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Impact of peristaltic inclined Tube of the Newtonian viscous fluid flow with different wavelengths described by linear Navier-Stokes equations

M. S. Ali

Mathematics Department, Faculty of Science, Tanta University, Tanta, Egypt
Email: maha.mohamadain@science.tanta.edu.eg

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Abstract: On the basis of the physical concept for a linear velocity operator ($\underline{\hat{v}}\ .\ \underline{\nabla}$), the unstable and nonlinear Navier-Stokes equations in cartesian coordinates are transformed into the linear diffusion equations. In this paper, the non-dimensional continuity and the linear Navier-Stokes' equations are used to explain the Newtonian viscous fluid flow in a two-dimensional peristaltic inclined tube with respect to the y-axis. The linear differential equations of the problem are solved analytically by using Picard method. The velocity and the stream function of the fluid are obtained as functions of the physical parameters like time, wavelengths, and Reynolds numbers for the first time. Several graphs for these results of physical interest are displayed and discussed in detail. The obtained analytical solutions satisfy the linear and nonlinear Navier-stokes and continuity equations for all values of physical parameters for a first time in the periodical journals up to date. The author considered this work as a millennium problem; which proposed by clay institute.

Keywords: Operator for linear velocity, 2D linear Navier-Stokes equations, Inclined tube, Peristaltic flow, Acceleration of linear convection, Transit, Laminar, Turbulent flow.

1. Introduction

The motion of peristaltic fluid flow in tubes is applied in different branches of engineering and medicine. Most of nondimensional nonlinear Navier-Stokes, Burger and Korteweg-deVries (KDV) Equations; which are roughly solved for large wavelength $\delta = 0$ and small Reynolds number describe the prior issues [1-4]. Additionally, the large values of wave lengths and small Reynolds numbers are used to determine the stream function and fluid velocity components in a special case. The numerical and approximate solutions of Navier-Stokes equations are obtained for a special case of fluid and flow [5-8]. The viscous non-nano and nanofluid flow in a vertical and horizontal peristaltic cylinderiical Tubes is studied between two-phase bubbly flow by many authors [9-14]. Christianto and Vladimir interpret the Schroedinger wave function to derive the precise solution to the nonlinear Naiver-Stokes equation for a viscous fluid flow [6,15]. The obstacles of analytical solutions are appeared in the nonlinear terms of partial differential equations (PDE) [16-19]. Recently, Mohammadein examines a novel treatment of fluid mechanics with heat mass transfer [20]. The nonlinear Navier-Stokes, Burger and Korteweg-deVries (KDV) equations are converted to the linear equatiions; which are valid for all values of wavelength δ and Reynolds number with simple solutions [3,4,16,19].

On the basis of the Bernoulli and Mohammadein theory, the pressure gradient notion is developed [17]. For the first time, the linear Navier-Stokes equations are used to describe the unsteady, incompressible, and viscous Newtonian fluid flow in a horizontal tube for various wave lengths and Reynolds numbers [16-17]. Additionally, the resulting analytical solutions satisfy both linear and nonlinear Navier-Stokes equations [11,20]. Moreover, the continuity and linear Navier-Stokes equations are used to describe the incompressible and viscous Newtonian fluid in a peristaltic flow horizontal tube at various Reynolds numbers, wave lengths, and flow patterns (laminar, transit, and turbulent flow).

Pressure gradient concept

A surface force, a key factor in fluid flow, is represented by the pressure gradient. Based on the Bernoulli equation, the fluid pressure gradient takes the following form.

$$\underline{\nabla}P = -\rho\left(\underline{v}.\underline{\nabla}\right)\underline{v} - \rho g\ \underline{\hat{n}}. \tag{1}$$

Based on the Mohammadein hypothesis [17], the pressure gradient formula above becomes

$$\underline{\nabla}P = \eta\underline{\nabla}^2\underline{V} - \rho\ g\ \underline{\hat{n}}. \tag{2}$$

