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On some classes of nearly open sets in nano topological spaces

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Abstract One of the aims of this paper is to study some near nano open sets in nano topological spaces. Secondly, some properties for near nano open (closed) sets. Also, we introduce the notion of nano β-continuity and we study the relationships between some types of nano continuous functions between nano topological spaces. Finally, we introduce two application examples in nano topology.

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1. Introduction and preliminaries

L. Thivagar [1] introduced the concept of nano topological spaces with respect to a subset X of a universe U. We study the relationships between some near nano open sets in nano topological spaces. In this paper we study the relationships between some weak forms of nano open sets in nano topological spaces. Also, we introduce the notion of nano β-continuity between nano topological spaces and we investigate several properties of these types of near nano continuity. Finally, we introduce two examples as an applications in nano topological spaces.

Definition 1.1. [2] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the in discernibility relation. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

(i) The lower approximation of X with respect to R is denoted by $L_R(X)$. That is, $L_R(X) = \cup_{x\in U}\{R(x) : R(x) \subset X\}$ where $R(x)$ denotes the equivalence class determined by x.
(ii) The upper approximation of X with respect to R is denoted by $H_R(X)$. That is, $H_R(X) = \cup_{x\in U}\{R(x) : R(x) \cap X \neq \phi\}$.

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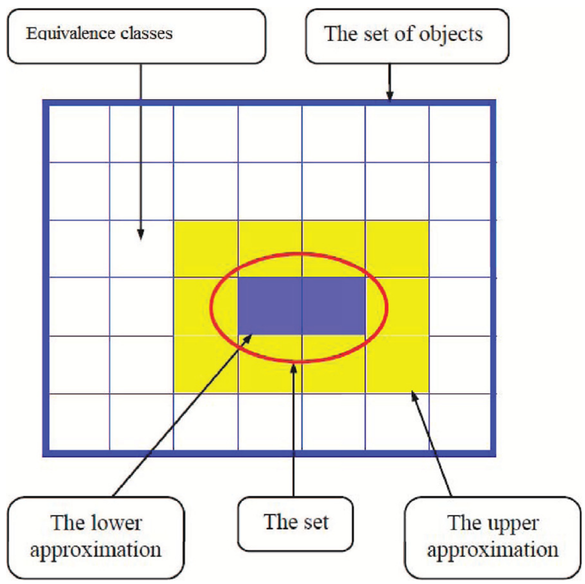

Fig. 1

(iii) The boundary region of X with respect to R is denoted by $B_R(X)$. That is, $B_R(X) = H_R(X) - L_R(X)$.

According to Pawlak's definitions, X is called a rough set if $L_R(X) \neq H_R(X)$.

Definition 1.2. [1] Let U be the universe and R be an equivalence relation on U. Then for $X \subseteq U$, $\tau_R(X) = \{U, \phi, L_R(X), H_R(X), B_R(X)\}$ is called the nano topology on U .We call $(U, \tau_R(X))$ is a nano topological space.

The elements of $\tau_R(X)$ are called a nano open sets and the complement of a nano open sets is called nano closed sets.

Definition 1.3. [3] Let $(U, \tau_R(X))$ be a nano topological space, the set $\beta = \{U, L_R(X), B_R(X)\}$ is called a bases for the nano topology $\tau_R(X)$ on U with respect to X.

Definition 1.4. [1] If $(U, \tau_R(X))$ is a nano topological space with respect to X, where $X \subseteq U$ and if $A \subseteq U$, then

(i) The nano interior of the set A is defined as the union of all nano open subsets contained in A, and is denoted by $nint(A)$.
(ii) The nano closure of the set A is defined as the intersection of all nano closed subsets containing A, and is denoted by $ncl(A)$.

Definition 1.5. [1,4] Let $(U, \tau_R(X))$ be a nano topological space and $A \subseteq U$. Then A is said to be:

(i) Nano regular open if $A = nint(ncl(A))$,
(ii) Nano α-open if $A \subseteq nint(ncl(nint(A)))$,
(iii) Nano semi-open if $A \subseteq ncl(nint(A))$,
(iv) Nano preopen if $A \subseteq nint(ncl(A))$,
(v) Nano γ-open (or nano b-open) if $A \subseteq ncl(nint(A)) \cup nint(ncl(A))$,
(vi) Nano β-open (or nano semi- preopen) if $A \subseteq ncl(nint(ncl(A)))$.

