3D Finite Element Modeling of Dowel Jointed Plain Concrete Pavement

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Abstract
The objective of this study is to develop three dimensional finite elements models to investigate the effect of many factors on the behavior of dowel Jointed Portland Cement Concrete Pavement (JPCCP). The 3D-FE model is applied to analyze the critical bending stresses and deflections for understanding some modes of distress. Moreover, developed equation for predicting the non-uniform distribution of the modulus of subgrade reaction beneath concrete slab subjected to concentrated load is achieved. The accuracy of the developed equation is verified by a comparison with Winkler approach such as SAP program. Winkler’s solutions are found to overestimate the maximum deflections by about 7.5% more than 3D-FE solutions up to a horizontal distance of 60 inch from the load center. In general, the results show that the developed 3D-FE is suitable for identifying the effect of different design features on the structural response of rigid pavements.

Keywords
"Rigid pavements; Finite element analysis; Modulus of sub-grade reaction; Deflection; Bending stresses; Winkler foundation."

1. Introduction
The Westergaard idealization has been the basis for the Federal Aviation Administration's (FAA) concrete pavement structural design procedure. Westergaard’s theory is limited by two significant shortcomings: (a) only a single slab panel is accommodated in the analysis; therefore, load transfer at joints is not accounted for, and (b) the layered nature of the pavement foundation is not explicitly reflected in the Winkler foundation model. To account for the increased capacity of the foundation caused by a stabilized layer, the modulus of subgrade reaction is inflated [1]. To address the limitations of the available concrete pavement response models, the FAA initiated a research effort to develop a three-dimensional (3D) finite element model of the concrete pavement slab-joint-foundation system that can be implemented in advanced pavement design concepts currently under development by the FAA. The use of three-dimensional (3D) finite element (FE) methods for analyzing rigid pavements subjected to mechanical and environmental loadings has grown significantly over the past decade. The increased use of 3D FE analysis has provided pavement researchers and designers with a better understanding of critical aspects of pavement response that cannot be captured with analytical solutions, such as joint load transfer, the effect of slab support on stresses (3), and pavement response under dynamic loads [2]. Stresses are developed in rigid pavements as a result of several factors, including the traffic wheel loads, the expansion and contraction of the concrete due to temperature changes, yielding of the subbase or subgrade supporting the concrete pavement and volumetric changes. For example, traffic wheel loads will induce flexural stresses that are dependent on the location of the vehicle wheels relative to the edge of the pavement, whereas expansion and contraction may induce tensile and compressive stresses, which are dependent on the range of temperature changes in the concrete pavement. These different factors that can induce stress in concrete pavement have made the theoretical determination of stresses rather complex. The supporting subbase and/or subgrade layer acts as an elastic material so that it deflects at the application of the traffic load and recovers at the removal of the load [3].

2. Problem statement and study objective.
Many researches were conducted on the optimal design of rigid pavement. Most of these researches used the conventional Winkler approach, (such as SAP program) in which the subgrade is represented by independent springs having a constant coefficient called the modulus of subgrade reaction. This assumption lads to many errors in the design parameters (i.e., deflections, shears, and bending
moment) [4]. Moreover there is no easy way to determine the modulus of subgrade reaction (Ksg ) because its value is not unique for a given type of soil. Recently, the use of three dimensional finite element method has provided more accurate results. However, there are many aspects of rigid pavement behavior that have not been thoroughly studied with 3D FE analysis. This can be attributed to several factors, including the complexity of concrete pavement structures (especially joint load transfer mechanisms), the need to consider both environmental and mechanical load effects, the difficulty of model generation and result interpretation, and the relatively long solution times required for large 3D FE analyses. These factors become especially challenging for the analyst when general purpose FE programs are used. To circumvent these issues, 3D FE analysis packages have been developed specifically for analyzing rigid pavements [5]. In this study, A comprehensive 3D finite element modeling technique is achieved to provide a rational approach for the structural response factors such as deflection and bending stresses of the jointed rigid pavement system using different parameters through two cases of loading (interior case, and edge case). These parameters are: modulus of elasticity of subgrade, dowel bars spacing, dowel bars diameter, and slab thickness. Moreover developing a mathematical equation using the finite element code EverFE is used compared with the commercial program SAP2000 for calculating the non-uniform distribution of winker coefficient of elastic subgrade beneath slab foundation subjected to concentrated load.