NonLinear Navier-Stokes Equations

Think about a flow of an incompressible viscous fluid influenced by body and surface forces, which are represented by continuous and vector nonlinear Navier-Stokes equations as follows:

$$\underline{\nabla}.\underline{v} = 0, \tag{3}$$

$$\rho\left(\frac{\partial v}{\partial t} + \left(\underline{v}.\underline{\nabla}\right)\underline{v}\right) = -\underline{\nabla}P + \underline{\nabla}.\tau_{ij}, \tag{4}$$

where $\underline{\nabla}P$ is the fluid pressure gradient, and $\underline{\nabla}.\tau_{ij} = \eta\ \underline{\nabla}^2\underline{v}$ is the shearing stress for a viscous Newtonian fluid.

In this study, the effect of surface and body forces on the flow of an incompressible and viscous Newtonian fluid in a peristaltic inclined tube ($0 \le \varphi < \frac{\pi}{2}$) in a two dimensional Cartesian coordinates is investigated for a first time. The linear Navier-Stokes equations and continuity are used to create the linear mathematical model in a simplest form. For two different values of wave lengths (($\lambda \neq 0$ *and* $\delta \neq 0$).), the unsteady analytical solution is obtained in terms of two velocity components and the stream function in a two-dimensional form. The results are plotted thoroughly explored. And the final comments are tallied. The concept of this work is based on novel treatment theory [16,17,20].

Nomenclatures

$\boldsymbol{a}$	is indicated to the tube half width
$\boldsymbol{\varphi}$	angle measured from y-axis
$\boldsymbol{A_1}$	constant
$\boldsymbol{\delta}$	ratio of vein dimeter and wave length
$\boldsymbol{b}$	wave amplitude
$\boldsymbol{\lambda}$	wavelength of fluid flow
$\boldsymbol{c_1, c_2}$	constants
$\boldsymbol{\rho}$	density of fluid
c	sound wave in blood
$\boldsymbol{\hat{n}}$	normal unit vector
$\boldsymbol{\nu}$	kinematic viscosity of fluid
$\boldsymbol{G_C}$	modified Grashof number
e	amplitude ratio
g	the gravitational acceleration
$\boldsymbol{\gamma}$	constant
H	peristaltic curve around wall
$\boldsymbol{\eta}$	viscosity of fluid
$\boldsymbol{h_1, h_2}$	arbitrary constant values
$\boldsymbol{\tau_{ij}}$	shearing stress
$\boldsymbol{L_1, L_2}$	arbitrary constant values
$\boldsymbol{\psi(x,y,t)}$	stream function
$\boldsymbol{q_b}$	volume flow rate
u, v	fluid velocity components
x, y	Cartesian coordinates
$\boldsymbol{R_e}$	Reynolds number
t	time

2. Physical model

Numerous publications [8,9,11,17,20] have discussed the peristaltic motion of fluid flow in tubes when there are lengthy wave durations. Here, we'll take into account the incompressible Newtonian viscous fluid flow in a peristaltic tube that is inclined to the y-axis at an angle similar to that in Figure 1.

An endless sinusoidal wave train traveling along the tube walls at a constant speed of c is what generates the flow. In our scenario, the force of gravity is taken into account. The form of the peristaltic border condition is

$$H = a + b \sin\left(\frac{2\pi}{\lambda}\overline{y}\right), \tag{5}$$

where a is indicated to the tube half width, b is indicated to the wave amplitude, λ is indicated to the wave length and t is the time.

Mathematical model

The following form can be used to represent the mathematical model for a viscous and incompressible Newtonian fluid flow in a peristaltic inclined tube in a two-dimensional cartesian coordinates under the influence of surface and body forces:

Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial \mathrm{v}}{\partial y} = 0, \tag{6}$$

Navier-Stokes equations

$$\text{x:}\ \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \mathrm{v}\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right), \tag{7}$$

$$\text{y:}\ \frac{\partial \mathrm{v}}{\partial t} + u\frac{\partial \mathrm{v}}{\partial x} + \mathrm{v}\frac{\partial \mathrm{v}}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 \mathrm{v}}{\partial x^2} + \frac{\partial^2 \mathrm{v}}{\partial y^2}\right) + g\ \cos\ \varphi, \tag{8}$$

where $0 \le \varphi < \frac{\pi}{2}$.