The family of all nano regular open (resp. nano α-open, nano semi-open, nano preopen, nano γ-open and nano β-open) sets in a nano topological space $(U, \tau_R(X))$ is denoted by $NRO(U, X)$ (resp. $N\alpha O(U, X)$, $NSO(U, X)$, $NPO(U, X)$, $N\gamma O(U, X)$ and $N\beta O(U, X)$).

Definition 1.6. [3] A subset K of a nano topological space $(U, \tau_R(X))$ is called nano regular closed (resp. nano α-closed, nano semi-closed, nano preclosed, nano γ-closed and nano β-closed) if its complements is nano regular open (resp. nano α-open, nano semi-open, nano preopen, nano γ-open and nano β-open).

Definition 1.7. [1] A nano topological space $(U, \tau_R(X))$ is called nano extremally disconnected if the nano closure of each nano open subset of U is nano open, or equivalently, if every nano regular closed subset of U is nano open.

Definition 1.8. Let $(U, \tau_R(X))$ and $(V, \tau^*_{R'}(Y))$ be nano topological spaces. A mapping f: $(U, \tau_R(X)) \rightarrow (V, \tau^*_{R'}(Y))$ is said to be:

(1) nano continuous [5] if $f^{-1}(B)$ is nano open set in U for every nano open set B in V.
(2) nano α-continuous [6] if $f^{-1}(B)$ is nano α-open set in U for every nano open set B in V.
(3) nano semi-continuous [6] if $f^{-1}(B)$ is nano semi-open set in U for every nano open set B in V.
(4) nano pre-continuous [6] if $f^{-1}(B)$ is nano preopen set in U for every nano open set B in V.
(5) nano γ-continuous [7] (or nano b-continuous) if $f^{-1}(B)$ is nano γ-open (or nano b-open) set in U for every nano open set B in V.

2. Fundamental properties of nano near open sets

The following diagram holds for a subset A of a nano topological space $(U, \tau_R(X))$.

The following examples show that, none of these implications is reversible.

Example 2.1. Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{d\}, \{b, c\}\}$ and $A = \{a, d\}$. Then one can deduce that $\tau_R(A) =$

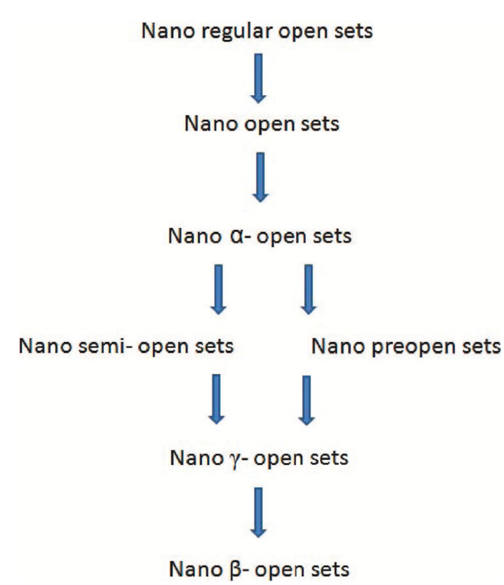

Fig. 2

$\{U, \phi, \{a, d\}\}$. Here, the set $\{a, b, d\}$ is nano α-open but not nano open in $(U, \tau_R(A))$.

Example 2.2. Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{c\}, \{b, d\}\}$ and $A = \{a, b\}$. Then the nano topology is defined as $\tau_R(A) = \{U, \phi, \{a\}, \{b, d\}, \{a, b, d\}\}$. Then, we have the following:-

(1) If $B = \{a, b, d\}$, then B is nano open but not nano regular open.
(2) If $C = \{a, c\}$, then C is nano semi-open but not nano α-open.
(3) If $D = \{a, b\}$, then D is nano preopen but not nano α-open.
(4) If $E = \{a, b, c\}$, then E is nano γ-open but not nano semi-open.
(5) If $G = \{b, c\}$, then G is nano β-open but not nano γ-open.
(6) If $F = \{b, c, d\}$, then F is nano γ-open but not nano pre-open.