**Literature review**

The ability of rigid pavement to sustain a beam like action across irregularities in the underlying materials suggests that the theory of bending is fundamental to the analysis of stresses in such pavements. The theory of a beam supported on an elastic foundation can therefore be used to analyze the stresses in the pavement when it is externally loaded. This pressure is given in Eq. (1) [5] as:

\[ p = K_{sg} y \]

Where:

\[ P = \text{reactive pressure at any point beneath the beam (lb/in)²);} \]

\[ y = \text{deflection at the point (in);} \]

\[ K_{sg} = \text{modulus of subgrade reaction (lb/in³).} \]

Kamyar et al.[6] have operated an experimental project to study the effect of environmental loads (i.e., temperature, moisture, precipitation and frost heave). They developed a three dimension finite element pavement model using ANSYS program to predict the mechanical behavior of the rigid pavement. It was demonstrated that the three dimensional finite element pavement models were successfully correlated to the stresses and staines in the concrete slabs. George et al. [7] highlighted the features of the program EverFE2.2, which has been developed specifically for the 3D finite-element analysis of jointed plain concrete pavements. They concluded that Slab stresses can be highly affected by shear transfer between the slab and base. In turn, the degree of slab-base shear depends on base type and the particular environmental loading (combination of temperature gradient and uniform shrinkage) considered in an analysis. Moreover, they obtained that the effect of dowel locking on stresses due to pure shrinkage and combined shrinkage and thermal gradients is significant for the range of slab-base shear transfer values considered here. The solution of a six-slab thick-plate model for jointed concrete pavements subjected to vertical loads have been developed by Liu Wei [8]. The theoretical solution based on the six-slab model is superior to the Westergaard solutions in two main aspects: (a) the explicit consideration of the finite dimensions of the slab panels in a jointed pavement system, and (b) the presence of joints and their load transfer efficiency are included in the formulation and solution of the six slab model. Analyses by the six-slab model have presented the trends of changes in critical bending stresses and deflections with two major characteristics of jointed pavement systems: slab size and load transfer efficiency of pavement joints. Yunus et al. [9] made a comparison between FEM and conventional method and concluded that the values of stresses obtained by Westergaard's method and the finite element method weren't different if the same foundation model (Winkler foundation) was considered in both cases. Davids [10] examined issues related to the finite element modeling of base and subgrade materials under jointed plain concrete pavements using the 3D FE program EverFE. Based on these results, use of an equivalent dense liquid foundation modulus in mechanistic rigid pavement analysis or design wasn’t recommended when stiff base layers were present. A rational, 3D finite element modeling technique has been developed by Patil et al. [11] to predict the response of the jointed rigid airport pavement system to aircraft loadings. There are some notable differences in the magnitudes of the predicted and observed deflection-based joint efficiencies. The finite element model more accurately predicts stress-based load transfer efficiency. This is encouraging, because current rigid pavement design methodologies are based upon a design stress that is calculated by reducing the critical edge stress by a stressbased load transfer factor. The literature review shows that many pavement computer response models based on the FE method were developed for the analysis of jointed pavement slabs, however, important considerations were overlooked. The major assumptions which has been used in the conventional methods of design of rigid pavements is Winkler Zimmerman’s assumption, in which the subgrade is represented by springs called the modulus of subgrade reaction (Ksg). According to this assumption, the subgrade reaction is proportional to the vertical deflection at that point, but independent of the vertical deflection at any other point. In the case of an elastic continuum, which is the more realistic situation for the subgrade underneath the pavement, this assumption may produce serious
errors. A vertical force at any point actually produces deflections at all other points. Thus the stiffness of the subgrade should be represented by a stiffness matrix rather than one stiffness coefficient as has been done in the conventional methods. Moreover there is no easy way to determine this value (Ksg) because its value is not unique for a given type of soil [12]. If the analysis is performed for a slab subjected to uniformly distributed load, there is no provision for differential settlement, bending moments, or shear forces in the slab, in disregard of reality. Many researchers have proved this lack of uniqueness of Ksg.

