and (3) give the following non-linear ordinary differential equations:

$$f''' + f f'' - f'^2 + M\left(1 - f'\right) + 1 = 0 \tag{8}$$

$$\frac{1}{\mathrm{Pr}}\theta'' + f\theta' - f'\theta + \frac{1}{\mathrm{Pr}}\left(A^* e^{-\eta} + B^*\theta\right) = 0 \tag{9}$$

and

$$f(0) = 0, \qquad f'(0) = \lambda, \qquad \theta'(0) = -1 \tag{10}$$

$$f'(\infty) \to 1, \qquad \theta(\infty) \to 0 \tag{11}$$

where prime denotes ordinary differentiation with respect toη, $M = \sigma B_0^2/\rho\, a$ is the magnetic field parameter, $\mathrm{Pr} = \mu\, c_p/k$ is the Prandtl number and $\lambda = c/a$ is the stretching parameter when $\lambda > 0$ and shrinking parameter when $\lambda < 0$ while, $\lambda = 0$ is the planar stagnation flow towards a stationary sheet. Moreover, $\lambda = 1$ ($u_e = u_w$) corresponds to the flow with no boundary layer.

From the engineering point of view, the most important characteristics of the flow are the local skin-friction coefficient C_{fx} and the local Nusselt Nu_x which are, respectively, defined by

$$C_{fx} = \frac{2\mu(\partial u/\partial y)_{y=0}}{\rho\, u_w^2} = 2\mathrm{Re}_x^{-1/2} f''(0) \text{ and}$$
$$Nu_x = -\frac{x}{T_w - T_\infty}\left(\frac{\partial T}{\partial y}\right)_{y=0} = \frac{\mathrm{Re}_x^{1/2}}{\theta(0)} \tag{12}$$

3. Numerical solution

The set of non-linear differential Eqs. (8) and (9) subject to the boundary conditions (10) and (11) constitute a two-point boundary value problem. In order to solve these equations numerically we follow most efficient numerical shooting technique with fifth-order Runge–Kutta–Fehlberg integration scheme. In this method it is most important to choose the appropriate finite values of $\eta \to \infty$. To select η_∞ we begin with some initial guess value and solve the problem with some particular set of parameters to obtain $f''(0)$ and $\theta(0)$. The solution process is repeated with another large value of η_∞ until two successive values of $f''(0)$ and $\theta(0)$ differ only after desired digit signifying the limit of the boundary along η. The last value of η_∞ is chosen as appropriate value of the limit $\eta \to \infty$ for that particular set of parameters. The two ordinary differential Eqs. (8) and (9) were first formulated as a set of five first-order simultaneous equations of five unknowns following the method of superposition [35]. Thus, we set $y_1 = f, y_2 = f', y_3 = f'', y_4 = \theta, y_5 = \theta'$, Eqs. (8) and (9) then reduced into a system of ordinary differential equations:

$$\begin{aligned}
&y'_1 = y_2, \quad y_1(0) = 0, \quad y'_2 = y_3, \quad y_2(0) = \lambda,\\
&y'_3 = y_2^2 - y_1 y_3 - M(1 - y_2) - 1, \qquad y_3(0) = \delta_1,\\
&y'_4 = y_5, \quad y_4(0) = \delta_2, \quad y'_5 = \mathrm{Pr}\,(y_2 y_4 - y_1 y_5) - A^* e^{-\eta} - B^* y_4,\\
&y_5(0) = -1,
\end{aligned}$$

where δ_1 and δ_2 are determined such that it satisfies $y_2(\infty) = 1$ and $y_4(\infty) = 0$. The shooting method is used to guess δ_1 and δ_2 until the boundary conditions $y_2(\infty) = 1$ and $y_4(\infty) = 0$ are satisfied. Then the resulting differential equations can be integrated by fifth-order Runge–Kutta–Fehlberg integration scheme. The above procedure is repeated until we get the results up to the desired degree of accuracy 10^{-6}.

