



Evaluation and Improvement of Satellite based Global Gravity Models in Egypt

By

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ABSTRACT

Nowadays, it is importantly and urgently needed in Egypt the existence of precise geoid for many applications. For that reason, lot of terrestrial gravity data could be collected as gravity values, gravity anomalies, and GPS leveling undulations. These data might be collected from governmental agencies, petroleum companies, and research institutes and centers. These data cover large area of the Egyptian territory, but they have been taken with different references, instruments, accuracies, and methodologies.

At the same time, satellite missions concerning the gravity field of the earth started to yield global gravity models free from the mentioned defects of the terrestrial observations. Recently, many global geoid models have been computed based on satellite data. Meanwhile global satellite only models still suffer from the resolution issue. Those global models need to be verified, investigated, and improved using terrestrial (GPS/Leveling) data. Therefore, this thesis considers two main parts. The first one is evaluating the whole released, until 2019, satellite-based data global models against GPS/Leveling data. The second part is improving some of the best resulted models. EGM2008 and two low d/o satellite data models are also improved using the proposed improving method.

Concerning the first part; ninety-three satellite data based global models are evaluated against three GPS/Leveling data sets. EGM2008 as the famous common global model is also evaluated. The results showed that the best ten satellites-based data global models have data either from Goce only or Goce plus other missions like Grace and SLR. Three of them are Goce only data, three of them are (Goce + Grace), and four of them are (**Goce + Grace + SLR**). The best ten models have d/o ranges from 240 to 300. They considered recent models released from 2014 to 2019. The results showed also that the mean value of the undulation differences ranged from 53 to 56 cm for all the ten models while it was 63 cm in the case of EGM2008. The standard deviation values are ranged from 24.2 to 28.1 cm while it is 25.8 cm in the case of EGM2008. The best ten models have close results to each other and most of them are better than EGM2008 in the test areas.

Concerning the second part, three of the ten best models are subjected to the improvement process. Two methods are applied, one is the traditional first order polynomial and the second is proposed simple shifting method. Regarding that the GPS/leveling points in the three data sets have a longitudinal extension, the results showed that the proposed simple shift method with the three data sets gave around 8 cm as a mean and 11 cm as a standard deviation using one seventh of all data points corresponding to 15, 30, and 25 km as radii around the center point in the three data sets, respectively. The results of the simple shift method when using one eleventh and one fifteenth of the data points corresponding to 50 and 60 km as radii around the center point are not very far from the results in the

previous notice. The same can be said about the results of the polynomial method but the mean values in the shifting solutions are noticeably smaller than their corresponding values from the polynomial solutions.

Finally, the proposed simple shift method improved EGM2008 and other two low resolution models very significantly.

1. Introduction

The determination of the Earth's global gravity field is one of the main tasks of physical geodesy. Global gravity field models provide information about the Earth, its shape, its interior and fluid envelope, [1]. For instance, orthometric height and normal height systems are the most widely used systems in the world, these heights can be determined by traditional spirit leveling, a precise but inefficient method. With the contribution of satellite positioning and an accurate geoid model, the orthometric or normal heights can be determined at any point and at any time very efficiently, [2]. Obviously, the definition of a global height reference system is only possible with an accurate global gravity field model. If we speak about "heights above sea level", we simply refer to the geoid as a reference surface which is accessible at any point globally. The geoid, in turn, is an equipotential surface of the gravity potential [3].

Global gravity satellite missions have presented a series of noticeable activities, starting with Challenging Mini satellite Payload (CHAMP), followed by Gravity Recovery and Climate Experiment (GRACE) and the Gravity field and steady state Ocean Circulation Explorer (GOCE). One of the principal scientific objectives of GOCE satellite mission were to recover the global gravity field with an expected accuracy of about 1–2 cm (in terms of geoid undulation) or 1 mGal (in terms of gravity) about degree 200 in terms of spherical harmonics, which corresponds to about 100 km at the equator [4].

Ninety-three satellites only global geo-potential models released between 1996 and 2019 were chosen for the study. They are obtained from the models released by the International Center for Global gravity field Earth Models (ICGEM) web site, [5]. The geoid undulations of those models are extracted and tested against the corresponding (observed) geoid undulations from GPS and spirit leveling. The best satellite only global gravity models are chosen according to three observed data sets in different places in Egypt.

Two main objectives are assigned to be verified in this research. The first one is to define the best global

geoid model, among the global satellite only models, which gives the best representation of the study area in Egypt. Many of such study (evaluation) are done, they can be found in e.g., [6], [7], [8], [9], and [10].

The second one is to improve the accuracy of the global geoid models. Assessment should be done firstly to the GGMs against local undulations calculated from observed ground data from GPS/Leveling stations. Until now, ninety-three satellite only global gravity field models are released [5]. They will be tested against three terrestrial data sets in different locations in Egypt. The obtained values of the differences indicate the accuracy of the tested GGMs. In this study 121 ground stations extended along River Nile with known values of geoid undulations (N) obtained from observed GPS/Leveling data surveys are used to assess 93 satellite only global geoid models. Other two observed data sets in different places in Egypt are used to confirm the obtained results. The results are compared to the results of EGM2008 which is known as one of the best known GGM until now. The research aims also to improve the global models in the study area using the traditional first order polynomial and once more using a proposed simple shift method.

2. Research Methodology

The following methodology will be done to reach and achieve the target of the research. It can be depicted in the following steps:

- 1- The observed undulations are obtained for the observed data sets points by subtracting the orthometric height of the points from their corresponding GPS ellipsoidal heights.
- 2- Obtaining the undulations of all satellite only models (93 models) and EGM2008 for the 121 points of data set 1.
- 3- Evaluating of 93 satellites only models and EGM2008 against data set 1.
- 4- Choosing the best models, comparing to data set 1.
- 5- Evaluating the chosen best models against the other observed 2 data sets.

- 6- Applying first order polynomial and simple shifting proposal to improve three satellite only global models, to assure the obtained results.
- 7- Testing the proposed simple shift method to improve the highest d/o model (EGM2008) and two low d/o models.

