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Modified Maximum Likelihood Estimation For The Inverse Weibull Parameters Based On Ranked Set Sampling

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Abstract

In this paper, estimation of the parameters of inverse Wiebull (IW) distribution is considered using method of maximum likelihood (ML) and method of modified maximum likelihood (MML) based on ranked set sampling (RSS).

1. Introduction

Ranked set sampling is recognized as a useful sampling technique for improving the precision and increasing the efficiency of estimation when the variable under consideration is expensive to measure or difficult to obtain but cheap and easy to rank. Ranked set sampling has been suggested by Mcintyre (1952) in relation to estimating pasture yields. Takahasi and Wakimoto (1968) established the theory of RSS, they showed that the sample mean of RSS is an unbiased estimator for the population mean and is more efficient than the sample mean of simple random sampling (SRS). The problems of estimation based on RSS from some distribution are discussed by several researchers, for example, Abu-Dayyeh et al. (2013)

considered estimation of the shape and location parameters of the Pareto distribution based on SRS and RSS. Hussian (2014) discussed Kumaraswamy distribution using SRS and RSS techniques based on maximum likelihood and Bayesian estimation methods. Helu et al. (2010) studied estimation of the parameters of Weibull distribution using different methods of estimation based on SRS, RSS and modified RSS. Sadek and Alharbi (2014) studied the problem of Bayesian estimation of the parameters for Weibull distribution based on RSS. Al-Saleh et al. (2003b) studied maximum likelihood estimation (MLE) and modified maximum likelihood estimation (MMLE) of the mean of exponential distribution based on moving extreme ranked set sampling (MERSS) under both perfect and imperfect ranking.

Zheng and Al-Saleh (2002) showed that the MMLE for the location parameter was always more efficient than MLE using SRS. For the scale parameter, the MMLE was at least as efficient as the MLE using SRS, when the same sample size was used. Balci et al. (2013) derived MMLEs for the population mean and variance of normal distribution under RSS and showed that they are considerably more efficient than RSS estimators. Also they suggested two new estimators for the unknown parameters using two modified RSS methods and showed that these methods make the variances of both MMLE and RSS estimators smaller. For some usual scale distributions, Chen et al. (2014) obtained an explicit form of the MMLE and proved that the MMLE is an unbiased estimator under MERSS. They also showed that the MMLE using MERSS is always more efficient than the MLE using SRS, when the same sample size is used. Al-Saleh and Al-Hadrami (2003a) studied the MMLE of location parameter and they showed that MMLE using MERSS is always more efficient than MLE using SRS for location parameter of normal distribution.

The paper aims to obtain the MLEs and MMLEs for the scale and the shape parameters of the IW distribution based on RSS. It is observed that the MLEs cannot be obtained in closed forms so, the MMLE is used. The rest of this paper is organized as follows: Section 2, is about types of RSS. The IW distribution is illustrated in Section 3. In Section 4, the MLEs are obtained based on RSS. The MMLEs based on RSS are derived in Section 5. Finally, a real data set is analyzed in Section 6 for illustrative purposes.

2. Ranked set sampling

According to RSS, we first select m^2 elements denoted by $x_{i(j)}$, $(i=1,2,\ldots,m; j=1,2,\ldots,m)$ from the population at random. These elements are then randomly splitted into m sets of m units each. On each set, we rank the m units by judgment or a supporting variable according to the characteristic of interest. We select the element with the smallest ranking, $x_{1(1)}$, for measurement from the first set. From the second set we select the element with the second smallest ranking, $x_{2(2)}$. We continue in this way until we have ranked the elements in the mth set and selected the element with the largest ranking, $x_{m(m)}$, as in Figure 1. This complete procedure, called a cycle which is repeated independently k times to obtain a ranked set sample of size $n = mk$ (see Chen et al. (2004)).

