Original Article

Some studies of the interaction between N-two level atoms and three level atom

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Abstract   In this paper, we present the analytical solution for the model that describes the interaction between a three level atom and two systems of N-two level atoms. The effect of the quantum numbers on the atomic inversion and the purity, for some special cases of the initial states, are investigated. We observe that the atomic inversion and the purity change remarkably by the change of the quantum numbers.

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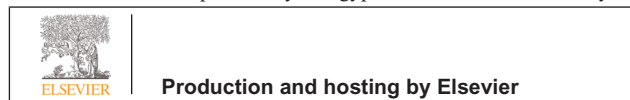
1. Introduction

The use of statistical mechanics is fundamental to the concepts of quantum optics: light is described in terms of field operators for creation and annihilation of photons [1–7]. The development of quantum optics started in the 50s and 60s, motivated by the need to understand photodetection, statistics and coherence of light within the framework of quantum theory. In the late 50s,

the seminal Hanbury-Brown and Twiss (HBT) experiment [8,9] demonstrated the bunching of photons emitted by a classical chaotic source and stimulated the development of the quantum theory of optical coherences by Glauber [10–12]. An important milestone in quantum optics was the demonstration of the non-classical character of fluorescence radiation emitted by a single atom and, later, the demonstration of photons sources showing photon antibunching [13–15]. In recent years the quantum optics of attenuating [16–22] and amplifying [23–27] dielectric media was developed, where optical modes are described as open quantum systems. Features of quantum optics are mainly based on three different types of interaction, namely, field–field, atom–atom, atom–field interaction. These interactions have been extensively considered in a huge number of papers; see for example Refs. [28–44]. Each one of these interactions represents a certain type of physical phenomena [45–50]. In fact these types of interactions have a strong relation between them so that one

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can transform one type to another [51]. This particular kind of the transformation would enable us to transform field–field interaction into atom–field interaction or atom–atom interaction. This depends on the nature of which kind of the Hamiltonian we use to carry out the transformation. The studying of any kind of these interaction would surely lead to more progress in the field of quantum optics and consequently it may open the door to observe or to report a new phenomenon in this field [52]. In the present communication we are concerned with the type of atom–atom (spin–spin) interaction, the interaction between a three level atom and two systems of N-two level atoms. The time evolution of dynamical systems has attracted considerable attention over the past several decades because of its various applications [53]. An important aspect in this regard is the atomic inversion and the purity. The atomic population inversion can be considered as one of the simplest important quantities, it is defined as "the difference between the probabilities of finding the atom in its exited state and in its ground state". The purity can range between zero, corresponding to a completely pure state, and $(1 - \frac{1}{d})$, corresponding to a completely mixed state, (here, d is the dimension of the density matrix). Hence, we study the atomic inversion and the purity of a three level atom coupled to two systems of N-two level atoms as an application.

This paper is organized as follows: in Section 2, we will describe the Hamiltonian of the system of interest, and obtain the explicit analytical solution of the model describing the interaction between a three level atom and two systems of N-two level atoms. Different cases are studied to demonstrate the effects due to the quantum numbers m_1, m_2 on the atomic inversion $\langle S_z \rangle$ and the purity $P_S(t)$ of the model. Conclusions are summarized at the end of the paper.

2. The model

The Hamiltonian of our model describes the interaction between a three level atom coupled to two systems of N-two level atoms. In this case the Hamiltonian of the whole system can be written in the form:

$$H = H_0 + H_{int.}, \tag{1}$$

where

$$H_0 = \sum_{\alpha=1}^{\alpha=2} \omega_\alpha J_z^{(\alpha)} + \sum_{\beta=1}^{\beta=3} \Omega_\beta S_{\beta\beta}, \tag{2}$$

$$H_0 = \sum_{\gamma=1}^{\gamma=2} \omega_\gamma J_z^{(\gamma)} + \sum_{\beta=1}^{\beta=3} \Omega_\beta S_{\beta\beta}, \tag{3}$$