3. Data Used Description

Available materials which will be used in this study can be classified into two main categories:

3-1. The observed Undulation data sets:

Dual frequency GPS receivers have been used during the observation. The stations relate to the nearest ESA GPS stations as reference stations. The orthometric heights of the stations were obtained by

spirit leveling from the nearest benchmarks in the study area.

The three observed undulation data sets used in this study are as follows:

Data set 1: 121 GPS/leveling stations were observed every about 5 km covering about 600 km distance from Assuit in the south to Domiat in the north. The stations are starting at Latitude 27° N to Latitude 31° N along the Nile River, figure 1.

Data set 2: 91 GPS/Leveling points along the North Coast from Port Said till Marsa Matroh with about 385 km as illustrated in figure 1.

Data set 3: 51 GPS/Leveling points along the Red Sea coast starting north of Bernece to Horgada with about 400 km length and they are shown in figure 1.



Figure 1: Three GPS/Leveling data sets

3-2. Website for downloading and interpolating the undulations of the global geoid models

All released satellite only models which developed based on the following satellite missions (CHAMP, GRACE, GOCE, LAGEOS, GRACE and GOCE, GRACE and CHAMP, GOCE and GRACE and LAGEOS); are downloaded from (<http://icgem.gfz-potsdam.de/ICGEM>), details of all released global models are existed in ICGEM website. EGM2008 Model, as the highest resolution combined global model until now, is also downloaded.

4. The Evaluation Process

To evaluate the satellite only global geoid models, the above-mentioned data are handled as follows:

- 1- The observed undulations are obtained for the observed data sets points by subtracting the orthometric height of the point from its corresponding GPS ellipsoidal height as:

$$N_i = h_i - H_i \quad (1)$$

Where N represents the geoid undulation and h refers to ellipsoidal height which observed by GPS and H refers to orthometric height which observed by spirit leveling. This has been done to all points in the three observed data sets.

Obtaining the undulations of all satellite only models (93 models) and EGM2008 for the 121 points of data set 1 from ICGEM website.

- 2- Evaluation of 93 satellites only models and EGM2008 against data set 1. Every observed undulation value is subtracted from its corresponding value of the 93 models to be evaluated. The undulation differences (ΔN) are obtained as:

$$\Delta N = N_{\text{model}} - N_{\text{observed}} \quad (2)$$

Where:

N_{observed} is the observed undulation from GPS/Leveling

N_{model} is obtained undulation from GGMs

ΔN is the difference in geoid undulation

Minimum, maximum, mean, and standard deviation of the obtained undulation differences are considered

Table (1): Statistical values of the best satellite only models based on data set 1.

MODEL NAME	MAX	MIN	MEAN	ST.DV
EGM 2008	1.75	0.34	0.63	0.166
EIGEN-GRGS.RL04.MEAN-FIELD	1.11	0.15	0.67	0.256
GGM05G	1.02	0.13	0.61	0.220
GO_CONS_GCF_2_DIR_R5	1.11	0.15	0.66	0.261
GO_CONS_GCF_2_DIR_R6	1.07	0.24	0.67	0.226
GO_CONS_GCF_2_TIM_R5	1.01	0.16	0.63	0.234
GO_CONS_GCF_2_TIM_R6	1.05	0.27	0.66	0.215
GOCO05s	1.05	0.17	0.63	0.235
GOCO06s	1.06	0.28	0.67	0.214
IfE_GOCE05s	1.20	0.14	0.66	0.259
ITU_GGC16	1.03	0.17	0.64	0.234

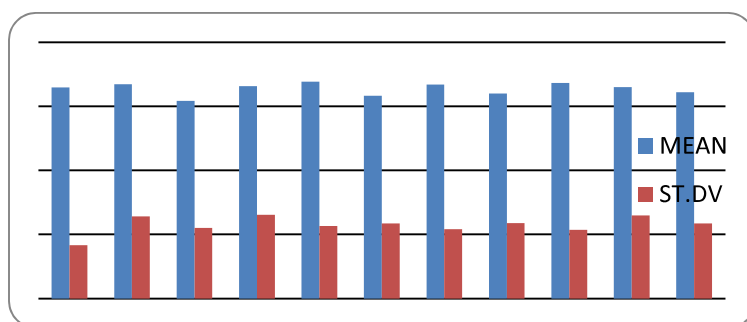


Figure (2): mean and standard deviations of the best ten models based on data set 1

The standard deviation values of the best ten models range from 21.4 to 26.1 cm while the mean value ranges from 61 to 67 cm which indicates that they are close to each other's. They are not so far from EGM2008. They are all recent models released from 2014 to 2019, d/o of these models ranges from 240 to 300.

the measures in the evaluation process. The results showed that the standard deviation values of all 93 satellite only models range from 0.215 to 2.57 m while that of EGM2008 is 0.166 m.

4-1. Choosing the best models

The best models are chosen according to min, max, mean and standard deviation of the resulted undulation differences. Because 93 models are too much to be evaluated against the three data sets points, the best models are chosen among the ninety-three tested models and they are shown in table (1) and figure (2)

4-2. Evaluating the chosen best models against the other observed 2 data sets

The chosen best GGM's are evaluated against the other 2 data sets points and the undulation differences are obtained and then their statistical values are computed.

Table (2): Statistical values of the chosen models when tested with data set 2

no	model name	Max (m)	Min (m)	MEAN (m)	ST.DV (m)
1	EGM2008	1.23	0.06	0.76	0.23
2	EIGEN-GRGS.RL04.MEAN-FIELD	0.93	0.04	0.57	0.18
3	GGM05G	1.03	0.03	0.52	0.22
4	GO_CONS_GCF_2_DIR_R5	0.93	0.04	0.57	0.18
5	GO_CONS_GCF_2_DIR_R6	0.91	0.05	0.57	0.17
6	GO_CONS_GCF_2_TIM_R5	1.02	0.10	0.58	0.21
7	GO_CONS_GCF_2_TIM_R6	0.89	0.05	0.57	0.16
8	GOCO05s	1.00	0.09	0.58	0.21
9	GOCO06s	1.93	0.05	0.59	0.21
10	Ife_GOCE05s	1.06	0.07	0.54	0.22
11	ITU_GGC16	1.02	0.10	0.59	0.21

