Published by: ***Higher Institute of Engineering and Technology, Kafrelsheikh*** **(KFS-HIET)**

Journal homepage: https://jiet.journals.ekb.eg/

Print ISSN 3009-7207 **Online ISSN** 3009-7568

More on Pre-Generalized Closed Sets

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Received: 29 April 2024; Revised: 7 May 2024; Accepted: 9 June 2024

ABSTRACT: Topology studies properties of spaces that are fixed under any continuous deformation. It is the area of mathematics which investigates continuity and related concepts. We extend the notion of closed sets in ordinary topological spaces to pre-generalized - closed sets in generalized topological spaces. In topological spaces, generalized topology is an important generalization. According to the definition of generalized topological spaces, f may not be come in closed set. In the same way, X may not be come in the open set of generalized topological spaces. In this paper a new class of sets in topological spaces is studied and some of their properties are investigated. A closed set of pre-generalized a topological space is pre - $T_{1/2}$, The relationships among t_n - closed sets, existing classes of generalized closed sets. Also, some different classes of continuity, irresoluteness, compactness, and connectedness via t_n - closed sets. We discuss some of their properties and investigate the relations between the associated topologies.

KEYWORDS: n-closed sets, t_n -closed sets, t_n -continuous maps, t_n -irresolute maps, t_n -closed maps, t_n -compact spaces and t_n -connected spaces

1. INTRODUCTION

In recent years, the exploration of closed sets has garnered significant attention within the field of general topology. This interest stems from their role as natural extensions of closed sets and their potential to introduce novel separation axioms, particularly relevant in digital topology studies such as those concerning the digital line [1, 2]. The investigation into generalized closed sets began with work in 1970 as discussed in [3-5]. Subsequently, generalized - closed sets in [6, 7]. In 1995, further expanded this area by defining and examining generalized semi-preclosed sets, the semi-preopen sets by [8-10]. The class of generalized preclosed sets by [11] in 1990, and more recently, researcher in [12] presented the generalized preregular closed sets. In [13-15] presented a comprehensive diagram illustrating various closed sets, along with two accompanying questions, which were later addressed in [16-20] presented the class of - closed sets, while in Section 4. Additionally, Section 5 introduces the notion of pre-generalized closed mappings and investigates.

2. PRELIMINARIES

(K,τ), (M,σ) and (Z,θ), represent nonempty topological spaces on a subset H of (K,τ), $Cl(H)$, $\operatorname{Int}(H)$ indicated the H closure with τ , H .

For some unknown concepts concerning the following definitions, the reader may refer to [21- 31].

Definition 2.1 (K,τ) is a topological - space then a subset H is:

(1) If $Cl(\operatorname{Int}(H)) \supseteq H$ then it is a set of semi open [21] and if $H \supseteq \operatorname{Int}(Cl(\operatorname{Int}(H))$ then it is a set of a semi closed,

(2) If $\operatorname{Int}(Cl(\operatorname{Int}(H)) \supseteq H$ then it is a preopen set [27], and if $H \supseteq \operatorname{Int}(Cl(\operatorname{Int}(H))$ then it is a semi closed set,

(3) an γ-open- set [30,32,33], if $\operatorname{Int}(Cl(\operatorname{Int}(H))) \supseteq H$ then γ-open set [30], and an if $H \supseteq Cl(\operatorname{Int}(Cl(H)))$ then γ-closed set [28,29,31],

Definition 2.2 A subset H of a space (K,τ) is said to be pre-generalized closed (abbreviated by tn -closed) [32] if $tCl(H) \subseteq S$ whenever $H \subseteq S$ and $S \in PO(K,\tau)$.