Sample					
1	$x_{1(1)}$	$x_{1(2)}$	....	$x_{1(m-1)}$	$x_{1(m)}$
2	$x_{2(1)}$	$x_{2(2)}$	....	$x_{2(m-1)}$	$x_{2(m)}$
.	.	.		.	.
.	.	.		.	.
.	.	.		.	.
m	$x_{m(1)}$	$x_{m(2)}$	....	$x_{m(m-1)}$	$x_{m(m)}$

Figure (1) Ranked set sampling

Marginally $x_{i(j)}$ have the same distribution with pdf given by (see David and Nagaraja (2003)).

$$f_m\left(x_{i(j)};\theta\right)=\frac{m!}{(i-1)!(m-i)!}\left[F(x_{i(j)};\theta)\right]^{i-1}\left[1-F(x_{i(j)};\theta)\right]^{m-i} f(x_{i(j)};\theta), -\infty<x<\infty$$

There are some methods of modified ranked set sampling, which are explained as follows:

a- Ranked set sampling by choosing both diagonal elements

Suppose that, m samples of size m are taken, and every sample is ranked in itself as in RSS design:

Sample					
1	$x_{1(1)}$	$x_{1(2)}$	....	$x_{1(m-1)}$	$x_{1(m)}$
2	$x_{2(1)}$	$x_{2(2)}$	....	$x_{2(m-1)}$	$x_{2(m)}$
.	.	.		.	.
.	.	.		.	.
m	$x_{m(1)}$	$x_{m(2)}$	....	$x_{m(m-1)}$	$x_{m(m)}$

Figure (2) RSS by choosing both diagonal elements

In this method, both diagonal elements are chosen as follows; the first and the mth order statistics are taken from the first sample ($x_{1(1)}$ and $x_{1(m)}$). From the second sample, the second and the (m-1)th order statistics are chosen ($x_{2(2)}$ and $x_{2(m-1)}$) , and so on, then the mth and the first order statistics are selected from the mth sample $x_{m(m)}$ and $x_{m(1)}$. The joint pdf of $x_{i(i)}$ and $x_{i(m-i+1)}$ is given by (see David and Nagaraja (2003)).

$$f_m(x_{i(i)},x_{i(m-i+1)})=\frac{m!}{(i-1)!(m-2i)!(i-1)!}\left[F(x_{i(i)})\right]^{i-1}\left[F(x_{i(m-i+1)})-F(x_{i(i)})\right]^{m-2i}$$
$$\times\left[1-F(x_{i(m-i+1)})\right]^{i-1}f(x_{i(i)})f(x_{i(m-i+1)}),$$

b- Ranked set sampling by choosing extremes of the samples

Consider m samples of size m are selected and every sample is ranked in itself as in RSS design;

$$\begin{array}{lcccc} \text{Sample} & & & & \\ 1 & x_{1(1)} & x_{1(2)} \;\cdots & x_{1(m-1)} & x_{1(m)} \\ 2 & x_{2(1)} & x_{2(2)} \;\cdots & x_{2(m-1)} & x_{2(m)} \\ . & . & . & . & . \\ . & . & . & . & . \\ m & x_{m(1)} & x_{m(2)} \;\cdots & x_{m(m-1)} & x_{m(m)} \end{array}$$

Figure (3) RSS by choosing extremes of the samples

According to this method, the smallest and largest order statistics from each sample are selected, which $x_{i(1)}$ and $x_{i(m)}$, $(i = 1,2,\ldots,m)$ are used as the random sample, the joint pdf of $x_{i(1)}$ and $x_{i(m)}$ is given by (see David and Nagaraja (2003)).

$$f_m\left(x_{i(1)}, x_{i(m)}\right) = m(m-1)\left[F\left(x_{i(m)}\right) - F\left(x_{i(1)}\right)\right]^{m-2} f\left(x_{i(1)}\right) f\left(x_{i(m)}\right)$$

c- Moving extremes ranked set sampling

The MERSS procedure is described as follows: first, choose m random samples of size $1,2,\ldots,m$, respectively. Second, identify the maximum of each sample by visual inspection or by some other relatively inexpensive method, without actual measurement of the characteristic of interest. Third, repeat the previous steps but for minimum. Finally, repeat the previous steps k times until the desired sample size, $n = 2km$ is obtained. The sample of these units is called the MERSS (see Al-Saleh and Al-Hadrami (2003a)).