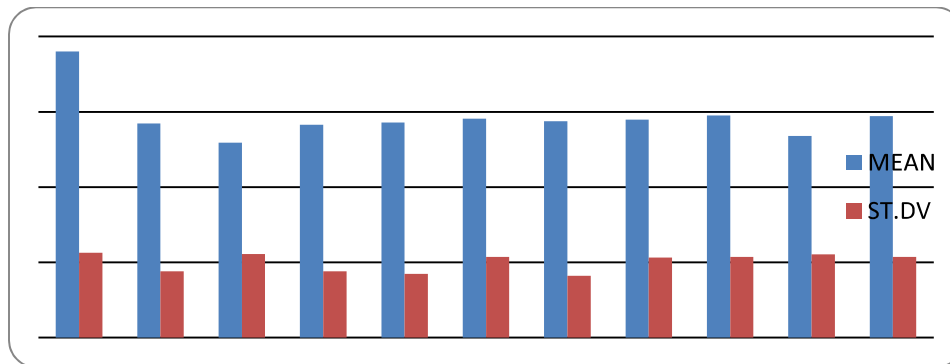


Figure (3): shows the mean and standard deviations of the chosen models when tested with data set 2.

Standard deviation values range from 16 to 22 cm while that of EGM2008 is 23 cm. The mean values range from 52 to 59 cm while that of EGM2008 is 76 cm.

Table (3): Statistical values of the chosen models when tested with data set 3

model name	Max (m)	Min (m)	MEAN (m)	ST.DV (m)
EGM2008	1.55	0.00	0.38	0.33
EIGEN-GRGS.RL04.MEAN-FIELD	0.83	0.01	0.30	0.18
GGM05G	1.12	0.00	0.35	0.22
GO_CONS_GCF_2_DIR_R5	0.83	0.01	0.30	0.18
GO_CONS_GCF_2_DIR_R6	0.88	0.01	0.28	0.18
GO_CONS_GCF_2_TIM_R5	0.80	0.03	0.31	0.18
GO_CONS_GCF_2_TIM_R6	0.87	0.02	0.28	0.18
GOCO05s	0.81	0.03	0.31	0.18
GOCO06s	0.86	0.01	0.28	0.18
Ife_GOCE05s	0.74	0.02	0.23	0.17
ITU_GGC16	0.81	0.04	0.31	0.18

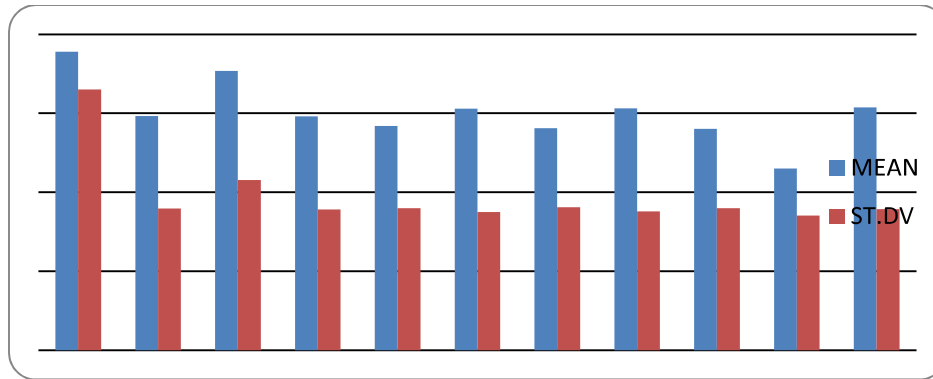


Figure (4): the mean and standard deviations of the chosen models when tested with data set 3

Standard deviations of 8 models are 18 cm and one model is 17 cm and another one is 22 cm while that of EGM2008 is 33 cm. The mean values range from 23 to 35 cm while that of EGM2008 is 38 cm.

The results obtained after evaluating the ninety-three models against data set 1 are classified according to the satellite data and illustrated as the following:

Table (4): Champ models assessment against data set 1

Model	Year	d/o	data	MAX m	MIN m	MEAN m	ST.DV m
EIGEN-CHAMP05S	2010	150	S(Champ)	5.45	0.21	0.97	0.51
AIUB-CHAMP03S	2010	100	S(Champ)	2.70	0.54	1.25	0.54
ULux_CHAMP2013s	2013	120	S(Champ)	2.45	0.00	1.25	0.65
TUM-2Sp	2003	60	S(Champ)	3.17	0.06	2.04	0.86
TUM-2S	2004	60	S(Champ)	3.20	0.10	2.05	0.86
TUM-1S	2003	60	S(Champ)	3.04	0.01	2.24	0.87
DEOS_CHAMP-01C	2004	70	S(Champ)	3.13	0.43	1.50	0.93
EIGEN-2	2003	140	S(Champ)	3.71	0.02	2.74	0.96
EIGEN-1	2002	119	S(Champ)	2.99	0.02	1.44	0.98
AIUB-CHAMP01S	2007	70	S(Champ)	3.27	0.02	1.72	0.99
ITG_Champ01K	2003	70	S(Champ)	3.43	0.01	1.96	1.04
EIGEN-CHAMP03Sp	2003	140	S(Champ)	3.11	0.00	1.27	1.10
ITG_Champ01E	2003	75	S(Champ)	3.69	0.20	1.62	1.15
ITG_Champ01S	2003	70	S(Champ)	3.64	0.15	1.85	1.17
EIGEN-CHAMP03S	2004	140	S(Champ)	3.32	0.01	1.48	1.20

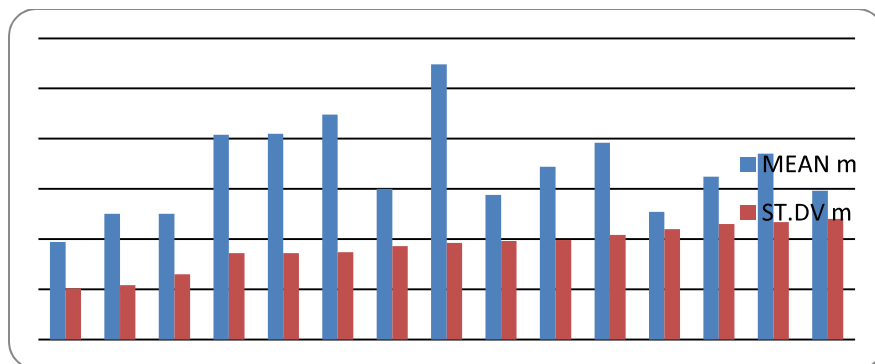


Figure (5): mean and standard deviations of CHAMP models based on data set 1

Fifteen CHAMP models are starting from 2002 till 2013. They have standard deviations from 0.51 to 1.20 m. They have low values of d/o from 60 to 150.