4. Results and discussion

For validation of the numerical method used in this study, results for the skin friction coefficient $f''(0)$ were compared with those reported by Wang [21], Bhattacharyya [36] and Bachok et al. [37] for hydrodynamic ($M = 0$) stagnation point flow toward a shrinking/stretching sheet in the absence of heat source/sink ($A^* = B^* = 0$). The quantitative comparison is shown in Tables 1–3 and it is found to be in excellent agreement.

Table 1 Comparison of values of the skin friction coefficient $f''(0)$ for M with previously published data.

λ	Present study		Wang [21]	
	First solution	Second solution	First solution	Second solution
−0.25	1.4022408		1.40224	
−0.50	1.4956698		1.49567	
−0.75	1.4892983		1.48930	
−1.00	1.3288169	0	1.32882	0
−1.10	1.1866804	0.0492289		
−1.15	1.0822314	0.1167021	1.08223	0.116702
−1.20	0.9324740	0.2336497		
−1.2465	0.5842915	0.5542856	0.55430	
−1.24657	0.5745290	0.5542954		
−1.2465798	0.5692646			

Table 2 Comparison of values of the skin friction coefficient $f''(0)$ for M with previously published data.

λ	Present study		K. Bhattacharyya [36]	
	First solution	Second solution	First solution	Second solution
−0.25	1.4022408		1.40224	
−0.50	1.4956698		1.49567	
−0.75	1.4892983		1.48930	
−1.00	1.3288169	0	1.32882	0
−1.10	1.1866804	0.0492289		
−1.15	1.0822314	0.1167021	1.08223	0.116702
−1.20	0.9324740	0.2336497		
−1.2465	0.5842915	0.5542856	0.55430	
−1.24657	0.5745290	0.5542954		
−1.2465798	0.5692646			

Table 3 Comparison of values of the skin friction coefficient $f''(0)$ for M with previously published data.

λ	Present study		N. Bachok et al. [37]	
	First solution	Second solution	First solution	Second solution
−0.25	1.4022408		1.4022408	
−0.50	1.4956698		1.4956698	
−0.75	1.4892983		1.4892983	
−1.00	1.3288169	0	1.3288170	0
−1.10	1.1866804	0.0492289	1.1866805	0.0492290
−1.15	1.0822314	0.1167021	1.0822315	0.1167022
−1.20	0.9324740	0.2336497	0.9324739	0.2336497
−1.2465	0.5842915	0.5542856	0.5842956	0.5542825
−1.24657	0.5745290	0.5542954	0.5639733	
−1.2465798	0.5692646			

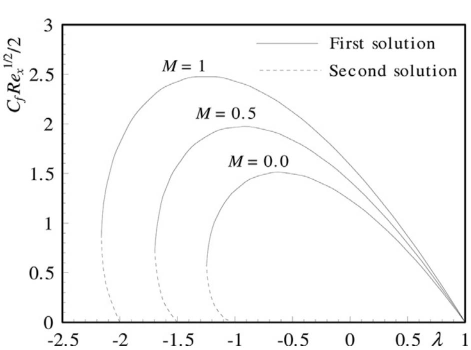

Fig. 2 Skin friction coefficient of $f''(0)$ with λ for several values of M when $A^* = B^* = 0.1$ and Pr = 0.72.

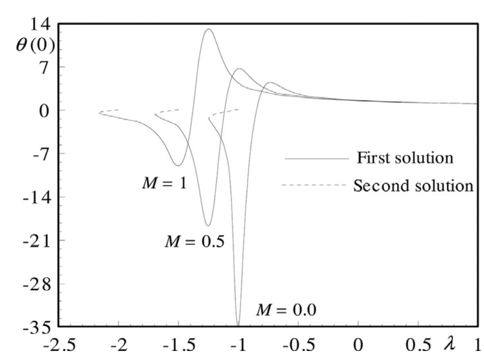

Fig. 3 Surface temperature θ (0) with λ for several values of M when $A^* = B^* = 0.1$ and Pr = 0.72.