Suppose that $(x_{i:1}, x_{i:2}, \ldots, x_{i:m})$ and $(y_{i:1}, y_{i:2}, \ldots, y_{i:m})$, $i = 1, 2, \ldots, m$, be $2m$ sets of random samples, they are independent and with pdf $f(x;\theta)$, cdf $F(x;\theta)$. Consider $x_{i:i} = max(x_{i:1}, x_{i:2}, \ldots, x_{i:m})$, $x_{i:i}$ is the ith order statistic of ith random sample, and $y_{1:i} = min(y_{i:1}, y_{i:2}, \ldots, y_{i:m})$, $y_{1:i}$ is the first order statistic of the ith random sample, $(i = 1, 2, \ldots, m)$, then $(x_{m:m}, x_{m-1:m-1}, \ldots, x_{1:1}, y_{1:m}, y_{1:m-1}, \ldots, y_{1:1})$ is a MERSS, then $x_{i:i}$ has the same distribution as the ith order statistics (maximum) in a sample of size i from $f(x;\theta)$, the pdf of $x_{i:i}$ is (see David and Nagaraja (2003)).

$$f_i(x_{i:i};\theta) = i\left[F(x_{i:i};\theta)\right]^{i-1} f(x_{i:i};\theta)$$

Also $y_{1:i}$ has the same distribution as 1st order statistics (minimum) in a sample of size i from $f(y;\theta)$, the pdf of $y_{1:i}$ is

$$f_i(y_{1:i};\theta) = i\left[1 - F(y_{1:i};\theta)\right]^{i-1} f(y_{1:i};\theta)$$

The likelihood function based on MERSS of size $2m$ is

$$L(\theta) = \prod_{i=1}^{m} i\left[F(x_{i:i};\theta)\right]^{i-1} f(x_{i:i};\theta)\, i\left[1 - F(y_{1:i};\theta)\right]^{i-1} f(y_{1:i};\theta)$$

3. Inverse Weibull

The Weibull distribution is one of the most common and great used models in life testing and reliability theory. However, it has been found that the Weibull distribution does not provide a satisfactory parametric fit for those lifetime distributions with non-monotone failure rate, such as the unimodal failure rate functions. The density and the hazard function of the IW distribution can be unimodal or decreasing, based on the choice of the shape parameter, and then the IW distribution is more appropriate model than the Weibull distribution. The IW distribution provides a good fit to several data such as the time to breakdown of an insulating fluid subjected to the action of a constant tension. Extensive work has been done on the IW distribution. The probability density function (pdf) and cumulative distribution function (cdf) of the IW distribution are, respectively, given by

$$f(y)=\alpha\beta y^{-(\beta+1)}e^{-\alpha y^{-\beta}},y>0,\alpha,\beta>0, \tag{1}$$

and

$$F(y)=e^{-\alpha y^{-\beta}}, \tag{2}$$

where α is the scale parameter and β is the shape parameter. The hazard rate function of the IW distribution is

$$h(y)=\alpha\beta y^{-(\beta+1)}\left(e^{\alpha y^{-\beta}}-1\right)^{-1}.$$

Many works have been suggested to estimate the unknown parameters of the IW distribution, see for example, Keller and Kamath (1982), Erto and Rapone (1984), Calabria and Pulcini (1994), Maswadah (2003) and Nassar and Abo-Kasem (2017).

4. Maximum likelihood estimation based on ranked set sampling

In this section, the ML estimation method is used to estimate the IW distribution parameters using RSS.