Table (5): (CHAMP + GRACE) models assessment against data set 1

Model	Year	d/o	data	MAX m	MIN m	MEAN m	ST.DV m
GOCO01S	2010	224	S(Champ), S(Grace)	1.81	0.00	0.62	0.46

One (CHAMP + GRACE) model has better standard deviation than CHAMP only models. More data increased the resolution and improved the ST DV.

Table (6): GOCE models assessment against data set 1

Model	Year	d/o	data	MAX m	MIN m	MEAN m	ST.DV m
GO_CONS_GCF_2_TIM_R6	2019	300	S(Goce)	1.76	0.27	0.67	0.215
GO_CONS_GCF_2_TIM_R5	2014	280	S(Goce)	1.63	0.16	0.63	0.23
GO_CONS_GCF_2_TIM_R4	2013	250	S(Goce)	1.50	0.01	0.54	0.30
GO_CONS_GCF_2_SPW_R5	2017	330	S(Goce)	1.80	0.15	0.64	0.31
GO_CONS_GCF_2_SPW_R4	2014	280	S(Goce)	1.59	0.13	0.57	0.31
JYY_GOCE04S	2014	230	S(Goce)	1.74	0.21	0.63	0.32
NULP-02s	2017	250	S(Goce)	1.62	0.00	0.58	0.35
GO_CONS_GCF_2_DIR_R1	2010	240	S(Goce)	1.50	0.07	0.65	0.35
GO_CONS_GCF_2_TIM_R3	2011	250	S(Goce)	1.71	0.00	0.61	0.35
JYY_GOCE02S	2013	230	S(Goce)	1.78	0.05	0.59	0.36
ITG-Goce02	2013	240	S(Goce)	1.68	0.01	0.63	0.38
GO_CONS_GCF_2_DIR_R2	2011	240	S(Goce)	1.50	0.06	0.56	0.39
GO_CONS_GCF_2_TIM_R2	2011	250	S(Goce)	1.77	0.00	0.60	0.41
IGGT_R1	2017	240	S(Goce)	1.34	0.00	0.62	0.42

GO_CONS_GCF_2_SPW_R2	2011	240	S(Goce)	1.90	0.02	0.64	0.44
GOSG01S	2018	220	S(Goce)	1.81	0.00	0.62	0.46
GO_CONS_GCF_2_TIM_R1	2010	224	S(Goce)	1.79	0.00	0.65	0.48
GO_CONS_GCF_2_SPW_R1	2010	210	S(Goce)	2.04	0.00	0.72	0.57

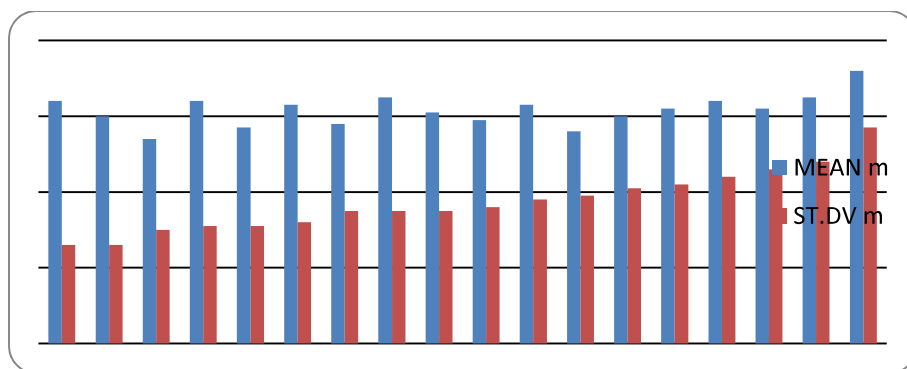


Figure (6): mean and standard deviations of GOCE only models based on data set 1

Eighteen GOCE only models have standard deviations from 21.5 to 57 cm. They are recent started from 2010 till 2019 with d/o from 210 to 330.

Table (7): (GOCE + Grace) models assessment against data set 1

Model	Year	d / o	d a t a	MAX m	MIN m	MEAN m	ST.DV m
GGM05G	2015	240	S(Goce), S(Grace)	1.47	0.13	0.61	0.22
ITU_GGC16	2016	280	S(Goce), S(Grace)	1.64	0.17	0.64	0.23
GOGRA04S	2014	230	S(Goce), S(Grace)	1.74	0.21	0.63	0.32
GOCO03s	2012	250	S(Goce), S(Grace)	1.75	0.00	0.62	0.36
GOGRA02S	2013	230	S(Goce), S(Grace)	1.78	0.05	0.59	0.36
GOCO02s	2011	250	S(Goce), S(Grace)	1.80	0.00	0.60	0.40
DGM-1S	2012	250	S(Goce), S(Grace)	1.70	0.01	0.64	0.40

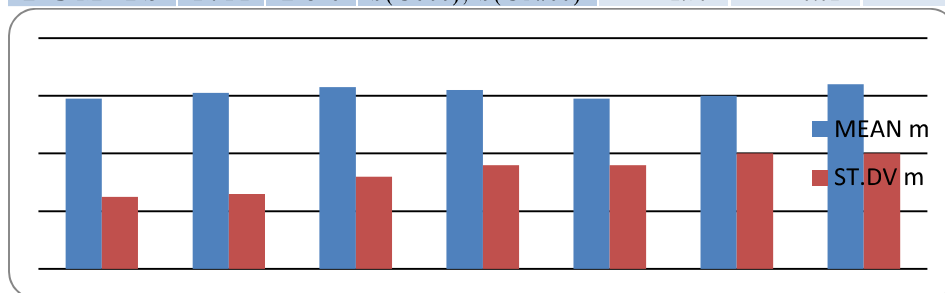


Figure (7): mean and standard deviations of (Goce + Grace) models based on data set 1

Seven (Goce + Grace) models have standard deviations from 22 to 40 cm. They are recent started from 2011 till 2016 with d/o from 230 to 280.

