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Efficient Quantum Algorithms for Set Operations on IBM Q: Design and Experimental Implementation

Rehab Elgendy*[1], Ahmed Younes[2, 3], H.M. Abu-Donia[1], R.M. Farouk[1]

[1]Department of Mathematics, Faculty of Science, Zagazig University, Zagazig, Egypt
[2]Department of Mathematics and Computer Science, Faculty of Science, Alexandria University, Alexandria, Egypt
[3]School of Computer Science, University of Birmingham, Birmingham, B15 2TT, United Kingdom

([*]**Corresponding author:** elgandyrehab@gmail.com)

***ABSTRACT*: Recent advances in the quantum computing field of computer science that apply the power of quantum mechanics to perform calculations have led many companies, such as IBM, Google, Microsoft, D-Wave Systems, and Xanadu Quantum Technologies, to invest in the field of quantum computing. The main goal of this paper is to apply the Elgendy et al. algorithm, which proposes quantum algorithms for set operations such as true intersection, false intersection, difference, and union, on the current viability of available IBM quantum computers. Moreover, this paper includes a new 5-qubit implementation of Elgendy et al., tests results showing the current capabilities of quantum computers, extends to greater scales, and evaluates algorithm performance on available noisy devices. Elgendy et al. algorithms use a two-stage quantum amplitude amplification technique: The first stage involves preparing the truth set of the first Boolean function using incomplete superpositions, which is done using the Younes et al. technique which performs quantum searching via entanglement and partial diffusion. In the second stage, set operations are handled using an oracle that represents the second Boolean function and a modified version of Arima's algorithm. Among the potential uses are machine learning, cryptography, pattern recognition, and database querying.**

***Keywords*: Quantum Set operations, Intersection, Difference, Union, IBM Quantum Experience, Qiskit**

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1. INTRODUCTION

Quantum computers use the essential laws of quantum physics to carry out more complicated computations that traditional computers solve with difficulty[1-3]. One of the basic features of quantum computers is superposition, in which qubits exist simultaneously in a state of 0, 1, allowing them to simultaneously apply multiple tasks and solve more efficient problems faster[4]; furthermore, entanglement is another basic feature in quantum computing, in which qubits are connected and their states become correlated, especially when they are separated by great distances[5]. Richard Feynman first presented the idea of using quantum mechanics to enhance computational power[6], but basic quantum bits, or qubits, to perform simple calculations weren't built until the late 1990s[6]. After that, scientists were encouraged to propose more complex quantum algorithms and increase the number of qubits in quantum computers[7]; for example, the Grover algorithm finds an item in a complete superposition; moreover, it runs quadratically faster than the best classical algorithm[8]. As a result, many researchers are motivated to generalize this algorithm, such as the Younes et al. algorithm[9-11], the Venture algorithm[12], and the Arima algorithm[13-15]. Furthermore, many companies, such as IBM[16], Google[17], Microsoft[18], D-Wave

Systems[19], and Xanadu Quantum Technologies[20] are currently making significant investments in the field of quantum computing research.

Set operations that include intersection, difference, complement, and union are fundamental mathematical operations; moreover, they are basic to different sciences and techniques, such as database management[21], signal processing[22], and image processing[23]. As a result, many researchers are motivated to propose many algorithms applied to set operations; for example, the quantum algorithm finds the matching element between two unsorted lists of length N, which was published in 2000 by Heiligman[24], the quantum algorithm enables us to present a hybrid quantum search engine and works extremely effectively in the case of several matches inside the search space, which was published in 2003 by Younes et al. [10], the quantum algorithm finds unique patterns in a given string (database) was published in 2005 by P. Mateus[25], the quantum algorithm locates a common element between two sets was published in 2012 by A. Tulsi[26], the quantum algorithm calculates the intersection between sets was published in 2013 by Pang et al.[27], the quantum algorithm finds the common entries between databases was published in 2017 by K. El-Wazan[28], the quantum algorithm combines two oracles and determines their similarity using the Deutsch-Jozsa algorithm was published in 2020 by A. Kiss and K. Varga[29], the quantum algorithm finds set operations utilizing Grover's quantum search algorithm and evaluates the proposed algorithms on various IBM-Q Experience systems was published in 2020 by S. Joczik and A. Kiss[30], and the quantum algorithm performs set operations on two Boolean functions was published in 2024 the Elgendy et al.[31].

IBM created the IBM Q quantum computing platform, which enables users to access and run quantum algorithms on quantum hardware[16]; moreover, IBM released an open-source software development kit called Qiskit for creating quantum circuits using Python[32].

The aim of this paper is to apply four quantum algorithms—true intersection, false intersection, difference, and union—that are proposed in this paper[31] to real quantum hardware. This paper implements and analyzes the results of these algorithms on Qiskit. Moreover, the applications of these algorithms in different fields, such as database management, signal processing, image processing, artificial intelligence, and cryptography, are discussed. This paper aims to develop efficient quantum algorithms for solving complex computational problems and to illustrate the feasibility and expandability of IBM Q and Qiskit for quantum experimental verification on real quantum computers.