4. Features of the EverFE Model

EverFE1.02, which was first made available in 1998, addressed these difficulties through the use of an interactive graphical user interface allowing easy model definition and visualization of results, specialized techniques for modeling both dowel and aggregate interlock joint load transfer, and fast iterative solution strategies that allow the inclusion of inequality constraints for modeling slab-base separation and material nonlinearity [13]. EverFE2.25 employs several element types to discretize concrete pavement systems having from one to nine slab/shoulder units. Up to three elastic base layers can be specified below the slab, and the subgrade is idealized as either a tensionless or tension-supporting dense liquid foundation. Twenty-noded quadratic hexahedral elements are used to discretize the slabs and elastic base layers, and the dense liquid foundation is incorporated via numerically integrated, 8-noded quadratic elements that are meshed with the bottom-most layer of solid elements. Linear or nonlinear aggregate interlock joint load transfer as well as dowel load transfer can be modeled at transverse joints. Load transfer across longitudinal joints via transverse tie bars can also be modeled [14].

4.1 Model description

The six-slab model was applied to analyze the critical stresses and deflections of an edge slab under two loading conditions commonly considered in concrete pavement analysis and design: interior and edge loadings. The length and width of the slab are a = 15 ft, b = 15 ft, with different thickness with a single base layer. The slab has an elastic modulus of E = 26000 MPa, and a Poisson’s ratio of u = 0.25 and density of 2400 kg/m3. the base layer of 6 in thickness has an elastic modulus of 350 MPa and a Poisson’s ratio of u = 0.25. The dense liquid foundation was assumed to have a modulus of subgrade reaction of 0.03 MPa/mm. dowels of 200000 MPa elastic modulus and 0.3 Poisson’s ratio and 460 mm long with different diameter and spacing are considered in this study.

4.2 Specification of loads

The loading condition considered in the analyses is a single, transversely centered axle consisting of two 40 kN wheel loads spaced at 1830 mm and applied at the joint. Each wheel load is idealized as a uniform pressure over a 150 mm by 300 mm long patch. This loading condition is chosen because it represents a common, repetitive loading most likely to result in dowel looseness and damage to the supporting soil [13].

4.3 Meshing and solution

EverFE employs an isoperimetric, 20-noded, quadratic hexahedral element, shown to be superior to an 8-noded linear brick element for modeling rigid pavement systems. The dense liquid foundation is meshed using isoperimetric, 8-noded, quadratic elements. By default, the horizontal boundary conditions are the minimum required to prevent rigid body motion of the system. A typical finite element mesh used in the analyses has 14,227 nodes and 43,365 degree of freedom where each slab has 432 brick elements.

5. Numerical Analysis

The EverFE code is used to perform a parametric study to show the effect of the different parameters on the behavior of rigid pavement. The different parameters used in this study are listed in Table 1.

Table 1: Different Parameters Used In This Study.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>modulus of elasticity of subgrade (Esg)</td>
<td>10000, 25000, 40000, 50000 and 75000 psi.</td>
</tr>
<tr>
<td>thickness of slabs (H)</td>
<td>8, 12, 16, 20 inch</td>
</tr>
<tr>
<td>dowel bar diameter (D)</td>
<td>0.75, 1.25 and 1.75 inch</td>
</tr>
<tr>
<td>dowel bar diameter (D)</td>
<td>0.75, 1.25 and 1.75 inch</td>
</tr>
<tr>
<td>dowel bars spacing (S)</td>
<td>7, 10 and 16 inch</td>
</tr>
<tr>
<td>Cases of loading</td>
<td>Interior and edge loading</td>
</tr>
</tbody>
</table>