$$H_{int.} = \lambda_1 \left[S_{21} J_+^{(1)} + J_-^{(1)} S_{12} \right] + \lambda_2 \left[S_{32} J_+^{(2)} + J_-^{(2)} S_{23} \right], \tag{4}$$

$$J_L^{(\gamma)} = \frac{1}{2} \sum_{k_\gamma=1}^{N_\gamma} \sigma_L^{k_\gamma}, \quad L = x, y, z, \tag{5}$$

$$\Omega_1 > \Omega_2 > \Omega_3, \tag{6}$$

$$J_\pm^{(\gamma)} = J_x^{(\gamma)} \pm i J_y^{(\gamma)}, \tag{7}$$

where ω_γ, $\gamma = 1, 2$ are the strength of the field (the two systems of N-two level atoms). The operators S_{ij} satisfy the commutation relation

$$[S_{kl}, S_{nm}] = S_{km}\delta_{nl} - S_{nl}\delta_{km}, \tag{8}$$

$J_\pm^{(\alpha)}$ and $J_z^{(\alpha)}$ are the collective angular momentum operators for N-two level atoms, which satisfy the relations, while $J_\pm^{(\gamma)}$ and $J_z^{(\gamma)}$ are the collective angular momentum operators for N-two level atoms, which satisfy the relations

$$\left[J_+^{(\gamma)}, J_-^{(\beta)} \right] = 2J_z^{(\gamma)}\delta_{\gamma\beta}, \tag{9}$$

$$\left[J_z^{(\gamma)}, J_\pm^{(\beta)} \right] = \pm J_\pm^{(\gamma)}\delta_{\gamma\beta}, \tag{10}$$

with the operator $\sigma_L^{k_\gamma}$ are the usual Pauli matrices.

We define

$$|\Psi(0)\rangle = |\Psi(0)\rangle_{j,m}|\Psi(0)\rangle_s \tag{11}$$

$$|\Psi(0)\rangle_{j,m} = |j_1, m_1, j_2, m_2\rangle \tag{12}$$

$$|\Psi(0)\rangle_s = a(0)|1\rangle + b(0)|2\rangle + c(0)|3\rangle \tag{13}$$

$$| a(0) |^2 + | b(0) |^2 + | c(0) |^2 = 1 \tag{14}$$

Let

$$\begin{aligned}|\Psi(t)\rangle = {} & A_{m_1,m_2}(t)|1, j_1, m_1, j_2, m_2\rangle \\ & + B_{m_1,m_2}(t)|2, j_1, m_1 + 1, j_2, m_2\rangle \\ & + C_{m_1,m_2}(t)|3, j_1, m_1 + 1, j_2, m_2 + 1\rangle. \end{aligned} \tag{15}$$

From Schrödinger equation

$$i\frac{\partial}{\partial t}|\Psi(t)\rangle = H|\Psi(t)\rangle, \tag{16}$$

we get from Eqs. (1) and (15)

$$i\frac{dA_{m_1,m_2}(t)}{dt} = \alpha_1 A_{m_1,m_2}(t) + \Lambda_1(m_1)B_{m_1,m_2}(t), \tag{17}$$

$$\begin{aligned}i\frac{dB_{m_1,m_2}(t)}{dt} = {} & \alpha_2 B_{m_1,m_2}(t) + \Lambda_1(m_1)A_{m_1,m_2}(t) \\ & + \Lambda_2(m_2)C_{m_1,m_2}(t), \end{aligned} \tag{18}$$

$$i\frac{dC_{m_1,m_2}(t)}{dt} = \alpha_3 C_{m_1,m_2}(t) + \Lambda_2(m_2)B_{m_1,m_2}(t), \tag{19}$$

where

$$\alpha_1 = (\omega_1 m_1 + \omega_2 m_2 + \Omega_1), \tag{20}$$

$$\alpha_2 = \omega_1(m_1 + 1) + \omega_2 m_2 + \Omega_2, \tag{21}$$

$$\alpha_3 = \omega_1(m_1 + 1) + \omega_2(m_2 + 1) + \Omega_3, \tag{22}$$

$$\Lambda_k = \lambda_k \sqrt{(j_k - m_k)(j_k + m_k + 1)} \quad k = 1, 2, \tag{23}$$

where λ_k the coupling parameters between spins.