Fig. 2 displays the variation of the skin friction coefficient in terms of $f''(0)$ with the velocity ratio parameter λ for Pr = 0.72 and various values of the magnetic field parameter. The results in Fig. 2 show that the existence and uniqueness of solution depend on the velocity ratio parameter λ as reported by Wang [21] who studied the hydrodynamic stagnation-point flow towards a shrinking sheet in the absence of magnetic field and heat source/sink. According to our computations in this problem, it is clear from Fig. 2 that there are regions of unique solution ($\lambda > -1$ when $M = 0$, $\lambda > -1.5$ when $M = 0.5$ and $\lambda > -2$ when $M = 1$), dual solutions ($-1.24658 \le \lambda \le -1$ when $M = 0$, $-1.69568 \le \lambda \le -1.5$ when $M = 0.5$ and $-2.15995 \le \lambda \le -2$ when $M = 1$) and no solutions for $\lambda > \lambda_c$ where $\lambda_c = -1.24658$, -1.69568 and -2.15995 for $M = 0$, 0.5 and 1, respectively. Accordingly, the solutions exist up to the critical value $\lambda = \lambda_c < 0$ beyond which the boundary layer separates from the surface and the solution based upon the boundary layer approximations are not possible. Further, it can be observed that the magnitude of the critical values λ_c ($\lambda_c < 0$) of the velocity ratio parameter λ for which the solution exists increases as the magnetic parameter M increases, i.e. introducing the magnetic effect enlarges the range of λ for which the solution exists.

Fig. 3 shows that the dual nature still stands in the case of thermal field for the same values of λ stated before. Further, for

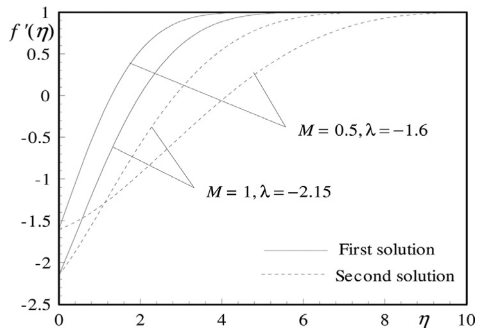

Fig. 4 Velocity profiles for various values of M and λ when $A^*= B^*= 0.1$ and $\Pr = 0.72$.

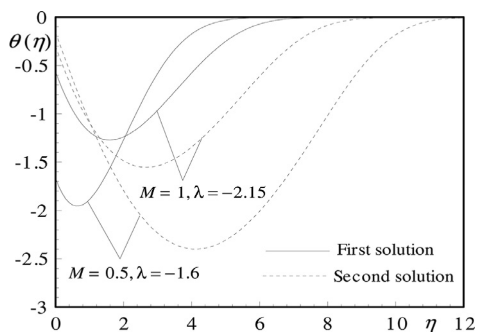

Fig. 5 Temperature profiles for various values of M and λ when $A^*= B^*= 0.1$ and $\Pr = 0.72$.

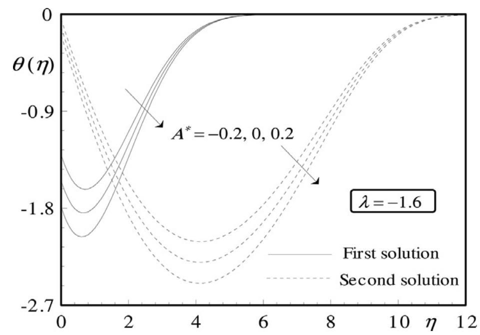

Fig. 6 Temperature profiles for various values of A^* when $M = 0.5$, $B^*= 0.1$ and $\Pr = 0.72$.