5.1 Effect of elastic modulus of Subgrade

Fig. (1a) shows the slab deflections in the case of interior load along x-axis at different values of elastic modulus of subgrade (H = 12 in, D = 1.25 in, S = 16 in). It is clearly noted that that as the elastic modulus of subgrade (Esg) decreases the slab deflection increases where the maximum deflection in the slab occurs at the slab center. The slab deflection in the case of edge load is shown in Fig. (1b). The computed ratio of maximum deflection of the unloaded slab to the loaded slab ranges from 0.82 to 0.9. Fig. (2a) shows the bending stresses in the slab in the case of interior load along x-axis at different values of modulus of elasticity of subgrade where the maximum stresses occurs below the point of application of loads. After the maximum point the bending stresses decreases with increasing the horizontal distance. For edge loaded slab, Fig. (2b) illustrates that the maximum bending stress occurs under the center of gear load wheel closest to the edge of the loaded slab. the ratio of maximum stress of the unloaded to the loaded slab ranges from 0.35 to 0.45.
5.2 Effect of Slab thickness

The effect of slab thickness on the slab deflection in the case of interior loading is as shown in Fig. 3a (Esg = 25000 Psi , D = 1.25 in , S= 16 in ) . The results illustrate that the slab deflection decreases as the slab thickness increases. While in the case of edge loading as shown in Fig. (3b), the maximum deflection occurs at the edge of the loaded slab. The ratio of maximum deflection of the unloaded slab to the loaded slab ranges from 0.85 to 0.91. Figs. (4a) and (4b) show the bending stresses in the slab for each studied cases of loading . It is clearly noted that the maximum bending stresses occurs below the point of application of loads. The computed ratio of maximum stress of the unloaded slab to the loaded slab in the case of edge loading ranged from 0.36 to 0.46.

5.3 Effect of Dowel Bars Diameter

Dowel bars are load-transfer devices, and, thus, they must be fairly heavy and spaced at close intervals to provide resistance to bending and shear on the concrete. Fig. (5a) illustrates that the slab deflection due to interior loading is not affected by the dowel bars diameter. The slab deflection decreases by a huge rate with increasing the distance from slab center in direction up to 120 inch , after that. The slab deflection decreases in a slight rate. Fig. (5b) shows the slab deflection for loaded and unloaded slab in the case of edge loading where the maximum deflection of the unloaded slab to the loaded slab ranges from 0.76 to 0.9. Fig. (6a) shows the effect dowel bars diameter on the bending stresses where it is observed that the bending stresses are not affected where the maximum bending stresses occurs below the point of application of gear loads. The slab bending stress is shown in Fig. (6b) which illustrates that the ratio of maximum bending stress of the unloaded slab to the loaded slab bending stress is approximately linear. Similarly, the rate of maximum bending stresses increasing with decreasing the dowel bars diameter is fairly linear. Moreover It is clearly noted that the difference between the maximum bending stresses in the loaded and unloaded slabs decrease as the diameter of dowel bars increase.

5.4 Effect of Dowel Bar Spacing

Figs. (7a) and (76) show the effect of dowel spacing (S) on the slab deflection in two cases of loading (H = 12 in , Esg = 23415 Psi, D = 1.25 in). Figs. 8a and 8b show the effect of (S) on the bending stress in two cases of loading. The result illustrates that the slab deflection and stress are not affected by the of dowel bars spacing. The slab deflection decreases with increasing the distance from slab center in direction. The maximum bending stresses occurs below the point of application of loads. Moreover, the maximum deflection occurs at the edge of the loaded slab while the maximum bending stress occurs under the point of application of gear load. The ratio of maximum slab deflection ranges from 0.86 to 0.92 while the ratio of maximum bending stress ranges from 0.39 to 0.49. Moreover the difference between the maximum bending stresses of the loaded and unloaded slabs decreases as the dowel bars spacing increases. From the previous results it can be concluded that the maximum slab bending stress in case of edge load is higher than it in case of interior load by approximately 24% for the studied cases. While the maximum slab deflection in the case of edge loading is higher than it in case of interior load by approximately 20% for the studied cases.