Define

$$A_{m_1,m_2}(t) = a(t)e^{-i\alpha_1 t}, \quad B_{m_1,m_2}(t) = b(t)e^{-i\alpha_2 t}, \quad C_{m_1,m_2}(t) = c(t)e^{-i\alpha_3 t} \tag{24}$$

by substituting from Eq. (24) in Eqs. (17)–(19) we get the following equations:

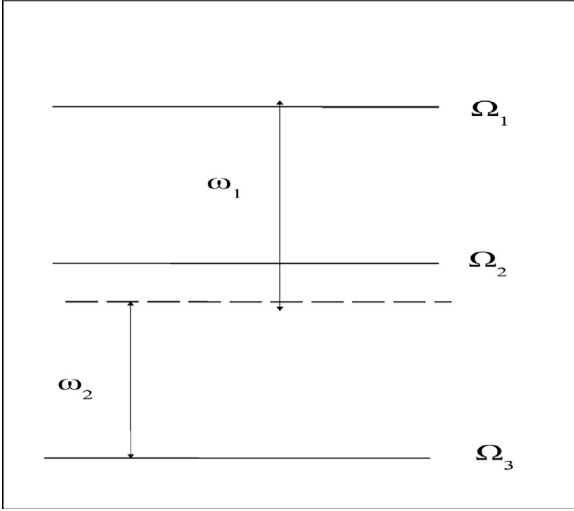$$i\frac{da(t)}{dt}e^{-i\alpha_1 t} = \Lambda_1(m_1)b(t)e^{-i\alpha_2 t}, \tag{25}$$

Fig. 1   Scheme of the interaction between a three level atom coupled to two systems of N-two level atoms.
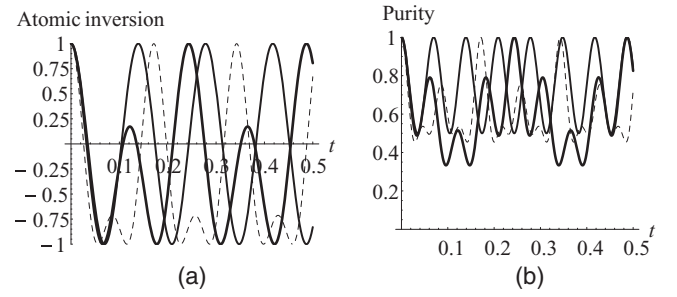

Fig. 2   Figures of the case in which $\Delta = 0$, $\lambda_1 = \lambda_2 = 1$ and $j_1 = 30$, $j_2 = 20$ and the initial state is $|\Psi(0)\rangle_s = |1\rangle$ where the dashed, bold solid, gray solid curves correspond, respectively, to $m_1 = m_2 = 1, 18, 20$.

$$i\frac{db(t)}{dt}e^{-i\alpha_2 t} = \Lambda_1(m_1)a(t)e^{-i\alpha_1 t} + \Lambda_2(m_2)c(t)e^{-i\alpha_3 t}, \qquad (26)$$

$$i\frac{dc(t)}{dt}e^{-i\alpha_3 t} = \Lambda_2(m_2)b(t)e^{-i\alpha_2 t}, \qquad (27)$$

from Fig. 1 we set

$$\Delta = [\omega_1 - (\Omega_1 - \Omega_2)] = [(\Omega_2 - \Omega_3) - \omega_2]. \qquad (28)$$