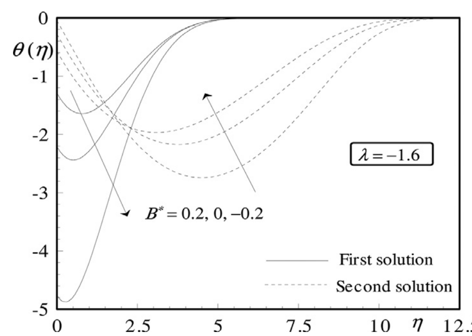

Fig. 7 Temperature profiles for various values of B^* when $M = 0.5$, $A^*= 0.1$ and $\Pr = 0.72$.

the stretching case ($\lambda > 0$) Fig. 3 shows that the effect of magnetic field M on the wall temperature θ (0) is insensible due to the fact that Eqs. (8) and (9) are uncoupled and the thermal field is not affected by the flow field while for the shrinking case ($\lambda < 0$), an interesting result occurs that is the magnetic field M has a strong effect on the wall temperature θ (0) in spite of the flow and thermal fields are uncoupled. From this result we can conclude for the shrinking case that the parameters of the flow field can affect the thermal field even though the flow and thermal fields are uncoupled. On the other hand, for given M as λ decreases the wall temperature θ (0) for the first solution increases monotonically, reaching a maximum value, then decreases rapidly to a minimum value and then increases quickly being equal $\theta_c(0)$ where $\theta_c(0)$ is the value of wall temperature corresponding to λ_c while for the second solution the wall temperature increases slowly with λ from $\lambda = \lambda_c$ reaching a maximum value when $\lambda \to \lambda_0{}^-$ where $\lambda_0 = -1$, -1.5 and -2 for $M = 0$, 0.5 and 1, respectively.

Figs. 4 and 5, respectively show the effect of magnetic parameter M on the velocity and temperature characteristics inside the boundary layer for two shrinking cases ($\lambda = -1.6$ and -2.15) when $A^*=B^*= 0.1$ and $\Pr = 0.72$. For the first solution, it is observed from these figures that the momentum and the thermal boundary layer thicknesses increase with increasing M values, while the opposite behavior is noticed for the second solution. Further as stated before the magnetic field has a remarkable effect on the fluid temperature $\theta(\eta)$ in the case of shrinking sheet.

The effect of space-dependent and temperature-dependent internal heat source/sink parameter A^* and B^* on the temperature profile is illustrated in Figs. 6 and 7, respectively for the shrinking case ($\lambda = -1.6$) when $M = 0.5$ and $\Pr = 0.72$. For both the first and second solutions, the profiles in Fig. 5 reveals that the dimensionless fluid temperature $\theta(\eta)$ is increased (in absolute sense) with a positive A^*(heat source) while it is decreased (in absolute sense) with a negative A^*(heat sink) as compared to the case of no heat source/sink ($A^*= 0$). This means that the heat transfer rate from the surface to the fluid is reduced with positive A^* values but increases with negative A^* values. Also, the thermal boundary layer thickness for the first solutions is smaller than that of the second solutions. Further, Fig. 5 shows that the maximum effect of A^* for the first solutions occurs in the vicinity of the sheet surface while that of the second solutions occurs inside the boundary layer at a distance somewhat away from the sheet. Fig. 6 shows for the first solutions that the effect of temperature-dependent heat source/sink parameter B^* on the dimensionless fluid temperature $\theta(\eta)$ in the first solutions is opposite to that of space-dependent heat source/sink parameter A^* but with markedly increasing magnitude, namely the temperature decreases (in absolute sense) with a positive B^*(heat source) but increases (in absolute sense) with a negative B^*(heat sink). On the other hand, for the second solutions the temperature $\theta(\eta)$ decreases with a positive B^*(heat source) and increases with a negative B^*(heat sink) near the sheet where $0 \le \eta < \eta_0 \cong 2$ whereas for $\eta > \eta_0$, the opposite trend is noticed before it dies out at the end of the boundary layer.

Figs 8 and 9 depict the variation of the wall temperature $\theta(\eta)$with λ for various values of A^* and B^*, respectively when $M = 0.5$ and $\Pr = 0.72$. It is observed for both the first and second solutions that the magnitude of wall temperature $|\theta(0)|$ increases with positive values of A^* (heat source) but decreases with negative values of A^*(heat sink). Also, Fig. 8 shows for the first solution and for all values of A^* that the wall temperature

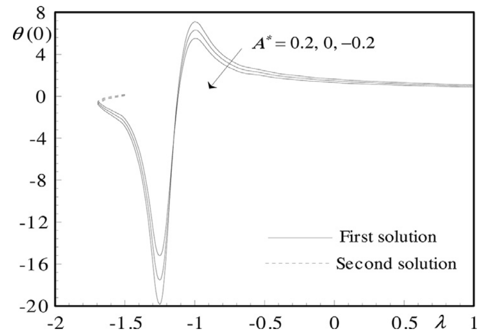

Fig. 8 Surface temperature θ (0) with λ for several values of A^* when $M = 0.5$, $B^*= 0.1$ and $\Pr = 0.72$.