Proposition 2.1. *For every nano topological space $(U, \tau_R(X))$, we have that: $NPO(U, X) \cup NSO(U, X) \subseteq N\gamma O(U, X) \subseteq N\beta O(U, X)$ holds but none of these implications can be reversed.*

Proposition 2.2. *Let $(U, \tau_R(X))$ be a nano topological space then:*

(i) *If $A \subseteq U$ is nano open and $B \subseteq U$ is nano semi-open (resp. nano preopen, nano β-open, nano γ-open) then $A \cap B$ is nano semi-open (resp, nano preopen, nano β-open, nano γ-open).*
(ii) *For every subset $A \subseteq U$, $A \cap nint(ncl(A))$ is nano preopen.*
(iii) *$A \subseteq U$ is nano γ-open, if and only if A is the union of a nano semi-open set and a nano preopen set.*

Proposition 2.3. *If $(U, \tau_R(X))$ is nano extremally disconnected space. Then, the following statements hold:*

(1) *Each nano β-open set is nano preopen.*
(2) *Each nano β-closed set is nano preclosed.*
(3) *Each nano semi-open set is nano α-open.*
(4) *Each nano semi-closed set is nano α-closed.*

Proof. It follows from the fact that if $(U, \tau_R(X))$ is nano extremally disconnected, then the notions of nano α-open sets, nano semi-open sets ,nano preopen sets and nano β-open sets are equivalent. □

Proposition 2.4. For a nano topological space $(U, \tau_R(X))$ the following properties are equivalent:

(1) $(U, \tau_R(X))$ is nano extremally disconnected.
(2) $NSO(U, X) \subseteq NPO(U, X)$.
(3) $N\beta O(U, X) = NPO(U, X)$.
(4) $N\gamma O(U, X) = NPO(U, X)$.

Proof. (1)⇔(2) and (1)⇔(3) these are obvious. Clearly, (3)⇒(4) and (4)⇒(1) follow immediately from Proposition 2.1 □

Proposition 2.5. *The intersection of a nano preopen set and a nano α-open set is nano preopen.*

Proof. Let $A \in NPO(U, X)$ and $B \in N\alpha O(U, X)$, then $A \subset nint(ncl(A))$, $B \subset nint(ncl(nint(B)))$. So, $A \cap B \subset nint(ncl(A)) \cap nint(ncl(nint(B))) \subset nint(nint(ncl(A)) \cap ncl(nint(B))) \subset nint(ncl(ncl(A) \cap nint(B))) \subset nint(ncl(ncl(A \cap B))) = nint(ncl(A \cap B))$. Hence, $A \cap B$ is nano preopen. □

Corollary 2.6. *The union of a nano preclosed set and a nano α-closed set is nano preclosed set.*

Proposition 2.7. *The intersection of a nano α-open set and a nano β-open set is nano β-open.*

Proof. Obvious. □

Corollary 2.8. *The union of a nano α-closed set and a nano β-closed set is nano β-closed.*

Remark 2.1. The arbitrary intersection of nano β-closed sets is nano β-closed but the union of two nano β-closed sets may not be nano β-closed set. This is clearly by the following example.

Example 2.3. Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{c\}, \{b, d\}\}$ and $A = \{a, b\}$. Then $\tau_R(A) = \{U, \phi, \{a\}, \{b, d\}, \{a, b, d\}\}$, the subsets $F = \{b\}$ and $W = \{a, d\}$ are nano β-closed sets but $F \cup W = \{a, b, d\}$ is not nano β-closed set.