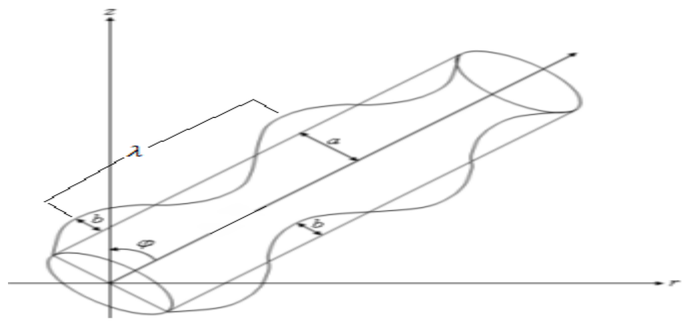

Fig.1. Sketch of the problem.

Applying the novel treatment theory [20] to the nonlinear system (6–8) in the previous sentence in the frame $(\overline{x}, \overline{y})$, the result is

$$\frac{\partial \overline{u}}{\partial \overline{x}} + \frac{\partial \overline{\mathrm{v}}}{\partial \overline{y}} = 0, \tag{9}$$

$$\frac{\partial \overline{u}}{\partial \overline{t}} = \nu\left(\frac{\partial^2 \overline{u}}{\partial \overline{x}^2} + \frac{\partial^2 \overline{u}}{\partial \overline{y}^2}\right), \tag{10}$$

$$\frac{\partial \overline{\mathrm{v}}}{\partial \overline{t}} = \nu\left(\frac{\partial^2 \overline{\mathrm{v}}}{\partial \overline{x}^2} + \frac{\partial^2 \overline{\mathrm{v}}}{\partial \overline{y}^2}\right) + g\ \cos\ \varphi. \tag{11}$$

In terms of dimensional ones, the nondimensional parameters take the following form.

$$\overline{x} = a\,x,\ \overline{y} = \lambda y,\ \overline{u} = c\delta u,\ \ \overline{\mathrm{v}} = c\mathrm{v},\ \delta = \frac{a}{\lambda},\ \overline{t} = \frac{\lambda}{c}\mathrm{t},$$

$$\overline{\psi} = a\,c\psi\,,\ e = \frac{b}{a},\ \ \gamma = \frac{a}{c^2}\ \text{and}\ \ h = \frac{H}{a}1 + e\sin(2\pi y). \tag{12}$$

The aforementioned equations (9–11) create a linear partial differential equation in terms of fluid velocity components u and v by applying the aforementioned transformations (12) in frame (x, y) in the form

$$Re\ \delta\ \mathrm{u}_t = \left(\mathrm{u}_{xx} + \delta^2\mathrm{u}_{yy}\right), \tag{13}$$

$$Re\ \delta\ \mathrm{v}_t = \left(\mathrm{v}_{xx} + \delta^2\mathrm{v}_{yy}\right) + Re\ g\ \gamma\cos\ \varphi. \tag{14}$$

The analytical solution of the aforementioned linear partial differential problem (13-14) obtained using the Picard technique [16] has the following form

$$v(x,y,t) = A_1 \quad e^{\frac{t}{R_e\,\delta}(c_1^2+\delta^2 c_2^2)-(c_1x+c_2y)} + R_e g\,\gamma\,t\,\cos\,\varphi, \quad (15)$$

and

$$u(x,y,t) = -\frac{c_2 A_1}{c_1} e^{\frac{t}{Re\,\delta}(c_1^2+\delta^2 c_2^2)-(c_1x+c_2y)}. \quad (16)$$