So, we can write Eqs. (25)–(27) as the following:

$$i\frac{da(t)}{dt} = \Lambda_1 b(t)e^{-i\Delta t}, \qquad (29)$$

$$i\frac{db(t)}{dt} = \Lambda_1 a(t)e^{i\Delta t} + \Lambda_2 c(t)e^{i\Delta t}, \qquad (30)$$

$$i\frac{dc(t)}{dt} = \Lambda_2 b(t)e^{-i\Delta t}. \qquad (31)$$

We solve Eqs. (29)–(31) analytically, we get:

$$b(t) = e^{\frac{i\Delta}{2}t}(q_1 e^{i\mu t} + q_2 e^{-i\mu t}), \qquad (32)$$

where

$$\mu = \sqrt{\Delta^2 + 4(\Lambda_1^2 + \Lambda_2^2)}, \qquad (33)$$

$$q_1 = \frac{\Lambda_1 a(0) + \Lambda_2 c(0) + (\frac{\Delta}{2} + \mu)b(0)}{2\mu}, \qquad (34)$$

$$q_2 = \frac{-\Lambda_1 a(0) - \Lambda_2 c(0) - (\frac{\Delta}{2} - \mu)b(0)}{2\mu}, \qquad (35)$$

similarly

$$a(t) = -\Lambda_1 e^{-\frac{i\Delta}{2}t}\left(\frac{q_1 e^{i\mu t}}{\mu - \frac{\Delta}{2}} - \frac{q_2 e^{-i\mu t}}{\mu + \frac{\Delta}{2}}\right) + q_3, \qquad (36)$$

$$q_3 = a(0) + \Lambda_1\left(\frac{q_1}{\mu - \frac{\Delta}{2}} - \frac{q_2}{\mu + \frac{\Delta}{2}}\right) \qquad (37)$$

and

$$c(t) = -\Lambda_2 e^{-\frac{i\Delta}{2}t}\left(\frac{q_1 e^{i\mu t}}{\mu - \frac{\Delta}{2}} - \frac{q_2 e^{-i\mu t}}{\mu + \frac{\Delta}{2}}\right) + q_4, \qquad (38)$$

$$q_4 = c(0) + \Lambda_2\left(\frac{q_1}{\mu - \frac{\Delta}{2}} - \frac{q_2}{\mu + \frac{\Delta}{2}}\right) \qquad (39)$$

So from Eq. (24) we get $A_{m_1,m_2}(t)$, $B_{m_1,m_2}(t)$ and $C_{m_1,m_2}(t)$.

3. The atomic inversion and the purity

The atomic population inversion is given by

$$\langle S_z \rangle = \left|A_{m_1,m_2}(t)\right|^2 - \left|B_{m_1,m_2}(t)\right|^2 - \left|C_{m_1,m_2}(t)\right|^2. \qquad (40)$$

The evolution of the purity $P_S(t)$ is given by

$$P_S(t) = 1 - Tr_S(\rho_S^2(t)) \qquad (41)$$

where $\rho_S(t)$ is the reduced density matrix of the system which is defined by $\rho_S(t) = Tr_F \rho(t)$.

From Eq. (41) the purity can range between zero, corresponding to a completely pure state, and $(1 - \frac{1}{d})$, corresponding to a completely mixed state. (Here, d is the dimension of the density matrix.)