Proposition 2.9. *Each nano β-open set which is nano semi-closed is nano semi-open.*

Proof. Let A be a nano β-open set and nano semi-closed. Then, $A \subseteq ncl(nint(ncl(A)))$ and $nint(ncl(A)) \subseteq A$. Therefore, $nint(ncl(A)) \subseteq nint(A)$ and so, $ncl(nint(ncl(A))) \subseteq ncl(nint(A))$. Hence, $A \subseteq ncl(nint(ncl(A))) \subseteq ncl(nint(A))$. This means that A is nano semi-open. □

Proposition 2.10. *A subset F of a nano topological space $(U, \tau_R(X))$ is nano β-closed if and only if $ncl(U - ncl(nint(F))) - (U - ncl(F)) \supset ncl(F) - F$.*

Proof. $ncl(U - ncl(F)) - (U - ncl(F)) \supset ncl(F) - F$ if and only if $(U - nint(ncl(nint(F)))) - (U - ncl(F)) \supset ncl(F) - F$ if and only if $(U - nint(ncl(nint(F)))) \cap ncl(F) \supset ncl(F) - F$ if and only if $(U \cap ncl(F)) - (nint(ncl(nint(F))) \cap ncl(F)) \supset ncl(F) - F$ if and only if $ncl(F) - (nint(ncl(nint(F))) \supset ncl(F) - F$ if and only if $F \supset init(ncl(ninl(F)))$ if and only if F is nano β-closed. □

Proposition 2.11. *Let F be a subset of a nano topological space $(U, \tau_R(X))$. If F is nano β-closed and nano semi-open, then it is nano semi-closed.*

Proof. Since F is nano β-closed and nano semi-open then $U - F$ is nano β-open and nano semi-closed and so by Proposition 2.9. $U - F$ is nano semi-open. Therefore, F is nano semi-closed. □

Proposition 2.12. *Each nano β-open set and nano α-closed set is nano regular closed.*

Proof. Let $A \subseteq U$ be a nano β-open set and nano α-closed set. Then $A \subseteq ncl(nint(ncl(A)))$ and $ncl(nint(ncl(A))) \subset A$, which implies that $ncl(nint(ncl(A))) \subseteq A \subseteq ncl(nint(ncl(A)))$. So, $A = ncl(nint(ncl(A)))$. This means that A is nano closed, and so it is nano regular closed. □

Corollary 2.13. *Each nano β-closed set and nano α-open set is nano regular open.*

Proposition 2.14. *Let $\tau_R(X)$ be the class of nano open subsets of X then, $\tau_R(X) = nintN\beta O(X)$.*

Proof. If $G \in \tau_R(X)$ then $G \in N\beta O(X)$. Since $G = nint(G)$, then $G \in nintN\beta O(X)$.

Conversely, let $G \in nintN\beta O(X)$ then $G = nint(W)$ for some $W \in N\beta O(X)$. Thus G is nano open. □

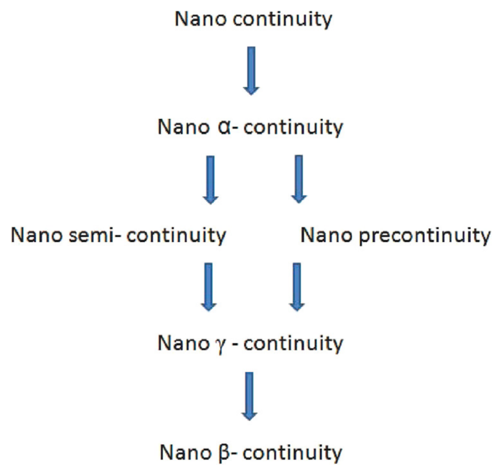

Fig. 3

3. Some classes of nano continuity

Definition 3.1. Let $(U, \tau_R(X))$ and $(V, \tau^*_{R'}(Y))$ be nano topological spaces. The mapping f: $(U, \tau_R(X)) \to (V, \tau^*_{R'}(Y))$ is said to be nano β-continuous or (nano semi-pre-continuous) if $f^{-1}(A)$ is nano β-open set in U for every nano open set A in V.

The relations between the above types of nano near continuous functions is clearly by the following diagram:

Now, we show that, none of these implications is reversible as shown by the following examples.

Example 3.1. Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{d\}, \{b, c\}\}$ and $X = \{a, d\}$. Then $\tau_R(X) = \{U, \phi, \{a, d\}\}$. Let $V = \{x, y, z, w\}$ with $V/R' = \{\{x\}, \{z\}, \{y, w\}\}$, $Y = \{x, y\}$ then $\tau^*_{R'}(Y) = \{V, \phi, \{x\}, \{y, w\}, \{x, y, z\}\}$. Define f: $U \to V$ as $f(a) = y$, $f(b) = y$, $f(c) = z$, $f(d) = w$, then f is nano α-continuous but not nano-continuous.