Consider $y_{1(1)}, y_{2(2)}, \dots, y_{n(n)}$ be a RSS of size n from the IW distribution with pdf (1) and cdf (2), then the log likelihood function can be written as

$$\ln L(\alpha,\beta) = n\ln(C\alpha\beta) - \alpha\sum_{i=1}^{n} i y_{i(i)}^{-\beta} - (\beta+1)\sum_{i=1}^{n}\ln y_{i(i)} + \sum_{i=1}^{n}(n-i)\ln\left(1-e^{-\alpha y_{i(i)}^{-\beta}}\right) \quad (3)$$

where

$$C = \frac{n!}{(i-1)!(n-i)!}$$

From (3), the likelihood equations can be written as follows

$$\frac{\partial \ln L}{\partial \alpha} = \frac{n}{\alpha} - \sum_{i=1}^{n} i y_{i(i)}^{-\beta} + \sum_{i=1}^{n}(n-i)\left(\frac{y_{i(i)}^{-\beta} e^{-\alpha y_{i(i)}^{-\beta}}}{1-e^{-\alpha y_{i(i)}^{-\beta}}}\right),$$

and

$$\frac{\partial \ln L}{\partial \beta} = \frac{n}{\beta} + \alpha\sum_{i=1}^{n} i y_{i(i)}^{-\beta}\ln y_{i(i)} - \sum_{i=1}^{n}\ln y_{i(i)} - \sum_{i=1}^{n}(n-i)\left(\frac{\alpha y_{i(i)}^{-\beta}\ln y_{i(i)} e^{-\alpha y_{i(i)}^{-\beta}}}{1-e^{-\alpha y_{i(i)}^{-\beta}}}\right) \quad (4)$$

It can be seen that the likelihood equations cannot be solved explicitly, so the MLEs of α and β can be obtained by using any numerical methods.

5. Modified maximum likelihood estimation based on ranked set sampling

In this section, the MMLs of the unknown parameters of the IW distribution are obtained using RSS.

It is observed that from (4) the ML equation of λ and θ cannot be obtained in explicit form, thus, the MMLEs which have closed form are obtained. Let $X = -\ln Y$, then X follows the extreme value distribution with pdf and cdf given, respectively, by

$$f(x) = \frac{1}{\sigma} \exp\left\{\left(\frac{x-\mu}{\sigma}\right) - \exp\left(\frac{x-\mu}{\sigma}\right)\right\}, \quad -\infty < x < \infty, \tag{5}$$

and

$$F(x) = 1 - \exp\left\{-\exp\left(\frac{x-\mu}{\sigma}\right)\right\} \tag{6}$$

where $\mu = -\ln \alpha/\beta$ and $\sigma = 1/\beta$ are the location and the scale parameters, respectively. From (5) and (6) and using RSS the log-likelihood function is given by

$$\ln L^*(\mu,\sigma) = -n \ln \sigma + \sum_{i=1}^{n} z_{i(i)} - \sum_{i=1}^{n} (n-i+1) e^{z_{i(i)}} + \sum_{i=1}^{n} (i-1) \ln\left(1 - e^{-e^{z_{i(i)}}}\right)$$

where

$$z_{i(i)} = \frac{x_{i(i)} - \mu}{\sigma}.$$

To obtain the MMLEs, we linearized the expression $g_1(z_{i(i)}) = e^{z_{i(i)}}$ and $g_2(z_{i(i)}) = \frac{e^{z_{i(i)} - e^{z_{i(i)}}}}{1 - e^{-e^{z_{i(i)}}}}$ in Taylor series around the points $\upsilon_i = \ln(-\ln(1 - p_i))$ and $p_i = i / n + 1$. Using only the first two terms, We get

$$g_1(z_{i(i)}) = \alpha_{1i} + \beta_{1i} z_{i(i)}$$

and

$$g_2(z_{i(i)}) = \alpha_{2i} + \beta_{2i} z_{i(i)}$$

where

$$\alpha_{1i} = e^{\upsilon_i}(1 - \upsilon_i), \qquad \beta_{1i} = e^{\upsilon_i}$$

and

$$\alpha_{2i} = \frac{e^{\upsilon_i - e^{\upsilon_i}}}{1 - e^{-e^{\upsilon_i}}} - \upsilon_i \beta_{2i}, \qquad \beta_{2i} = \frac{\left[e^{\upsilon_i - e^{\upsilon_i}}\left(1 - e^{-e^{\upsilon_i}}\right)\left(1 - e^{\upsilon_i}\right)\right] - e^{2(\upsilon_i - e^{\upsilon_i})}}{\left(1 - e^{-e^{\upsilon_i}}\right)^2}$$