Table (8): (Goce + Grace + Lageos) models assessment against data set 1

Model	Year	d / o	data	MAX _m	MIN _m	MEAN _m	ST.DV _m
GO_CONS_GCF_2_DIR_R5	2014	300	S(Goce), S(Grace), S(Lageos)	1.75	0.15	0.66	0.26
EIGEN-6S2	2014	260	S(Goce), S(Grace), S(Lageos)	1.60	0.05	0.58	0.32
GO_CONS_GCF_2_DIR_R4	2013	260	S(Goce), S(Grace), S(Lageos)	1.60	0.05	0.58	0.32
GO_CONS_GCF_2_DIR_R3	2011	240	S(Goce), S(Grace), S(Lageos)	1.67	0.04	0.66	0.36
EIGEN-6S4 (v2)	2016	300	S(Goce), S(Grace), S(Lageos)	1.57	0.17	0.63	0.39
EIGEN-6S	2011	240	S(Goce), S(Grace), S(Lageos)	1.57	0.17	0.63	0.39

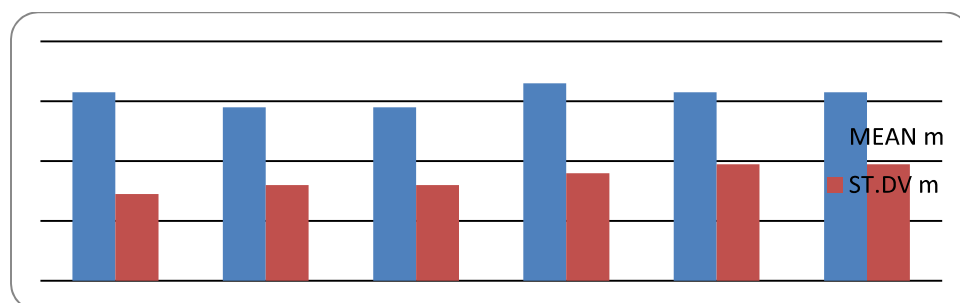


Figure (8): mean and standard deviations of (Goce + Grace + Lageos) models based on data set1

Six (Goce + Grace + Lageos) models have standard deviations from 26 to 39 cm. They are recent started from 2011 till 2014 with d/o from 240 to 300

Table (9): Grace only models assessment against data set 1

Model	Year	d/o	data	MAX _m	MIN _m	MEAN _m	ST.DV _m
AIUB-GRACE02S	2009	150	S(Grace)	1.81	0.13	0.61	0.35
ITSG-Grace2018s	2019	200	S(Grace)	1.92	0.26	0.80	0.36
EIGEN-GRACE02S	2004	150	S(Grace)	2.01	0.19	0.76	0.39
HUST-Grace2016s	2016	160	S(Grace)	1.60	0.00	0.53	0.40
AIUB-GRACE03S	2011	160	S(Grace)	1.91	0.20	0.85	0.40
Tongji-GRACE01	2013	160	S(Grace)	1.79	0.12	0.72	0.41
ITSG-Grace2014k	2014	200	S(Grace)	1.73	0.01	0.66	0.41
ITG-Grace02s	2006	170	S(Grace)	1.65	0.01	0.61	0.41
Tongji-Grace02s	2017	180	S(Grace)	2.01	0.16	0.71	0.44
Tongji-Grace02k	2018	180	S(Grace)	2.00	0.19	0.73	0.46
EIGEN-GRACE01S	2003	140	S(Grace)	2.24	0.30	0.80	0.47
ITG-Grace2010s	2010	180	S(Grace)	2.22	0.17	0.76	0.48
GGM01S	2003	120	S(Grace)	2.36	0.35	0.84	0.51
ITG-Grace03	2007	180	S(Grace)	2.55	0.00	0.85	0.56
GGM05S	2014	180	S(Grace)	2.45	0.00	0.70	0.59

GGM03S	2008	180	S(Grace)	3.17	0.01	0.88	0.78
ITSG-Grace2014s	2014	200	S(Grace)	3.08	0.00	1.15	0.84
AIUB-GRACE01S	2008	120	S(Grace)	3.27	0.01	1.72	0.99
ITU_GRACE16	2016	180	S(Grace)	5.70	0.20	2.67	1.14

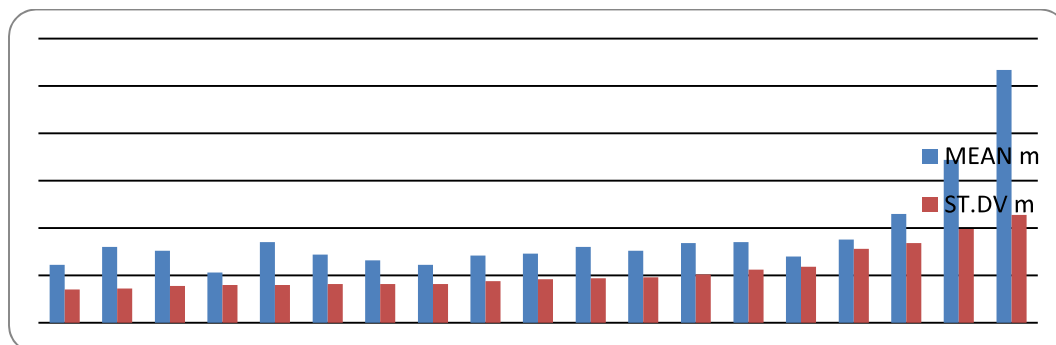


Figure (9): mean and standard deviations of Grace only models based on data set 1

Nineteen Grace only models released from 2003 till 2019 with d/o ranges from 120 to 200. They have standard deviations from 0.35 to 1.14 m.