Example 3.2. Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{c\}, \{b, d\}\}$ and $X = \{a, b\}$. Then $\tau_R(X) = \{U, \phi, \{b, d\}, \{a, b, d\}\}$. Let $V = \{x, y, z, w\}$ with $V/R' = \{\{x\}, \{w\}, \{y, z\}\}$. Define f as; $f(a) = y$, $f(b) = y$, $f(c) = z$, $f(d) = w$, then:

(1) f is nano semi-continuous but not nano α-continuous.
(2) f is nano γ-continuous but not nano pre-continuous.

Example 3.3. Let $(U, \tau_R(X))$ and $(V, \tau^*_{R'}(Y))$ be nano topological spaces defined as in Example 3.2 and let g: $U \to V$ be defined as follows $g(a) = w$, $g(b) = y$, $g(c) = z$, $g(d) = w$. Then g is nano β-continuous but not nano γ-continuous.

Example 3.4. Let $(U, \tau_R(X))$ and $(V, \tau^*_{R'}(Y))$ be nano topological spaces defined as in Example 3.2 and let h: $U \to V$ be defined as follows $h(a) = y$, $h(b) = x$, $h(c) = z$, $h(d) = w$. Then h is nano γ-continuous but not nano semi-continuous.

Example 3.5. Let $(U, \tau_R(X))$ and $(V, \tau^*_{R'}(Y))$ be nano topological spaces defined as in Example 3.2 and let f: $U \to V$ be a mapping defined as follows $f(a) = w$, $f(b) = x$, $f(c) = w$, $f(d) = x$. Then f is nano pre-continuous but not nano α-continuous.

Theorem 3.1. *Let $(U, \tau_R(X))$ and $(V, \tau^*_{R'}(Y))$ be nano topological spaces, and let f: $(U, \tau_R(X)) \to (V, \tau^*_{R'}(Y))$ be a mapping. Then, the following statements are equivalent:*

(1) *f is nano β-continuous.*
(2) *The inverse image of every nano closed set G in V is nano β-closed in U.*
(3) *$f(n\beta cl(A)) \subseteq ncl(f(A))$, for every subset A of U.*
(4) *$n\beta cl(f^{-1}(F)) \subseteq f^{-1}(ncl(F))$, for every subset F of V.*
(5) *$f^{-1}(nint(F)) \subseteq n\beta int(f^{-1}(F))$, for every subset F of V.*

Proof. (1)⇒(2): Let f be nano β-continuous and let F be nano closed set in V. That is $V - F$ is nano open in V. Since f is nano β-continuous mapping. Then $f^{-1}(V - F)$ is nano β-open in U. Then $f^{-1}(V - F) = U - f^{-1}(F)$ which means that, $f^{-1}(F)$ is nano β-closed set in U.

(2)⇒(1): Let G be nano open set in V. Then, $f^{-1}(V - G)$ is nano β-closed in U. Then $f^{-1}(G)$ is nano β-open in U. Therefore, f is nano β-continuous mapping.

(1)⇒(3): Let f be nano β-continuous and let $A \subseteq U$. Since f is nano β-continuous and $ncl(f(A))$ is nano closed in V, $f^{-1}(ncl(f(A)))$ is nano β-closed in U. Since $f(A) \subseteq ncl(f(A))$, $f^{-1}(f(A)) \subseteq f^{-1}(ncl(f(A)))$, then $n\beta cl(A) \subseteq n\beta cl[f^{-1}(ncl(f(A)))] = f^{-1}(ncl(f(A)))$. Thus $n\beta cl(A) \subseteq f^{-1}(ncl(f(A)))$. Therefore, $f(n\beta cl(f(A))) \subseteq ncl(f(A))$ for every subset A of U.