On basis of fluid velocity (13) and the following conditions

$$u(x,y,0) = f(x,y) = e^{-(c_1 x + c_2 y)},$$

$$u(x,L_1,t) = 3, \qquad u(x,L_2,t) = 1,$$

$$u(h_1,y,t) = 5, \qquad u(h_2,y,t) = 4. \quad (17)$$

On the basis of initial and boundary conditions (17) and equation (16), the constants c_1, c_2 and A_1 are estimated as follows:

$$c_1 = \frac{1}{h_2-h_1}\ln 5/4, \qquad c_2 = \frac{1}{L_1-L_2}\ln 1/3, \quad , A_1 = 1,$$

$$h = 1 + e\sin(2\pi y). \quad (18)$$

on basis of equations (15), (16) and (6), the stream function becomes

$$\psi(x,y,t) = \frac{A_1}{c_1} e^{\frac{t}{Re\,\delta}(c_1^2+\delta^2 c_2^2)-(c_1x+c_2y)} + Re\,g\,\gamma\,x\,t\,\cos\,\varphi. \quad (19)$$

The relations (15), (16) and (19) represent the analytical solutions of the present model (9-11).

3. Results and Discussion:

The incompressible Newtonian fluid in a peristaltic inclined tube with angle φ with axis y is studied. The linear continuity and Navier-Stokes equations based on the notion of fluid pressure gradient (2) are described the current problem. The non-dimensional linear equations (13-14) is created from the set of linear partial differential equations (6–8). The Picard approach is used to arrive to the simplest solution [13] in terms of fluid velocity components (15-16) and stream function (19). All wavelength and Reynolds number values can be satisfied using the obtained solutions. In other words, the fluid velocity components can be estimated for all values wave lengths and Reynolds numbers in fluid mechanics. Moreover, the fluid flow is valid in a vertical Tube ($\varphi = 0$) and in case of inclined Tube ($0 \le \varphi < \frac{\pi}{2}$).

In the follows, numerical values, which are used in calculations of solutions and graphs are considered for flow patterns as in In Table 1.

Table 1. The variable data

Symbol	a	b	e	Re	L_1	L_2
value	10	0.1	0.01	7	0	10

The tube radius a, amplitude e, Reynolds number Re, distances L_1 and L_1 takes a limited value. Moreover, it is shown that in Figures 2-6 such that, each group of alphabetically lettered figures are put in one row so that all parameters are fixed except one parameter.

The fluid velocity for various values of $\delta = 0.01$ and $\delta = 0.1$ is shown in Fig.2. It is observed that, the velocity of fluid proportional with parameter "δ". The velocity for various values of inclined angles $\varphi = 30$ and $\varphi = 30.002$ is shown in Fig.3. It is observed that, the velocity of fluid proportional with parameter "φ". The velocity for different values of $t = 0.5$ and $t = 0.51$ is shown in Fig.4. It is observed that, the velocity of fluid proportional with interval time "t". The velocity for various values of $e = 0.5$ and $e = 0.6$ is shown in Fig.5. It is observed that, the velocity of fluid proportional with amplitude ratio parameter"e". For instance, when ($\varphi = 0$), the current problem represents the fluid flow in a vertical Tube and affected by gravity force.

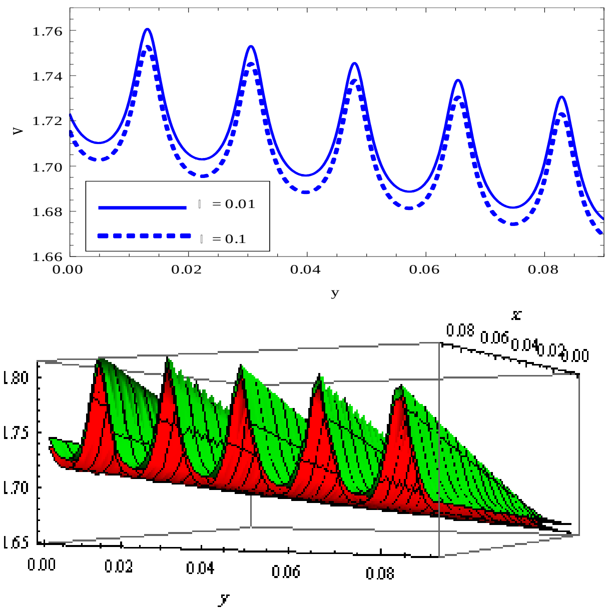

Fig. 2. The velocity for different values of $\delta = 0.01$ and $\delta = 0.1$.