6. Comparison between EverEF model and Winkler model

For analyzing slabs resting on a soil medium, engineers have been using a classical mathematical model called the Winkler model, where the behavior of the soil is simplified by means of independent springs placed continuously underneath the foundation. The corresponding spring constant is called the modulus of subgrade reaction (Ksg). Using this concept, many computer programs have been developed for the analysis of slabs on elastic foundation. There is no easy way to determine this value (Ksg ) because its value is not unique for a given type of soil. Thus, the finite elements code EverEF is used along with the commercial program SAP2000 to develop a mathematical equation for calculating the non-uniform distribution of Winkler coefficient of elastic subgrade beneath slab foundation subjected to loads.

6.1 Winkler coefficient distribution

The distribution of the coefficient of subgrade reaction is considered non-uniform beneath the slab [7]. In this study, different values of (Ksg) is investigated to select the best distribution. Three cases for the distribution of Winkler coefficient are studied:

1-The modulus of subgrade reaction is constant under slab area of 0.98 x 0.9L and tripled at the edges ;
2-The modulus of subgrade reaction is constant under slab area of 0.75B x 0.75L and double at the edges ;
3-The modulus of subgrade reaction is constant under slab area of 0.7B x 0.7L and doubled at the edges.

Fig. (9) shows a comparison between deflection obtained by EverFE2.25 and SAP2000 programs based on non-uniform distribution of modulus of subgrade reaction for slab 12x12 feet subjected to concentrated load of 80000 lb and Ksg was taken as 125 pci . For the above three cases, the maximum deflection difference (Df %) percentage between EverFE2.25 and SAP2000 to the obtained value from EverFE code were calculated. Regarding to Fig. 9, the Df for each distribution case were 1.11% , 0.27% and 0.169% respectively, it is observed that the distribution of modulus of subgrade reaction of the third case produced the smaller percentage of error.

6.2 Developing equation

Ever FE and SAP2000 programs are used to perform a parametric study for the purpose of developing equation for predicting the uniform distribution of the modulus of subgrade reaction.
The Poisson’s ratios of the soil and the slab (usu) as well as the modulus of elasticity of the slab (E) assumed as 0.3, 0.15 and 4000000 Psi, respectively. Non-dimensional terms used in the analysis are given as follows [7]:

\[
K_r = \frac{4E \left(1 - \mu_{su}^2\right) h^3}{\left(3E_{sg} \left(1 - \mu_{sg}^2\right) B^3\right)} \quad (2)
\]

Where:

- \(K_r\): relative stiffness;
- \(E\): modulus of elasticity of the slab;
- \(E_{sg}\): modulus of elasticity of subgrade material;
- \(I\): moment of inertia of the slab;
- \(L\): slab length; \(B\): slab width;
- \(K_{sg}\): non-dimensional coefficient of subgrade reaction;
- \(K_{sg}\): coefficient of subgrade reaction.

To develop the equation, EverFE is used to analyze the slab and the maximum deflection is recorded. SAP2000 was used to analyze the same slab and the modulus of subgrade reaction (Ksg) is changed till the of the maximum deflection approximately equal to that obtained by EverFE .The values of the non-dimensional parameters \(K_r\) and \(K_n\) is calculated for each problem using Eqs. 2 and 3.

The result for slab dimensions of 12*12 ft and concentrated load of 80000 lb is presented in Table 2. The percentage of deflection difference (Df%) is calculated .The relationship between \(K_r\) and \(K_n\) are plotted as shown in Fig. 10. The relationships between non-dimensional coefficient of subgrade reaction, \(K_n\) and the relative stiffness \(K_r\), for each load are as following:

\[
K_r = 12.699*(K_n)-0.8302 \quad (4)
\]

### 6.3 Evaluation of the developed equation

A number of slabs (12*12ft) subjected to concentrated load of 80000 lb are analyzed using EverFE code and SAP2000 according to the following examples groups where \(K_{sg}\) is obtained from the developed Equation:

<table>
<thead>
<tr>
<th>Group</th>
<th>Thickness (in)</th>
<th>E (Psi)</th>
<th>(K_{sg}) (Pci)</th>
</tr>
</thead>
<tbody>
<tr>
<td>g1</td>
<td>12</td>
<td>10000</td>
<td>135.5</td>
</tr>
<tr>
<td>g2</td>
<td>12</td>
<td>20000</td>
<td>200.8</td>
</tr>
<tr>
<td>g3</td>
<td>14</td>
<td>20000</td>
<td>233.8</td>
</tr>
<tr>
<td>g4</td>
<td>18</td>
<td>30000</td>
<td>245.8</td>
</tr>
</tbody>
</table>