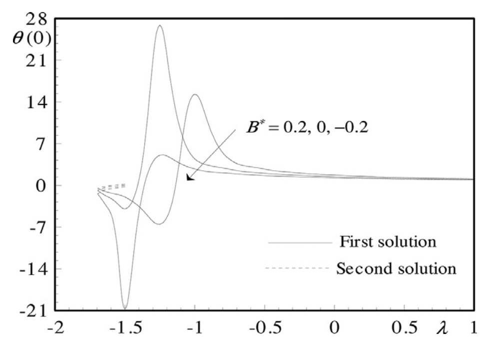

Fig. 9 Surface temperature θ (0) with λ for several values of B^* when $M = 0.5$, A^*=0.1 and $\Pr = 0.72$.

$\theta(\eta)$ increases gradually with λ reaching a maximum profile for $\lambda = -1$, then decreases greatly reaching a minimum profile for $\lambda = -1.25$ and then increases again for $\lambda < -1.25$ being equal $\theta_c(0)$ ($\theta_c(0) = -0.748212$, -0.619206 and -0.490202 for $A = 0.2$, 0.0 and -0.2, respectively) when $\lambda = \lambda_c = -1.69568$. On the other hand, for the second solution the wall temperature increases gradually with λ from $\lambda = \lambda_c$ approaching a maximum value $\theta_{\max}(0)$ ($\theta_{\max}(0) = 0.0545322$, 0.10939 and 0.164248 for $A = 0.2$, 0.0 and -0.2, respectively) when $\lambda \to \lambda_0$ where $\lambda_0 = -1.5$. From Fig. 9 it is observed that the effect of B^* on the wall temperature $\theta(0)$ is similar to that of A^* but with $\theta_c(0) = -0.52623$, -0.873807 and -1.43176 for the first solution and $\theta_{\max}(0) = 0.0545322$, 0.10939 and 0.164248 for the second solution. From Figs. 8 and 9 it is interesting to note that the change in the values of the wall temperature $\theta(0)$ from positive to negative indicates that the heat transfer process is reversed. i.e., the heat flow directed from the shrinking wall surface to the ambient fluid (positive values of $\theta(0)$) and then reversed from the ambient fluid to the shrinking wall surface (negative values of $\theta(0)$).

Finally, it is worth mentioning here that both the profiles of the first and second solution in Figs. 4–7 satisfy the far field boundary conditions asymptotically, thus support the validity of the numerical results obtained, besides supporting the dual nature of the solutions presented in Figs. 2 and 3 .

5. Conclusion

In this paper, the problem of a steady MHD stagnation point flow towards a stretching/shrinking sheet with a prescribed surface heat flux immersed in a viscous, electrically conducting and heat generating/absorbing fluid in the presence of a uniform transverse magnetic field and non-uniform heat source/sink was studied numerically. The governing partial differential equations were first transformed into a system of ordinary differential equations using a similarity transformation, before being solved numerically by the Runge–Kutta–Fehlberg method with shooting technique. The internal heat generation/absorption term is modeled according to the prescribed surface heat flux (PHF) case. The effects of the velocity ratio parameter λ, the magnetic field parameter M and the coefficient of space and temperature-dependent heat source/sink A^* and B^*, respectively on the fluid flow and heat transfer characteristics were obtained and discussed.