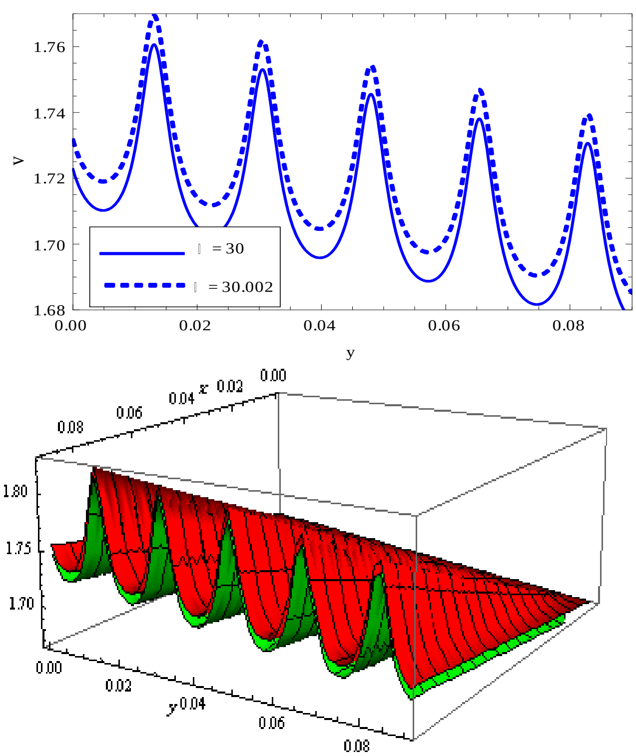

Fig.3. The velocity for different values of $\varphi = 30$ and $\varphi = 30.002$.

4. Conclusion

The linear equations (13-14) is created by transforming the system of linear partial differential equations (9–11). By using the Picard technique [20], the analytical solution of equation (13-14) are found in terms of the fluid velocity components and stream function (15-16) and (19) respectively. The results are valid for all wavelength and Reynolds number values. The following bullet points list the final observations:

1. The fluid velocity is increasing with the decreasing of wavelength parameter “λ".
2. The fluid velocity in the inclined tube with angle “ φ" is proportional with the values of angle parameter “φ" in the interval ($0 \leq \varphi < \frac{\pi}{2}$).
3. The velocity of fluid proportional with interval time "t".
4. The fluid velocity is proportional with amplitude ratio parameter"e".
5. The fluid flow is valid in a vertical and inclined directions of Tube ($\varphi = 0$) and ($0 \leq \varphi < \frac{\pi}{2}$) respectively, which can be considered as two coupled problems in same time.
6. The obtained solutions u (x, y, t), u(x, y, t), and $\psi(x, y, t)$represent the fluid flow in terms of wave function.
7. The motivation of this method introduced that, the obtained analytical solutions (15-16) and (19) satisfy the linear and non linear Navier-stokes and continuity equations (6) and (13-14) for all physical values for a first time in the periodical journals up to date.
8. This effort is the first treatment of a viscous and incompressible Newtonan fluid flow in an inclined tube ($0 \leq \varphi < \frac{\pi}{2}$) in two dimensions without ignoring wavelengths and Reynolds numbers.
9. The concluded results proved the validity of the proposed physical model. Moreover, this model can be modified for some properties of fluid and flow.

The author considered this work as an contribution of solving the millennium problem; which proposed by clay institute in this century.