For each data group, the value of relative stiffness \(K_r\) can be calculated from Eq. (2). Using Eq. (4), the corresponding value of non-dimensional coefficient of subgrade reaction \(K_n\) can be obtained. From Eq (3) the modulus of subgrade reaction \(K_{sg}\) can be calculated. This \(K_{sg}\) was used to represent the subgrade in SAP program. Figure 11 illustrates a comparison between slab deflection obtained by SAP2000 and by EverFE code. It can be indicated that the slab deflection values obtained by EverFE and by SAP2000 with \(K_{sg}\) calculated from the developed equation are in a good agreement, whereas, the maximum predicted slab deflection by SAP2000 are lower than predicted values by EverFE code by about 7.5% up to horizontal distance of 60 in for all studied groups. Thus it can be clearly noted that the developed 3D-FE is suitable for identifying the effect of different design features on the structural response of rigid pavements.

### 7. Conclusions

Three Dimensional Finite Elements Models is investigated to evaluate the dowel Jointed Portland Cement Concrete Pavement (JPCCP). The following conclusions are obtained:

1. In the case of interior loading, the maximum deflection and maximum bending stress is not affected by the dowel bars diameter and dowel bars spacing. While in the case of edge loading the ratio of maximum deflection in the unloaded slab to the loaded slab ranged due to dowel spacing variation from 0.86 to 0.92. However, for the maximum bending stresses, this ratio is ranged from 0.39 to 0.49.

2. The effect of increasing the slab thickness on reducing the maximum bending stress is more obvious than its effect on reducing the deflection in the case of interior loading . Increasing of modulus of elasticity of subgrade reduces the maximum deflection and bending stress significantly in both cases of loading. In the case of edge loading, The maximum deflection occurs at the edge of the loaded slab.

3. The maximum slab bending stress in case of edge load is higher than slab bending stress in case of interior load by approximately 24 % for the studied cases. While the maximum slab deflection in the case of edge loading is higher than the slab deflection in case of interior load by approximately 20 % for the studied cases.

4. From the studied three cases for the distribution of Winkler coefficient, the minimum error ratio (0.169%) is obtained when the modulus of subgrade reaction is constant beneath slab area of 0.78 x 0.71 and doubled at the edges. using this case any computer program based on Winkler approach (e.g., SAP2000) will produced maximum deflection in the concrete slab equal to the maximum deflection in the same concrete slab obtained by EverFE code.

5. The slab deflection values obtained by EverFE and by SAP2000 with \(K_{sg}\) calculated from the developed equation are in a good agreement, whereas, the maximum predicted slab deflection by SAP2000 are lower than predicted values by EverFE code by about 7.5% up to horizontal distance of 60 inch.

### References


temperatures. Cold Regions Science and Technology 57123–130.
Table 2: Comparison Between Maximum Deflection Obtained By ExoFE Code and SAP2000 Program for Slab 1x1.2 m Subjected to 1000 KN Concentrated Load.

<table>
<thead>
<tr>
<th>Thickness of Slab</th>
<th>$K_u$ (MPa)</th>
<th>Deflection (mm)</th>
<th>$E_u$ (MPa)</th>
<th>Deflection from SAP2000 (mm)</th>
<th>Deflection from ExoFE (mm)</th>
<th>Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 mm</td>
<td>5000</td>
<td>0.00015</td>
<td>100</td>
<td>0.00015</td>
<td>0.00015</td>
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<td></td>
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<td>0.00047</td>
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<tr>
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<td>20000</td>
<td>0.00063</td>
<td>100</td>
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</table>

Fig. 9. Comparison Between Slab Deflection.

Fig. 10. The relationship between $K_u$ and $K_e$.

Fig. 11. Comparison between slab deflection obtained by SAP2000 and ExoFE code.