As a summary, we can conclude that:

1. For the shrinking case, the magnetic field has a marked effect on the temperature and heat transfer even though the flow and thermal fields are uncoupled.
2. For the stretching case ($\lambda > 0$), the solutions for velocity and temperature distributions are unique and exist for all values of λ, whereas the dual solutions were found to exist only for a certain range of $\lambda < 0$, up to a certain critical value $\lambda_c < 0$ (depends on the values of M) for the shrinking case.
3. Increasing the values of the magnetic field is to increase the range of λ for which the solution exists.
4. The momentum and thermal boundary layer thickness for first solutions are always thinner than that of the second solution.
5. For both the first and second solutions, the temperature-dependent heat source/sink parameter B^* demonstrates a more significant influence on the surface temperature than that of space-dependent heat source/sink parameter B^*.

Acknowledgments

The author is very thankful to the reviewers for their encouraging comments and constructive suggestions to improve the presentation of this manuscript.

References

[1] L.J. Crane, Flow past a stretching plane, Z. Angew. Math. Phys. 21 (1970) 645–647.

[2] C.-K. Chen, M.-I. Char, Heat transfer of a continuous stretching surface with suction or blowing, J. Math. Anal. Appl. 135 (1988) 568–580.

[3] P.S. Datti, K.V. Prasad, M.S. Abel, A. Joshi, MHD viscoelastic fluid over a non-isothermal stretching sheet, Int. J. Eng. Sci. 42 (2004) 935–946.

[4] A. Ishak, R. Nazar, I. Pop, Mixed convection stagnation point flow of a micropolar fluid towards a stretching sheet, Meccanica 43 (2008) 411–418.

[5] M. Abd El-Aziz, Radiation effect on the flow and heat transfer over an unsteady stretching sheet, Int. Commun. Heat Mass Transfer 36 (2009) 521–524.

[6] T. Hayat, Z. Abbas, M. Sajid, Stagnation point flow of an upper convected Maxwell fluid over a stretching surface, Chaos Solitons Fract. 39 (2009) 840–848.

[7] M. Abd El-Aziz, Flow and heat transfer over an unsteady stretching surface with Hall effect, Meccanica 45 (2010) 97–109.

[8] A. Chakrabarti, A.S. Gupta, Hydromagnetic flow and heat transfer over a stretching sheet, Q. Appl. Math. 37 (1979) 73–78.