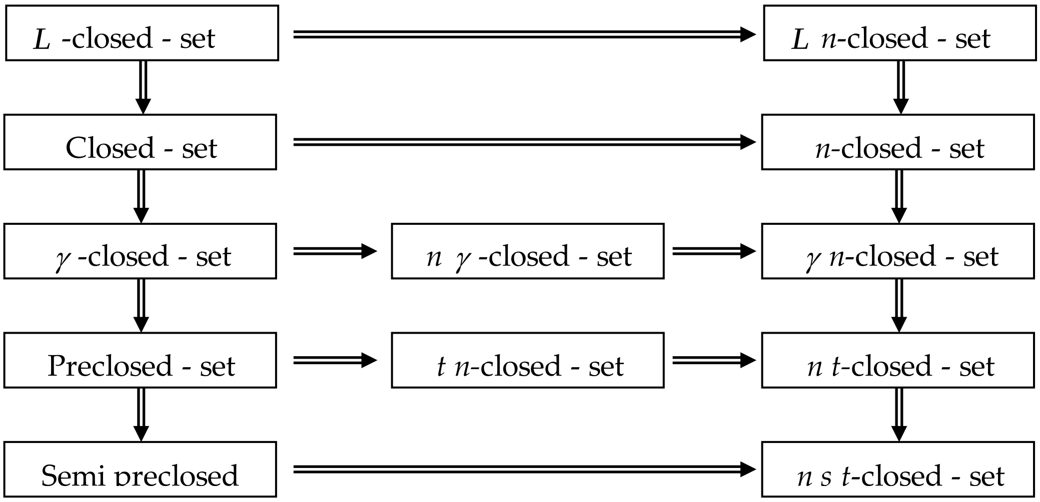

Fig. 1. Pre-generalized open.

However, by Example 3.1 of [13], Examples 3.2 of [33], Example 2.2 of [34], Example 2.3 of [35], Example 2.1 of [25].

Example 2.1 Let $H=\{u,v,h,m,e\}$ and $\tau=\{\phi,\{u\},\{v,h\},\{u,v,h\},\{v,h,m\},\{u,v,h,m\},K\}$. Set $H=\{v,h\}$. Observe that $tCl(H)=\{u,v,h,m,e\}$ and $stCl(H)=\{v,h,m\}$. Then H is nst-closed, neither nt-closed not nt-closed in (K,τ).

Example 2.2 Let $K=\{u,v,h,m,e\}$, $\tau=\{\phi,\{u,v\},\{h,m\},\{u,v,h,m\},K\}$. Let $H=\{u\}$ and $C=\{v\}$. Here H and C are nt-closed but $H\bigcup C=\{u,v\}$ is not nt-closed, since $\{u,v\}$ is preopen and $tCl\{u,v\}=\{u,v,e\}$.

Example 2.3 If $tCl(H)\setminus H$ contains (no non-empty) preclosed set, then H need not be nt-closed. Let $K=\{u,v,h,m,e\}$ and $\tau=\{\phi,\{u,v\},\{h,m\},\{u,v,h,m\},K\}$. Consider $H=\{u,v\}$. Then $tCl(H)=\{u,v,e\}$, $tCl(H)\setminus H=\{e\}$ which not contain non-empty preclosed set. Also, H is not nt-closed.

Sufficiency. Suppose O is preclosed and $O\subset H$ implies $H\subset t\,\mathrm{Int}(H)$. Let $K\setminus H\subset U$ where $U\in PO(K)$. Then $K\setminus U\subset H$ where $K\setminus U$ is preclosed. $K\setminus U\subset t\,\mathrm{Int}(H)$. $K\setminus t\,\mathrm{Int}(H)\subset U$. By Remark 3.3, we have $tCl(K\setminus H)\subset U$. This implies $K\setminus H$ is -closed, H is nt-open.

Example 2.4 Let $K=\{u,v,h\}$ and $\tau=\{\phi,\{u\},\{v\},\{u,v\},K\}$. Let $A=\{v\}$. Then $tCl(H)=\{v,h\}$ and $tCl(H)\setminus H=\{h\}$ which nt-open in (K,τ). While H is not nt-closed in (K,τ).

(K,τ) is called submaximal if each of its dense subsets are open.

3. Pre-Generalized Continuous, Pre-Generalized Irresolute Mapping

Definition 3.1 A mapping $f:(K,\tau)\rightarrow(M,\mu)$ is pre-generalized continuous (uv-breviated by tn-continuous) if $f^{-1}(I)$ is tn-closed in (K,τ) for every closed set I of (M,μ)

Example 3.1 Let $K=M=[0,1]$ and let $f:K\rightarrow M$ defined as: $f(x)=1$ if $x\in[0,1/2]$, $f(x)=0$ if $x\in[1/2,1]$. So one can deduce that f is precontinuous, therefore, by Remark 4.1, tn-continuous but from [6], Example 2.3 it is not continuous.

Remark 3.1 The following Fig. 2 of mappings.

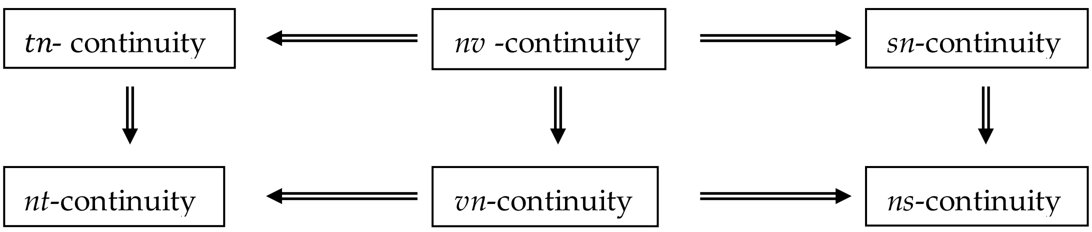

Fig. 2. Pre-Generalized Irresolute Mapping.