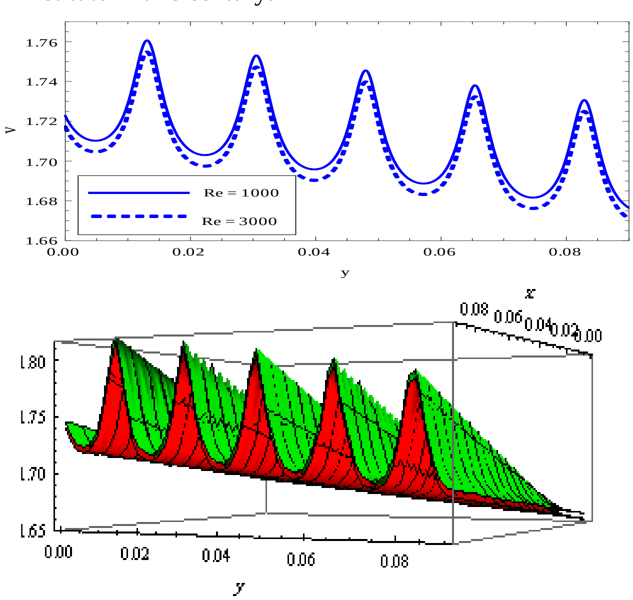

Fig.4. The velocity for different values of $Re = 1000$ and $Re = 3000$.

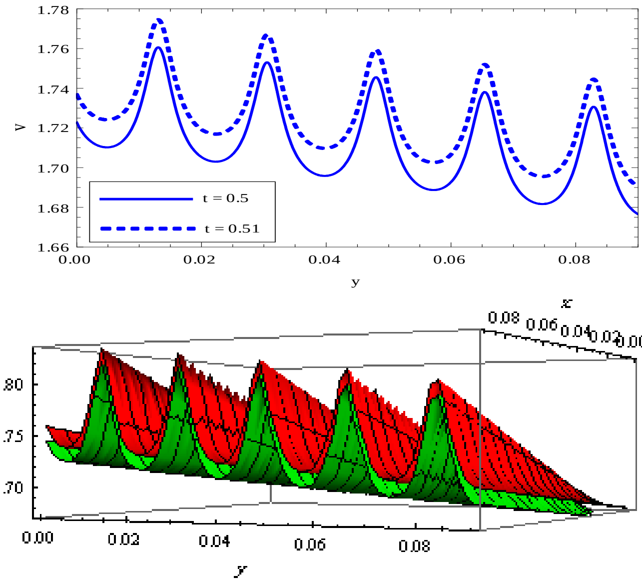

Fig.5. The velocity for different values of $t = 0.5$ and $t = 0.51$.

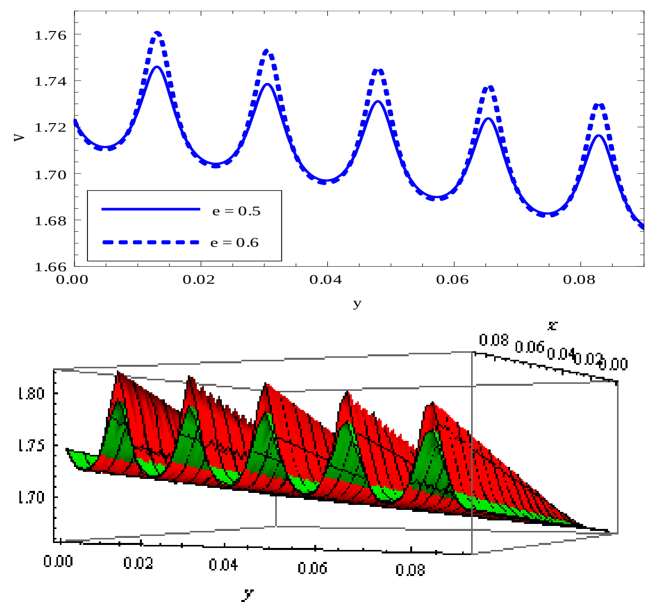

Fig.6. The velocity for different values of $e = 0.5$ and $e = 0.6$.

Data availability statement

The data used to support the findings of this study are available from the corresponding author upon request.

Declaration of competing interest

The author declare that she has no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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