[9] H.I. Andersson, MHD flow of a viscoelastic fluid past a stretching surface, ActaMech. 95 (1992) 227–230.
[10] T.C. Chiam, Hydromagnetic flow over a surface stretching with a power-law velocity, Int. J. Eng. Sci. 33 (1995) 429–435.
[11] M.A.A. Mahmoud, Thermal radiation effects on MHD flow of a micropolar fluid over a stretching surface with variable thermal conductivity, Physica A 375 (2007) 401–410.
[12] E.M. Abo-Eldahab, M. Abd El-Aziz, Flow and heat transfer in a micropolar fluid past a stretching surface embedded in a non-Darcian porous medium with uniform free stream, Appl. Math. Comput. 162 (2005) 881–899.
[13] E.M. Abo-Eldahab, M. Abd El-Aziz, Blowing/suction effect on hydromagnetic heat transfer by mixed convection from an inclined continuously stretching surface with internal heat generation/absorption, Int. J. Thermal. Sci. 43 (2004) 709–719.
[14] M. Abd El-Aziz, A.M. Salem, MHD-mixed convection and mass transfer from a vertical stretching sheet with diffusion of chemically reactive species and space or temperature dependent heat source, Can. J. Phys. 85 (4) (2007) 359–373.
[15] M. Abd El-Aziz, Thermal radiation effects on magnetohydrodynamic mixed convection flow of a micropolar fluid past a continuously moving semi-infinite plate for high temperature differences, Acta Mech. 187 (2006) 113–127.
[16] M. Abd El-Aziz, Thermal-diffusion and diffusion-thermo effects on combined heat and mass transfer by hydromagnetic three-dimensional free convection over a permeable stretching surface with radiation, Phys. Lett. A 372 (3) (2007) 263–272.
[17] C.Y. Wang, Liquid film on an unsteady stretching sheet, Q. Appl. Math. 48 (1990) 601–610.
[18] M. Miklavčič, C.Y. Wang, Viscous flow due to a shrinking sheet, Q. Appl. Math. 228 64 (2006) 283–290.
[19] T. Fang, Boundary layer flow over a shrinking sheet with power-law velocity, Int. J. Heat Mass Transf. 51 (2008) 5838–5843.
[20] T. Fang, W. Liang, C.F. Lee, A new solution branch for the Blasius equation -a shrinking sheet problem, Comput. Math. Appl. 56 (2008) 3088–3095.
[21] C.Y. Wang, Stagnation flow towards a shrinking sheet, Int. J. Non Linear Mech. 43 (2008) 377–382.
[22] T. Fang, J. Zhang, S.-S. Yao, Viscous flow over an unsteady shrinking sheet with mass 236 transfer, Chin. Phys. Lett. 26 (2009) 014703-1.
[23] N.A. Yacob, A. Ishak, I. Pop, Melting heat transfer in boundary layer stagnation-point flow towards a stretching/shrinking sheet in a micropolar fluid, Comput. Fluids 47 (2011) 16–21.
[24] K. Bhattacharyya, S. Mukhopadhyay, G.C. Layek, Slip effects on boundary layer stagnation-point flow and heat transfer towards a shrinking sheet, Int. J. Heat Mass Transf. 54 (2011) 308–313.
[25] S.V. Subhashini, R. Sumathi, Dual solutions of a mixed convection flow of nanofluids over a moving vertical plate, Int. J. Heat Mass Transf. 71 (2014) 117–124.
[26] N.C. Roşca, I. Pop, Unsteady boundary layer flow over a permeable curved stretching/shrinking surface, Eur. J. Mech. - B/Fluids 51 (2015) 61–67.
[27] Muhaimina, R. Kandasamy, I. Hashim, Effect of chemical reaction, heat and mass transfer on nonlinear boundary layer past a porous shrinking sheet in the presence of suction, Nucl. Eng. Design 240 (2010) 933–939.
[28] T. Hayat, Z. Abbas, T. Javed, M. Sajid, Three-dimensional rotating flow induced by a shrinking sheet for suction, Chaos Solitons Fract. 39 (2009) 1615–1626.
[29] T. Fang, J. Zhang, Closed-form exact solution of MHD viscous flow over a shrinking sheet, Commun. Nonlinear Sci. Numer. Simul. 14 (2009) 2853–2857.
[30] N.F.M. Noor, S.A. Kechil, I. Hashim, Simple non-perturbative solution for MHD viscous flow due to a shrinking sheet, Commun. Nonlinear Sci. Numer. Simul. 15 (2010) 144–148.
[31] N.S. Akbara, S. Nadeemb, R. Ul Haqb, S. Ye, MHD stagnation point flow of Carreau fluid toward a permeable shrinking sheet: dual solutions, Ain Shams Eng. J. 5 (2014) 1233–1239.
[32] Z. Abbas, M. Sheikh, I. Pop, Stagnation-point flow of a hydromagnetic viscous fluid over stretching/shrinking sheet with generalized slip condition in the presence of homogeneous-heterogeneous reactions, J. Taiwan Inst. Chem. Eng. 55 (2015) 69–75.
[33] J.C. Crepeau, R. Clarksean, Similarity solutions of natural convection with internal heat generation, J. Heat Transf. 119 (1997) 183–185.
[34] A.M. Salem, M. Abd El-Aziz, Effect of Hall currents and chemical reaction on hydromagnetic flow of a stretching vertical surface with internal heat generation/absorption, Appl. Math. Model. 32 (7) (2008) 1236–1254.
[35] T Y Na, Computational Methods in Engineering Boundary Value Problems, Academic Press, New York, 1979.
[36] K. Bhattacharyya, Dual solutions in boundary layer stagnation-point flow and mass transfer with chemical reaction past a stretching/shrinking sheet, Int. J. Heat Mass Transf. 38 (7) (2011) 917–922.
[37] N. Bachok, A. Ishak, I. Pop, Melting heat transfer in boundary layer stagnation-point flow towards a stretching/shrinking sheet, Phys. Lett. A 374 (2010) 4075–4079.