(3)⇒(1): Let $f(n\beta cl(A)) \subseteq ncl(f(A))$ for every subset A of U. Let F be nano closed in V, then $f(n\beta cl(F)) \subseteq ncl(f(f^{-1}(F))) = ncl(F) = F$ that is $f(n\beta cl(f^{-1}(F))) \subseteq F$. Thus $n\beta cl(f^{-1}(F)) \subseteq f^{-1}(F)$, but $f^{-1}(F) \subseteq n\beta cl(f^{-1}(F))$. Hence $n\beta cl(f^{-1}(F)) = f^{-1}(F)$. Therefore, $f^{-1}(F)$ is nano β-closed in U for every nano closed set F in V. That is f is nano β-continuous.

(1)⇒(4): Let f be a nano β-continuous and let $F \subseteq V$, then $ncl(F)$ is nano closed in V and hence $f^{-1}(ncl(F))$ is nano β-closed in U. Therefore, $n\beta cl[f^{-1}(ncl(F))] = f^{-1}(ncl(F))$. Since $F \subseteq n\beta cl(F)$, $f^{-1}(F) \subseteq f^{-1}(ncl(F))$. Then $n\beta cl(f^{-1}(F)) \subseteq n\beta cl(f^{-1}(ncl(F))) = f^{-1}(ncl(F))$. Thus $n\beta cl(f^{-1}(F)) \subseteq f^{-1}(ncl(F))$.

(4)⇒(1): Let $n\beta cl(f^{-1}(F)) \subseteq f^{-1}(ncl(F))$ for every subset F of V. If F be nano closed in V, then $ncl(F) = F$. By assumption, $n\beta cl(f^{-1}(F)) \subseteq f^{-1}(ncl(F)) = f^{-1}(F)$. But $f^{-1}(F) \subseteq n\beta cl(f^{-1}(F))$. Therefore, $n\beta cl(f^{-1}(F)) = f^{-1}(F)$. That is, $f^{-1}(F)$ is nano β-closed in U for every nano closed set F in V. Therefore f is nano β-continuous.

(1)⇒(5): Let f be a nano β-continuous and let $F \subseteq V$, then $nint(F)$ is nano open in V and hence $f^{-1}(nint(F))$ is nano β-open in U. Therefore $n\beta int[f^{-1}(nint(F))] = f^{-1}(nint(F))$. Also, $nint(F) \subseteq F$ implies that $f^{-1}(nint(F)) \subseteq f^{-1}(F)$. Therefore, $n\beta int(f^{-1}(nint(F))) \subseteq n\beta int(f^{-1}(F))$. That is, $f^{-1}(nint(F)) \subseteq n\beta int(f^{-1}(F))$

(5)⇒(1): Let $f^{-1}(nint(F)) \subseteq n\beta int(f^{-1}(F))$ for every $F \subseteq V$. If F is nano open in V, then $nint(F) = F$. By assumption, $f^{-1}(nint(F)) \subseteq n\beta int(f^{-1}(F))$. Thus $f^{-1}(F) \subseteq n\beta int(f^{-1}(F))$. But $n\beta int(f^{-1}(F)) \subseteq f^{-1}(F)$. Therefore, $f^{-1}(F) = n\beta int(f^{-1}(F))$. That is, $f^{-1}(F)$ is nano β-open in U for every nano open set F in V. Therefore, f is nano β-continuous. □

Remark 3.1. If f: $(U, \tau_R(X)) \to (V, \tau^*_{R'}(Y))$ is nano β-continuous where $(U, \tau_R(X))$ and $(V, \tau^*_{R'}(Y))$ are nano topological spaces. Then $f(nbcl(A))$ is not necessarily equal to $ncl(f(A))$. This is clearly by the following example:

Example 3.6. Let $U = \{a, b, c, d, e\}$ with $U/R = \{\{a, c\}, \{b\}, \{d\}, \{e\}\}$. Let $X = \{a, b, c\} \subseteq U$. Then $\tau_R(X) = \{U, \phi, \{a, b, c\}\}$. Let $V = \{u, v, z, y, z\}$ with $V/R' = \{\{u\}, \{z, v\}, \{x, y\}\}$ and $Y = \{u, v, z\} \subseteq V$. Then $\tau^*_{R'}(Y) = \{V, \phi, \{z, u, v\}\}$. Define f: $U \to V$ as

$f(a) = x, f(b) = x, f(c) = u, f(d) = v, f(e) = y$. Clearly, f is nano β-continuous. Let $A = \{a, b, c\} \subseteq V$. Then $f(n\beta cl(A)) = f(\{a, b, c, d, e\}) = \{u, v, x, y\}$. But, $ncl(f(A)) = ncl(\{x, u\}) = V$. Thus $f(n\beta cl(A)) \neq ncl(f(A))$.