Based on the analytical solution in the previous section, we shall examine the evolution in time of the atomic inversion and the purity. In the numerical results we consider ($\Delta = 0$, $\lambda_1 = \lambda_2 = 1$ and $j_1 = 30$, $j_2 = 20$). We investigate the effect of the quantum numbers m_1, m_2 on the atomic inversion and the purity. In Fig. 2, the initial state is $|\Psi(0)\rangle_s = |1\rangle$, The atomic inversion and the purity have regular and periodic oscillations, when the quantum numbers $m_1 = m_2 = 1$, we find generating small oscillation between each two oscillations. When $m_1(m_2)$ increases, the phase of the generating small oscillation increases gradually until it reaches its maximum value. This means that the number of oscillations increases. In Fig. 3, the initial state is $|\Psi(0)\rangle_s = \frac{1}{\sqrt{2}}|1\rangle + \frac{1}{\sqrt{2}}|2\rangle$, when the quantum numbers $m_1 =$
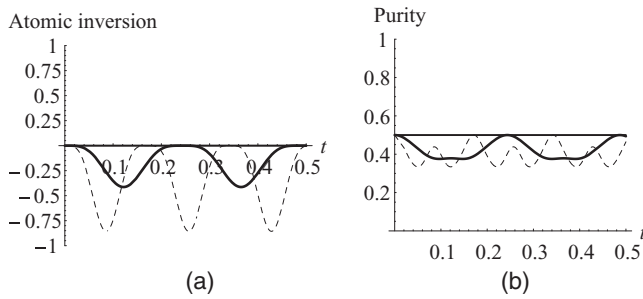
Atomic inversion

Purity

(a)   (b)

Fig. 3 Figures of the case in which $\Delta = 0, \lambda_1 = \lambda_2 = 1$ and $j_1 = 30, j_2 = 20$ and the initial state is $|\Psi(0)\rangle_s = \frac{1}{\sqrt{2}}|1\rangle + \frac{1}{\sqrt{2}}|2\rangle$ where the dashed, bold solid, gray solid curves correspond, respectively, to $m_1 = m_2 = 1, 18, 20$.
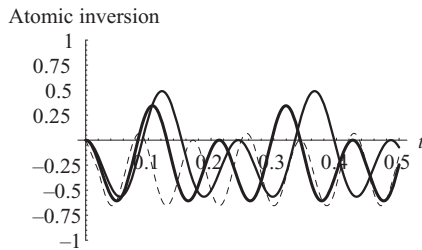
Atomic inversion

Fig. 4 Figure of the case in which $\Delta = 0, \lambda_1 = \lambda_2 = 1$ and $j_1 = 30, j_2 = 20$ and the initial state is $|\Psi(0)\rangle_s = \frac{1}{\sqrt{2}}|1\rangle + \frac{1}{2}|2\rangle + \frac{1}{2}|3\rangle$ where the dashed, bold solid, gray solid curves correspond, respectively, to $m_1 = m_2 = 1, 15, 18$.

Purity

Fig. 5 Figure of the case in which $\Delta = 0, \lambda_1 = \lambda_2 = 1$ and $j_1 = 30, j_2 = 20$ and the initial state is $|\Psi(0)\rangle_s = \frac{1}{\sqrt{2}}|1\rangle + \frac{1}{2}|2\rangle + \frac{1}{2}|3\rangle$ where the dashed, bold solid, gray solid curves correspond, respectively, to $m_1 = m_2 = 1, 18, 20$.
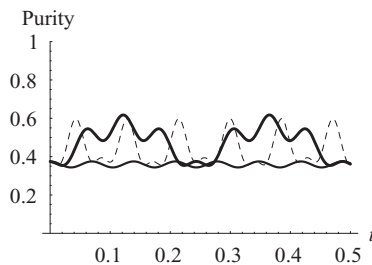
$m_2 = 1$, the atomic inversion has regular and periodic oscillations, and its minimum value is -0.9. When the quantum numbers $m_1(m_2)$ increases, the minimum value of the atomic inversion increases also until it reaches $\langle S_z \rangle = -0.4$. However, when the quantum numbers $m_1(m_2)$ increases until it reaches the quantum numbers $m_1 = m_2 = 20$, the atomic inversion is constant curve at $\langle S_z \rangle = 0$. When $m_1 = m_2 = 1$, the purity has regular and periodic oscillations, and its minimum value is 0.3. When the quantum numbers $m_1(m_2)$ increases, the minimum value of the purity increases also until it reaches $\langle S_z \rangle = 0.4$. However, when the quantum numbers $m_1(m_2)$ increases until it reaches the quantum numbers $m_1 = m_2 = 20$, the purity is constant curve at $P_S(t) = 0.5$. In Figs. 4 and 5 the initial state is $|\Psi(0)\rangle_s = \frac{1}{\sqrt{2}}|1\rangle + \frac{1}{2}|2\rangle + \frac{1}{2}|3\rangle$, the atomic inversion has regular and periodic oscillations, and its minimum value is -0.65. When the quantum numbers $m_1(m_2)$ increases, the maximum value of the atomic inversion increases also until it
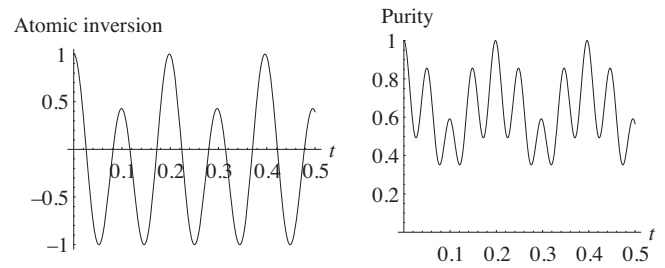
Atomic inversion