Table (10): (Grace + Lageos) models assessment against data set 1

Model	Year	d / o	data	MAX m	MIN m	MEAN m	ST.DV m
EIGEN-GL04S1	2006	150	S(Grace), S(Lageos)	2.59	0.49	1.20	0.44
EIGEN-5S	2008	150	S(Grace), S(Lageos)	2.21	0.27	0.80	0.45

Two models with no high resolution and standard deviations 44 and 45 cm.

4-3. Overall evaluation over the collected three data sets

The resulted undulation differences from all 3 data sets are collected and the statistical values of the overall data are obtained. The three data sets exist in

different places over the Egyptian territory, so it is expected that the results in this case will be more expressive and indicative. The results are as the following:

Table (11): the statistical values for the whole three data sets, units in m.

Model	Year	d/o	data	MAX	MIN	MEAN	ST.DV
EGM2008	2008	2159	combined	1.23	0.003	0.63	0.258
EIGEN-GRGS.RL04.MEAN-FIELD	2019	300	Grace, Goce, SLR	1.11	0.012	0.56	0.258
GOCO06s	2019	300	Goce, Grace, SLR	1.06	0.005	0.55	0.242
GO_CONS_GCF_2_DIR_R6	2019	300	Goce, Grace, SLR	1.08	0.003	0.56	0.250
IE_GOCE05s	2017	250	S (Goce)	1.20	0.020	0.53	0.281
GO_CONS_GCF_2_TIM_R6	2019	300	S(Goce)	1.05	0.01	0.56	0.243

GO_CONS_GCF_2_TIM_R5	2014	280	S(Goce)	1.02	0.03	0.55	0.250
ITU_GGC16	2016	280	S(Goce), S(Grace)	1.03	0.04	0.56	0.253
GGM05G	2015	240	S(Goce), S(Grace)	1.03	0.00	0.53	0.244
GO_CONS_GCF_2_DIR_R5	2014	300	S(Goce), S(Grace), S(Lageos)	1.11	0.01	0.55	0.261
GOCO05s	2015	280	Goce, Grace	1.05	0.03	0.55	0.252

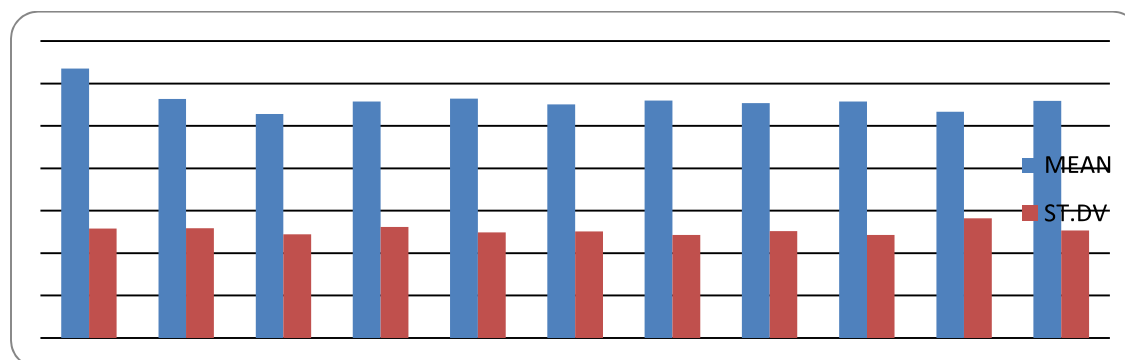


Figure (11): shows the mean and standard deviation values of the best ten models and EGM2008 over the three data sets.

The results show that the mean value ranges from 53 to 56 cm for all the models while it is 63 cm in the case of EGM2008. The standard deviation values are ranging from 24.2 to 28.1 cm while it is 25.8 cm in the case of EGM2008. The ten tested models have close results to each other and most of them are better than EGM2008.

5. Improving Process

5-1. first order polynomial improvement

First order polynomial in latitude and longitude using number of used common points is applied:

$$\Delta N = N_{\text{model}} - N_{\text{obs}} = a_0 + a_1\phi + a_2\lambda \quad (3)$$

Where:

N_{model} is the undulation at the point from the global model

N_{obs} is the observed undulation from the GPS leveling

Φ and λ are the latitude and longitude of the point

The last equation will be written at number of well distributed common points in the study area and they

will be solved for the three unknown coefficients (a_0 , a_1 , and a_2).

The obtained coefficients will be applied for all the stations of the study area to obtain the corresponding values of ΔN which will be subtracted from the model undulations to obtain the corresponding model (estimated) undulations.

The observed undulations will be subtracted from their corresponding estimated values and statistics will be done (max, min, mean, and St. Dv) for the differences to assess the improving process.

5-2. the proposed simple shift improvement

It is well known that the obtained undulations, which came from the orthometric heights and the GPS ellipsoidal heights, contain systematic errors. The global models also may have systematic effects and errors. It is well known also that the values of those undulations are correlated to the local topography and geology around the stations. So, the suggested proposal here depended on getting rid of those local systematic common errors as well as the common local part of topography and geology.

The undulation difference ($N_{\text{model}} - N_{\text{obs}}$) is obtained at one data point (center point) and the global model

undulations of all points around that (center point) within certain radius will be shifted with that difference. So, every data set will include number of center points. The radius around the center point will be changed and tested at different values. The modified (shifted) undulation values will be compared with the corresponding observed undulations. Three chosen global models are improved to be closer to the observed undulations in the three data sets cases. Three radii around center points are tested.

5-3. the results of the improving process

The results of the improving process showed that:

- The three polynomials and the three shifting solutions concerning every data set are better than the original global model, i.e., all of them improved the original model.

- The three polynomial solutions (nine solutions) concerning every data set are close to each other within few centimeters except few cases.
- The three shifting solutions concerning every data set (nine solutions) are also close to each other within few centimeters except few cases.
- The polynomial solutions in most cases are comparable to the corresponding shifting solutions.
- Most of the best solutions (mean and standard deviation) are resulted when using largest number of used solution points (smallest radius around the center point).
- Most of the best solutions are resulted from the shifting method, especially for its small mean values.