Using these linear approximations, we can obtain the modified likelihood equations as

$$\frac{\partial \ln L^*}{\partial \mu} \cong -\frac{n}{\sigma} + \frac{1}{\sigma}\sum_{i=1}^{n}(n - i + 1)\left(\alpha_{1i} + \beta_{1i} z_{i(i)}\right) - \frac{1}{\sigma}\sum_{i=1}^{n}(i - 1)\left(\alpha_{2i} + \beta_{2i} z_{i(i)}\right),$$

and

$$\frac{\partial \ln L^*}{\partial \sigma} \cong -\frac{n}{\sigma} - \frac{1}{\sigma}\sum_{i=1}^{n} z_{i(i)} + \frac{1}{\sigma}\sum_{i=1}^{n}(n-i+1)z_{i(i)}\left(\alpha_{1i} + \beta_{1i} z_{i(i)}\right) - \frac{1}{\sigma}\sum_{i=1}^{n}(i-1)z_{i(i)}\left(\alpha_{2i} + \beta_{2i} z_{i(i)}\right)$$

The solutions of these equations are

$$\hat{\mu}_{MML.RSS} = E + \hat{\sigma} F$$

and

$$\hat{\sigma}_{MML.RSS} = \frac{-H + \sqrt{H^2 + 4nJ}}{2n}$$

where

$$E = \frac{\left[\sum_{i=1}^{n}(i-1)\beta_{2i} x_{i(i)} - \sum_{i=1}^{n}(n-i+1)\beta_{1i} x_{i(i)}\right]}{\left[\sum_{i=1}^{n}(i-1)\beta_{2i} - \sum_{i=1}^{n}(n-i+1)\beta_{1i}\right]},$$

$$F = \frac{\left[n - \sum_{i=1}^{n}(n-i+1)\alpha_{1i} + \sum_{i=1}^{n}(i-1)\alpha_{2i}\right]}{\left[\sum_{i=1}^{n}(i-1)\beta_{2i} - \sum_{i=1}^{n}(n-i+1)\beta_{1i}\right]},$$

$$H = \sum_{i=1}^{n}\left[1 - (n-i+1)\alpha_{1i} + (i-1)\alpha_{2i}\right]\left(x_{i(i)} - \hat{\mu}\right),$$

and

$$J = \sum_{i=1}^{n}\left[(n-i+1)\beta_{1i} - (i-1)\beta_{2i}\right]\left(x_{i(i)} - \hat{\mu}\right)^2$$

6. Numerical example

In this section, the real data set of Dumonceaux and Antle (1973) is used to show the applicability of the proposed estimators. The data set represents the maximum flood levels of the Susquehenna River at Harrisburg, Pennsylvenia over 20 four-year periods (1890–1969). The same data set was mentioned by Maswadah (2003), and he illustrated the fits well for the IW distribution of the mentioned data set. He obtained the MLE of α and β from the complete data set as $\hat{\alpha} = 0.0119$ and $\hat{\beta} = 4.3138$. Here, we choose a random sample of size 15 by using SRS and RSS, sampling was done with replacement. In RSS method, five matrices 3 × 3 are drawn and then applying the method presented in Figure 1. Table 1 displays the different estimates of the unknown parameters under SRS and RSS as well as the corresponding confidence intervals bounds. From table 1, it is noted that for parameter α the MML method under SRS perform better than other methods in terms of confidence interval length, while for parameter β the ML method under RSS perform better than other methods in terms of confidence interval length (CL).

Table 1. The ML and MML estimates of α and β, the lower and upper confidence bounds and the corresponding 95% confidence intervals under SRS and RSS for real data set.

Sampling	Parameter	MLE	LB	UB	CL	MMLE	LB	UB	CL
SRS	α	0.002	-0.001	0.005	0.006	0.002	0.001	0.004	0.002
	β	5.641	4.515	6.767	2.252	5.635	4.079	9.111	5.032
RSS	α	0.035	0.010	0.060	0.050	0.036	0.030	0.044	0.014
	β	3.351	2.676	4.026	1.350	3.338	2.782	4.170	1.388

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