Theorem 3.1 $f:(K,\tau)\rightarrow(M,\mu)$ is a mapping then the condition is equivalent:

(1) O is tn-continuous,

Proof. (1) $\Leftrightarrow$ (2): From Definition 3.1.

Example 3.2 Let $K=M=\{u,v,h\}$, $\tau=\{\phi,\{u\},\{h\},\{u,h\},K\}$ and $\mu=\{\phi,\{u\},M\}$. Let $f:(K,\tau)\to(M,\mu)$ be a mapping defined as follows: $f(u)=v=f(h)$ and $f(v)=h$. It is easily observed that f is tn-continuous. However, $\{v\}$ is tn-closed in Y but $f^{-1}(\{v\})$ is not tn-closed in K. This shows that f in not tn-irresolute.

Theorem 3.2 Let $f:(K,\tau)\to(M,\mu)$ be tn-irresolute, then a subset H of K, $f(tnCl(H))\subset tCl(f(H))$.

Proof. If $H\subset K$, then $Cl(f(H))$ which is tn-closed in M., $f^{-1}(tCl(f(H)))$ is tn-closed in K. $H\subset f^{-1}(f(H))\subset f^{-1}(tCl(f(H)))$. Therefore by the definition of tn-closure $tnCl(H)\subset f^{-1}(tCl(f(H)))$, consequently, $f(tnCl(H))\subset f(f^{-1}(tCl(f(H))))\subset tCl(f(H))$.

4. Pre-Generalized Closed Mappings

Definition 4.1 $f:(K,\tau)\to(M,\mu)$ is a mapping that pre-generalized closed set if a closed set O of K. $f(O)$ is tn-closed (resp. tn-open) in M. We use thee abbreviations "tn-closed mapping" to mean pre-generalized closed mapping and "tn-open-mapping" to mean pre-generalized open mapping.

Example 4.1 Let $K=\{u,v\}$, $M=\{t,j,r\}$, $r=\{\phi,\{u\},K\}$ and $\mu=\{\phi,\{t,j\},M\}$. define a mapping $f:(K,\tau)\to(M,\mu)$ as: $f(u)=r$, $f(v)=j$. So f is tn-closed map but it is not nu-closed. Since $\{v\}\subset K$ is closed, $f(\{v\})=\{j\}$ and $\{j\}\subset\{t,j\}\in\mu^{\mu}$, but $Cl_{\gamma}(\{j\})=M\not\subset\{t,j\}$.

Example 4.2 Let $K=\{t,j,r\}$, $M=\{u,v,h\}$, $\tau=\{\phi,\{t\},K\}$ and $\mu=\{\phi,\{u\},M\}$. Let $f:(K,\tau)\to(M,\mu)$ a mapping defined as: $f(t)=c$ and $f(j)=v$ and $f(r)=u$. f is nt-closed map but it is not nt-closed.

Theorem 4.1 Let $f:(K,\tau)\to(M,\mu)$ is a bijective mapping. So the conditions are equivalent:

(1) f is a nt-closed mapping,

(2) f is a nt-open mapping,

(3) $f^{-1}:(M,\mu)\to(K,\tau)$ is a nt-continuous.

Theorem 4.2 A mapping $f:(K,\tau)\to(M,\mu)$ is tn-closed (resp. tn-open) if for a subset S of M and for an open (resp. closed) set U containing $f^{-1}(S)$, then a tn-open (resp. tn-closed) subset I of M that $S\subset I$, $f^{-1}(I)\subset U$.

Proof. If Let S a subset of M, U open set of K that $f^{-1}(S)\subset U$. Then $M\setminus f(K\setminus U)$, I, is a tn-open set that $f^{-1}(I)=K\setminus f^{-1}(f(K\setminus U))\subset U$. Let O a closed set of K. Then, $f^{-1}(M\setminus f(O))\subset K\setminus O$ and $K\setminus O$ is open. There is a tn-open subset I of M such that $M\setminus f(O)\subset I$ and $f^{-1}(I)\subset K\setminus O$. Then, $O\subset K\setminus f^{-1}(I)$ and $M\setminus I\subset f(O)\subset f(K\setminus f^{-1}(I))\subset M\setminus I$. Which implies $f(O)=M\setminus I$ is tn-closed, $f(O)$ is tn-closed, thus f is a tn-closed mapping.