Table (12): concludes the description of the best results of the improvement process

table	Model with data set	The best solution	Used solution pts (Radius)	Mean cm	St. Dv cm
1	IFE-GOCE05s with data set 1	shift	17 (15 km)	8	11
2	IFE-GOCE05s with data set 2	Shift	12 (30 km)	9	10.5
3	IFE-GOCE05s with data set 3	polynomial	4 (50 km)	11	8.9
4	GOCO06s with data set 1	shift	17 (15 km)	8	11
5	GOCO06s with data set 2	shift	12 (30 km)	8	9.8
6	GOCO06s with data set 3	polynomial	4 (50 km)	11	9
7	TIM-R6 with data set1	shift	17 (15 km)	7	10.8
8	TIM-R6 with data set 2	shift	12 (30 km)	8	10
9	TIM-R6 with data set 3	polynomial	4 (50 km)	11	8.8

The results (mean and standard deviations) of improving the three global models are close to each other because the differences between those models and the observed data sets (GPS leveling) were close to each other too. In the case of data set 1 and data set 2, the best solutions were of shifting using the bigger number of used solution points (smallest radius), while in the case of data set 3 the best solutions were of polynomial using the least number of used solution points.

The following figures are samples representing undulations from the global model, GPS/leveling, the best polynomial solution, and the best shift solution. It should be noted that the differences between solutions from polynomials and shift methods are small where they hardly clear on the vertical axis of the figures. i.e., all figures showing the solutions are almost identical.

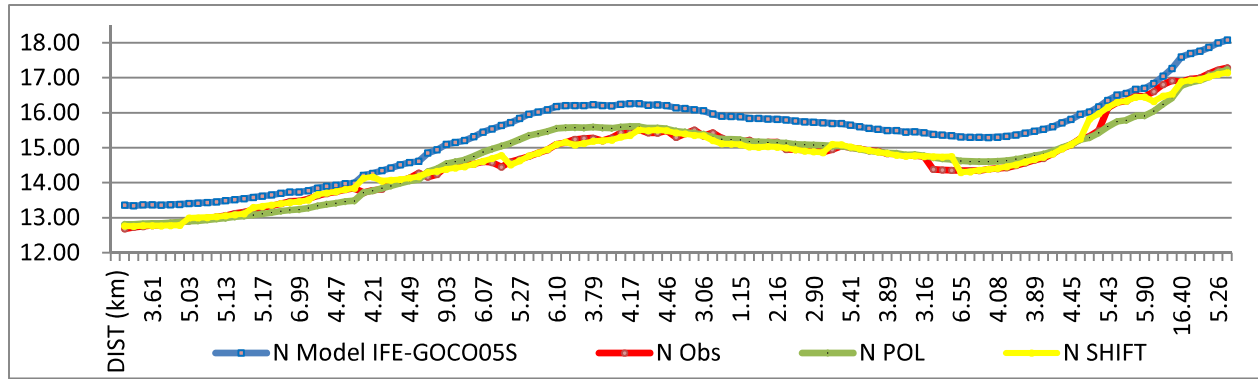


Figure (12): profile of IFE-GOCE05s model, observed undulations, the best shift, and the best polynomial solutions in case of data set 1.

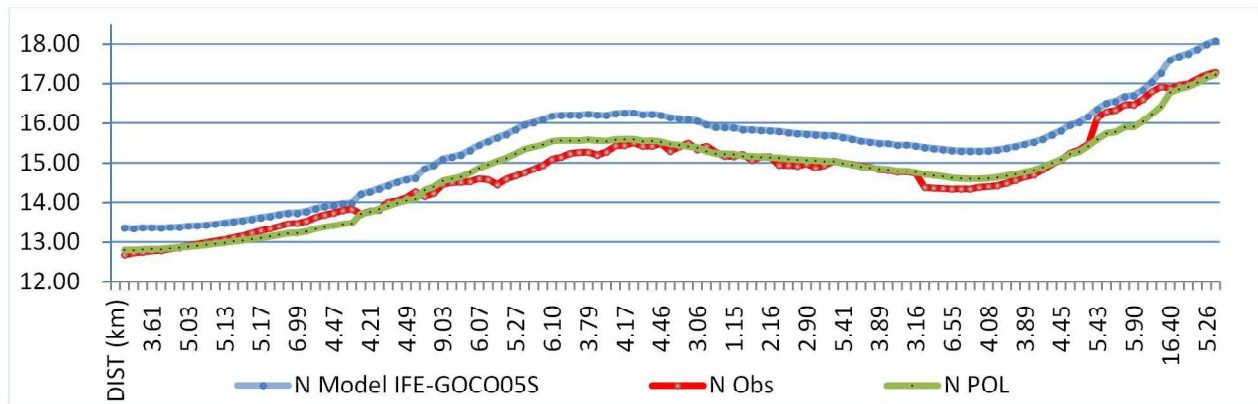


Figure (13): profile of IFE-GOCE05s model, observed undulations, and the best polynomial solution in case of data set 1.

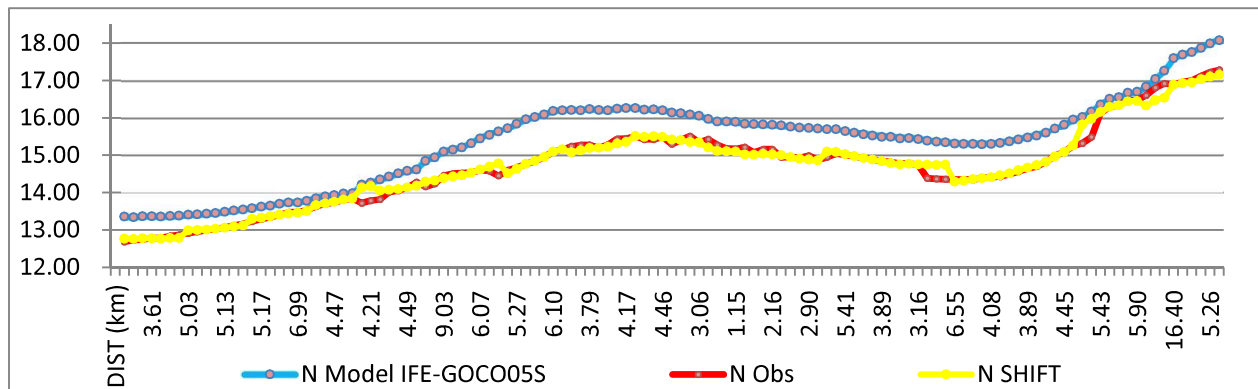


Figure (14): profile of IFE-GOCE05s model, observed undulations, the best shift solution in case of data set 1.