Purity

Fig. 6 Figures of the case in which $\Delta = 0, \lambda_1 = \lambda_2 = 1$ and $j_1 = 30, j_2 = 20, m_1 = 1, m_2 = 18$ and the initial state is $|\Psi(0)\rangle_s = |1\rangle$.

reaches $\langle S_z \rangle = 0.5$. While the increasing the quantum numbers $m_1(m_2)$ causes the purity to reach constant curve at $P_S(t) = 0.4$. This clearly shows the effect of the the quantum numbers $m_1(m_2)$.

In Fig. 6 the initial state is $|\Psi(0)\rangle_s = |1\rangle$, We investigate the effect of the quantum numbers $m_1, m_2, m_1 \neq m_2, m_1 = 1, m_2 = 18$, on the atomic inversion and the purity. The atomic inversion and the purity have regular and periodic oscillations. In case $m_1 = m_2 = 1$, Fig. 2, we find generating small oscillation between each two oscillations. In case $m_1 = 1, m_2 = 18$, Fig. 6, The phase of the generating small oscillation increases remarkably. So, if we compare between the case of $m_1 = m_2 = 18$, and the case of $m_1 = 1, m_2 = 18$, we find the phase of the generating small oscillation in the second case is bigger than the phase in first case.

4. Conclusion

In this paper, we solve the model that describes the interaction between a three level atom and two systems of N-two level atoms. We calculated the atomic inversion and the purity for some special cases of the initial states ($|\Psi(0)\rangle_s$) and special values of the quantum numbers $m_1(m_2)$. When the initial state is $|\Psi(0)\rangle_s = |1\rangle$, the increasing the quantum numbers $m_1(m_2)$ causes the number of oscillations of the atomic inversion and the purity to increase. When the initial state is $|\Psi(0)\rangle_s = \frac{1}{\sqrt{2}}|1\rangle + \frac{1}{\sqrt{2}}|2\rangle$, the increasing the quantum numbers $m_1(m_2)$ causes the minimum value of the atomic inversion and the purity to increase until they reach constant curves at $\langle S_z \rangle = 0$ and $P_S(t) = 0.5$. When the initial state is $|\Psi(0)\rangle_s = \frac{1}{\sqrt{2}}|1\rangle + \frac{1}{2}|2\rangle + \frac{1}{2}|3\rangle$, the increasing the quantum numbers $m_1(m_2)$ causes the maximum value of the atomic inversion to increase. While the increasing the quantum numbers $m_1(m_2)$ causes the purity to reach constant curve at $P_S(t) = 0.4$. This clearly shows the effect of the quantum numbers $m_1(m_2)$. So, we found the difference between the probabilities of finding the atom in its exited state and in its ground state depend on the values of the quantum number and the initial state of the model. Also, we can reaching to a completely pure state, or a completely mixed state by controlling of the quantum number and the initial state of the model.

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