Theorem 4.3 If $f:(K,\tau)\to(M,\mu)$ is $f:(K,\tau)\to(M,\mu)$, H is a closed set of X, so ($f_H:(H,\tau|H)\to(M,\mu)$) is tn-closed.

Proof. A closed set O of H. Since O is closed in K, so $(f|H)(O)=f(O)$ is a tn-(closed set of O). So, is a -tn closed map.

Theorem 4.4 If $f:(K,\tau)\to(M,\mu)$ is bijective preirresolute and M-preopen and if C is a $f:(K,\tau)\to(M,\mu)$ subset of Y, so $f^{-1}(C)$ is tn-open in K.

Proof. Suppose that C is tn-closed in M and that $f^{-1}(C)\subset S$ where S is preopen in K. We will show that $tCl(f^{-1}(C))\subset S$. That f is surjective, $C\subset f(S)$ holds. Hence $tCl(C)\subset f(S)$. Therefore, $f^{-1}(tCl(C))\subset S$. Since f is preirresolute and $tCl(C)$ is preclosed in M, $tCl(f^{-1}(tCl(C)))\subset S$ and so

$t\,Cl\,(f^{-1}(C)) \subset S$. Therefore $f^{-1}(C)$ is $t\,n$ -closed in K.

Remark 4.1 The following Fig. 3 Pre-Generalized Closed Mappings.

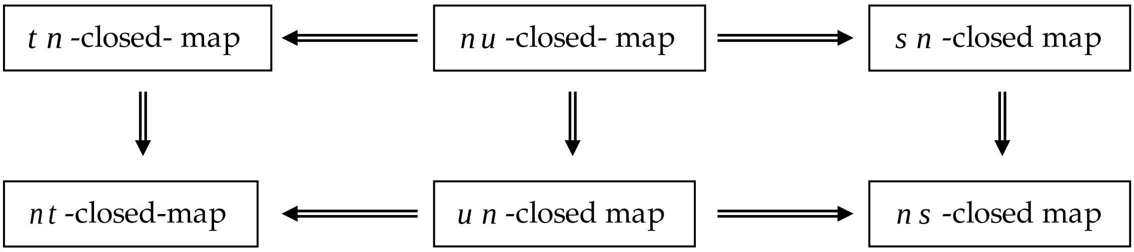

Fig. 3. Pre-Generalized Closed Mappings.

5. APPLICATION

Here, we show that a topological space via $t\,n$ -open sets

Definition 5.1 Let $\{H_\gamma : \gamma \in \nabla\}$ of $t\,n$-open sets is called $t\,n$ -open cover of a subset C of X if $C \subset \{H_\gamma : \gamma \in \nabla\}$ holds.

Definition 5.2 Let *K* be a topological space called compact pre-generalized (or $t\,n$ -compact) if finite subcover is $t\,n$ -open of *K*.

Theorem 5.1 Every $t\,n$ -closed subset of a $t\,n$ -compact space *K* is $t\,n$ -compact relative to *K*.

Proof. Every Let *H* be a $t\,n$ -closed subset of *K*. So $K \mid H$ is $t\,n$ -open in *K*. $W = \{S_u : u \in \nabla\}$ be a cover of *H* by $t\,n$-open subset in *K*. Then $W^* = W \cup (K \setminus H)$ is a $t\,n$-open cover of *K* i. e., $K = \left[\cup[S_\gamma : \gamma \in \nabla\}\right] \cup (K \setminus H)$

By hypothesis, *K* is $t\,n$ -compact, hence W^* is reducible of *K*, say $K = S_{\gamma_1} \cup S_{\gamma_2} \cup \ldots \cup G_{\alpha_m} \cup (X \setminus A)$, $G_{\alpha_k} \in W$. A , its complement disjoint, then $A \subset G_{\alpha_1} \cup G_{\alpha_2} \cup \ldots \cup S_{\gamma_m} \cup (K \setminus H)$, $S_{\gamma_k} \in W$. It was shown that $t\,n$ -open of *W* of *H* is a subcover, this completes the proof.

6. CONCLUSIONS

The purpose of this paper is to point out extremely elementary character of the proofs and to get unknown results by special choice of the generalized closed set. The theory of generalized closed sets has been discussed extensively in recent years by many topologists. More importantly, generalized closed sets suggest some new separation axioms which have been found to be very useful in the study of certain objects of digital topology, for example the digital line and the possible applications in computer graphs and quantum physics.

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