The figures of the results of the improvement process showed also that using the polynomial yielded, as a trend, improved field parallel to the original global model. It is expected, because the used polynomial is one homogeneous process for the whole global model field. On the contrary, simple shift method treated (improved) the global model in successive portions. So, the result was nearer to the observed field.

- The best solutions of the three improved global model gave almost the same mean

and standard deviation with the same data set, i.e., IFE-GOCE05s, GOCO06s, and TIM-R6 gave (8, 11 cm), (8, 11 cm), and (7, 10.8 cm) as (mean and standard deviation) with data set 1 after the improving process. They gave (9, 10.5 cm), (8, 9.8 cm), and (8, 10 cm) with data set 2, and they gave (11, 8.9 cm), (11, 9 cm), and (11, 8.8 cm) with data set 3. Remembering that the mean and standard deviation of the original three

global models were (66, 25.9 cm), (67, 21.4 cm), and (66, 21.6 cm) with data set 1 and they were (53, 21.8 cm), (57, 16.2 cm), and (57, 16.3 cm) with data set 2 and (22, 17 cm), (28, 18.1 cm), and (27, 18.9 cm) with data set 3. And that of EGM2008 were (63, 21.8 cm), (76, 23 cm), and (38, 33 cm) with data set 1, data set 2, and data set 3, respectively.

Regarding that the GPS/leveling points in the three data sets have a longitudinal extension, the results showed also:

- The simple shift method with the three data sets gave around 8 cm as a mean and 11 cm as a standard deviation using one seventh of all data points corresponding to 15, 30, and 25 km as radii around the center point in the three data sets, respectively.
- The results of the simple shift method when using one eleventh and one fifteenth of the data points corresponding to 50 and 60 km as radii around the center point are not very far from the results in the previous notice.
- The same can be said about the results of the polynomial method but the mean values in the shifting solutions are noticeably smaller than their corresponding values from the polynomial solutions.

To investigate the behavior of the proposed shift method on high and low d/o models, it is applied on EGM2008 (d/o 2190), ITU-GRACE16 (d/o 180) and AIUB-CHAMP01s (d/o 120). The results showed that the simple shift method improved the models very significantly, and they assured the same trend of the improvement like the previous results. The results were as:

EGM2008 with data set 1

The mean and standard deviation values of the undulation differences in case of EGM2008 itself were 63 and 21.8 cm while they improved to 6 and 8 cm in the case of shifting the model using (1/7) data points (radius 15 km). They are 9 and 11 cm in the case of shifting the model using (1/11) of the data points (radius 25 km) and 12.7 and 12 cm in the case of shifting the model using (1/15) of data points (radius 35 km).

EGM2008 with data set 2

The mean and standard deviation values of the undulation differences in case of EGM2008 itself were 76 and 23 cm while they improved to 9.6 and 12.9 cm in the case of shifting the model using (1/7) data points (radius 15 km). They are 14 and 14.8 cm in the case of shifting the model using (1/11) of the data points (radius 25 km) and 17 and 18.8 cm in the case of shifting the model using (1/15) of data points (radius 35 km).

EGM2008 with data set 3

The mean and standard deviation values of the undulation differences in case of EGM2008 itself were 38 and 33 cm while they improved to 9 and 10.6 cm in the case of shifting the model using (1/7) data points (radius 15 km). They are 11 and 10.6 cm in the case of shifting the model using (1/11) of the data points (radius 25 km) and 12.7 and 12.5 cm in the case of shifting the model using (1/15) of data points (radius 35 km).

ITU-GRACE16 with data set 1

The mean and standard deviation values of the undulation differences in case of ITU-GRACE16 itself were 2.67 and 1.135 m while they improved to 0.27 and 0.287 m in the case of shifting the model using (1/7) data points (radius 15 km). They are 0.40 and 0.39 m in the case of shifting the model using (1/11) of the data points (radius 25 km) and 0.625 and 0.495 m in the case of shifting the model using (1/15) of data points (radius 35 km).

AIUB-CHAMP01S with data set 1

The mean and standard deviation values of the undulation differences in case of AIUB-CHAMP01S itself were 1.716 and 0.991 m while they improved to 0.077 and 0.086 cm in the case of shifting the model using (1/7) data points (radius 15 km). They are 0.126 and 0.123 m in the case of shifting the model using (1/11) of the data points (radius 25 km) and 0.141 and 0.17 m in the case of shifting the model using (1/15) of data points (radius 35 km).

6. Conclusions

6.1 Conclusions concerning the evaluation process.

- The evaluation process revealed that at least ten of the satellite-based data models are comparable and some of them are better than EGM2008. Those models are close to each other's.

- The best ten global satellite-based data
GOCO06s - GO_CONS_GCF_2_TIM_R6 -
GGM05G - GO_CONS_GCF_2_DIR_R6 -
GO_CONS_GCF_2_TIM_R5 - GOCO05s -
ITU_GGC16 - EIGEN-GRGS.RL04.MEAN.FIELD -
GO_CONS_GCF_2_DIR_R5 - IfE_GOCE05s.

- The best ten satellites-based data global models have data either from Goce only or Goce plus other missions like Grace and SLR. Three of them are Goce only data, three of them are (Goce + Grace), and four of them are (Goce + Grace + SLR). The best ten models have d/o ranges from 240 to 300. They considered recent models released from 2014 to 2019. They all except two are better than EGM2008.

6.2 Conclusions concerning the improving process.

The results of the improving process showed that:

- The first order polynomial and shifting solutions are better than the original global model, i.e., all their solutions improved the original model.
- The polynomial solutions with every data set are close to each other within few centimeters except few cases.
- The shifting solutions with every data set are also close to each other within few centimeters except few cases.
- The polynomial solutions in most cases are comparable to their corresponding shifting solutions.
- Solutions are better when using largest number of used solution points (smallest radius around the center point).
- Most of the best solutions are resulted from the shifting method, especially for its small mean values.

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