Remark 3.2. In Theorem 3.1 equality of the statements 4 and 5 does not hold in general as shown by the following example:

Example 3.7. Let $U = \{a, b, c, d\}$ with $U/R = \{\{a, d\}, \{b\}, \{c\}\}$. Let $X = \{a, c\} \subseteq U$. Then $\tau_R(X) = \{U, \phi, \{c\}, \{a, d\}, \{a, c, d\}\}$. Let $V = \{x, y, z, w\}$ with $V/R' = \{\{x\}, \{y\}, \{z\}, \{w\}\}$ and $Y = \{x, w\} \subseteq V$. Then $\tau^*_R(Y) = \{V, \phi, \{x, w\}\}$. Define f: $U \to V$ as $f(a) = x, f(b) = y, f(c) = z, f(d) = w$. Then f is nano β-continuous.

(i) Let $F = \{z, w\} \subseteq V$. Then $f^{-1}(ncl(F)) = f^{-1}(V) = U$ and $n\beta cl(f^{-1}(F)) = n\beta cl(\{c, d\}) = \{b, c, d\}$. Therefore, $n\beta cl(f^{-1}(F)) \neq f^{-1}(ncl(F))$.

(ii) Let $F = \{z, w\} \subseteq V$. Then $f^{-1}(nint(\{x, y\})) = f^{-1}(\{x, y\}) = \{a, b\}$ and $n\beta int(f^{-1}(F)) = n\beta int(f^{-1}(\{x, y\})) = n\beta int(\{a, b\}) = \{a\}$. Therefore, $f^{-1}(nint(F)) \neq n\beta int(f^{-1}(F))$.

4. Application in nano topology

Example 4.1. Measles is an acute viral and infectious disease. It is more prevalent in childhood, but may infect adults as well and the cause of this disease is measles virus. It is spread by contact with infected person through coughing and, sneezing and is transmitted by droplet infection or air borne. The virus remains active and contagious on a contaminated surface for up to two hours. The incubation period ranging from 5 to 10 days. The symptoms of this disease are skin rashes, fatigue, dry cough, conjunctivitis and fever. The disease can be prevented through vaccination by measles vaccine. After recovery from measles person acquires immunity against infection for his life.

The next table gives data about 8 patients.

Patients	Skin rash (S)	Conjunctivitis (C)	dry cough (D)	Fatigue (F)	Temperature T	Measles
p_1	Yes	Yes	No	No	normal	No
p_2	Yes	Yes	No	No	very high	Yes
p_3	Yes	No	No	No	high	Yes
p_4	No	No	No	No	very high	No
p_5	No	Yes	Yes	Yes	high	No
p_6	Yes	No	No	No	high	No
p_7	Yes	Yes	Yes	Yes	high	Yes
p_8	Yes	Yes	No	No	very high	Yes

The columns represent the attributes (the symptoms for measles) and the rows represent the objects (the patients) in the above table.

Let $U = \{p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8\}$, then:

Case I: Let $X = \{p_2, p_3, p_7, p_8\}$ be the set of patient having measles. Let R be the equivalence relation on U with respect to the set of all condition attributes.

The set of equivalence classes corresponding to R is given by $U/I(R) = \{\{p_1\}, \{p_2, p_8\}, \{p_3, p_6\}, \{p_4\}, \{p_5\}, \{p_7\}\}$, therefore the nano topology on U with respect to X is given by $\tau_R(X) = \{U, \phi, \{p_2, p_7, p_8\}, \{p_2, p_3, p_6, p_7, p_8\}, \{p_3, p_6\}\}$. If we remove the attribute "Skin rash" we get

$U/I\ (R - (S)) = \{\{p_1\}, \{p_2, p_8\}, \{p_3, p_6\}, \{p_4\}, \{p_5, p_7\}\}$. Hence $\tau_{R-(S)}(X) = \{U, \{p_2, p_8\}, \{p_2, p_3, p_5, p_6, p_7, p_8\}, \{p_3, p_5, p_6, p_7\}\} \neq \tau_R(X)$. If we remove the attribute "conjunctivitis" we get $U/I(R - (C)) = U/I(R)$ and hence $\tau_{R-(C)}(X) = \tau_R(X)$. If we remove the attribute "dry cough" we get $U/I(R - (D)) = U/I(R)$ and hence $\tau_{R-(D)}(X) = \tau_R(X)$. If we remove the attribute "fatigue" we get $U/I(R - (F)) = U/I(R)$ and hence $\tau_{R-(F)}(X) = \tau_R(X)$. If we remove the attribute "temperature" we get, $U/I(R - T) = \{\{p_1, p_2, p_8\}, \{p_3, p_6\}\{p_4\}, \{p_5\}, \{p_7\}\}$. Therefore $\tau_{R-(T)}(X) = \{U, \phi, \{p_7\}, \{p_1, p_2, p_3, p_6, p_7, p_8\}, \{p_1, p_2, p_3, p_6, p_8\}\} \neq \tau_R(X)$. From Case I we get $core(R) = \{S, T\}$.

Case II: Let $X = \{p_1, p_4, p_5, p_6\}$ be the set of patients not having measles. Then $U/I(R) = \{\{p_1\}, \{p_3, p_6\}, \{p_4\}, \{p_5\}, \{p_2, p_8\}, \{p_7\}\}$, therefore $\tau_R(X) = \{U, \phi, \{p_1, p_4, p_5\}, \{p_1, p_3, p_4, p_5, p_6\}, \{p_3, p_6\}\}$. If we remove the attribute "Skin rash" we get, $U/I(R - (S)) = \{\{p_1\}, \{p_3, p_6\}, \{p_4\}, \{p_5, p_7\}, \{p_2, p_8\}\}$, and hence $\tau_{R-(S)}(X) = \{U, \phi, \{p_1, p_4\}, \{p_1, p_3, p_4, p_5, p_6, p_7\}, \{p_3, p_5, p_6, p_7\}\} \neq \tau_R(X)$. If the attribute "conjunctivitis" is removed we get,

$U/I(R - (C)) = \{\{p_1\}, \{p_4, p_6\}, \{p4\}, \{p_5\}, \{p_2, p_8\}, \{p_7\}\}$ which is the same as $U/I(R)$ and hence $\tau_{R-(C)}(X) = \tau_R(X)$. If the attribute "dry cough" is removed we get, $U/I(R - (D)) = \{\{p_1\}, \{p_3, p_6\}, \{p_4\}, \{p_5\}, \{p_2, p_8\}, \{p_7\}\}$, which is the same as $U/I(R)$ and hence $\tau_{R-(D)}(X) = \tau_R(X)$. When the attribute "fatigue" is omitted, $U/I(R - (F)) = \{\{p_1\}, \{p_3, p_6\}, \{p_4\}, \{p_5\}, \{p_2, p_8\}, \{p_7\}\}$, which is the same as $U/I(R)$ and hence $\tau_{R-(F)}(X) = \tau_R(X)$. If the attribute "Temperature" is removed we get, $U/I(R - (T)) = \{\{p_1, p_2, p_8\}, \{p_3, p_6\}, \{p_4\}, \{p_5\}, \{p_7\}\}$, therefore $\tau_{R-(T)}(X) = \{U, \phi, \{p_4, p_5\}, \{p_1, p_2, p_3, p_4, p_5, p_6, p_8\}, \{p_1, p_2, p_3, p_6, p_8\}\} \neq \tau_R(X)$. From Case II we get $core(R) = \{S, T\}$.

Observation: From the two cases above, we investigate that, "skin rash" and "Temperature" are the necessary and sufficient to say that a patient has measles.

5. Conclusion

In this paper some of the properties of nano near open sets and nano continuity are discussed. Also, we introduce an application example in nano topology. Thus it is advantageous to use nano topology in real life situations.

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