The paper is organized as follows: Section 2 provides an overview of quantum computer, IBM Q, and Qiskit. Section 3 describes three quantum algorithms - Grover, Younes et al., and Arima - including their implementation circuits and analysis of experimental results in graphical form. Section 4 introduces four new quantum algorithms - true intersection, false intersection, difference, and union - and illustrates their implementation circuits in Qiskit. Section 5 evaluates the proposed algorithms on IBM's quantum computers and analyzes the experimental results graphically. Section 6 evaluates the proposed algorithms using mathematical computations. Finally, Section 7 concludes the paper.

2. PRELIMINARIES

2.1. Quantum computation

In traditional computers, the bit is the basic unit, which can only exist in one of two states: 0 or 1. Nevertheless, quantum computers use the qubit, which can exist in both $|0\rangle$ and $|1\rangle$ simultaneously. The Dirac notation represents the qubit as $|0\rangle$ and $|1\rangle$, such that the vertical bar represents the state and the angle brackets indicate the Dirac notation.

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle. \tag{1}$$

The probability amplitudes of states 0 and 1 in quantum computing are denoted by α and β, respectively. Equation (2) shows how these probability amplitudes may be expressed as vectors.

$$|0\rangle = \begin{bmatrix}1\\0\end{bmatrix}, |1\rangle = \begin{bmatrix}0\\1\end{bmatrix}. \tag{2}$$

The following vector may be used to represent the superposition state:

$$\begin{bmatrix}\alpha\\\beta\end{bmatrix} = \alpha\begin{bmatrix}1\\0\end{bmatrix} + \beta\begin{bmatrix}0\\1\end{bmatrix}. \tag{3}$$

This Equation may be used to quantitatively illustrate the need for a qubit's state to be geometrically normalized to a length of 1 in quantum mechanics.

$$|\alpha|^2 + |\beta|^2 = 1, \tag{4}$$

A quantum circuit is composed of wires and simple quantum gates that are used to process quantum information on quantum computers. There are two types of quantum gates: single-qubit gates and multiple-qubit gates. According to Nielsen and Chuang [33], quantum gates must achieve the unitary condition, $U^{\dagger}U = I$, where$U^{\dagger}$ denotes the conjugate transpose of U. Hence, a unitary matrix may represent any quantum gate.

2.2. IBM Q

In 2016, IBM made its 5-qubit prototype IBM Quantum Experience device available to the general public. The next year saw the release of IBM Q, marking the beginning of efforts to create practical universal quantum computers for use in industry and academia [16]. Two 5-qubit devices, the ibmqx2 and ibmqx4, were released in the same year. In 2018, they created ibmqx5, a third device for public use that has 16 qubits and can be accessed using Qiskit. IBM has revealed that its simulator can handle up to 32 qubits and that they have successfully constructed and tested a 20-qubit device for one of their clients. Many basic gates, including the Toffoli gate (also known as the ccX-gate), X-gate, H-gate, cZ-gate (control-Z gate), and cX-gate (control-NOT gate), can be supported by IBM Q.

2.3. Qiskit

A software package called Qiskit (Quantum Information Software Kit) has been developed for use indeveloping applications and experiments on quantum computers [32]. A series of quantum circuits is referred to as a quantum program in Qiskit. There are three steps in the program process: building, compiling, and running. Users can construct various quantum circuits to represent the problem they are trying to solve through building. Users may modify the circuits to run on multiple backends, such as simulators or real chips with varied quantum volumes, sizes, and fidelity, by using compilation. Running executes the jobs, and once they have completed their work, the data is collected and assembled based on the program's specifications. Qiskit contains tools for characterizing quantum states as well as Python-based tools for generating, modifying, visualizing, and analysing quantum states.

Additionally, Qiskit includes a compiler that maps the desired experiment to real hardware. The Identity gate (I), Hadmard gate (H), Pauli-X gate (X), Pauli-Z gate (Z), U gate (U), and Toffoli gate are a few of the quantum gates in Qiskit that will be used in the research, as illustrated in Figure 1.

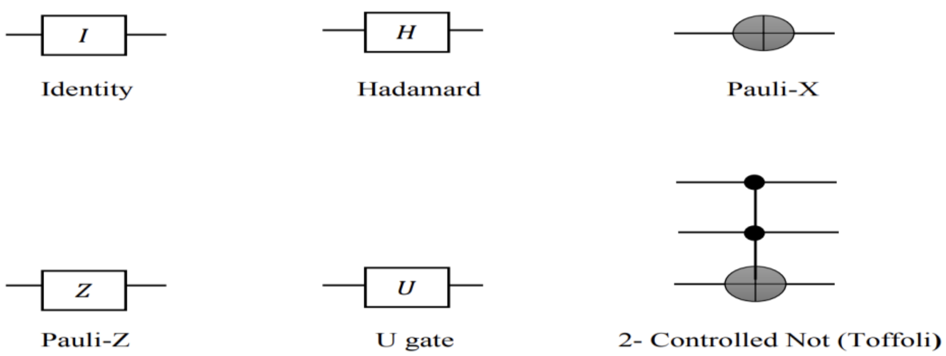


Figure 1: Some of the quantum gates in Qiskit.

3. Quantum algorithms and experiment implementations based on IBM Q

3.1. Grover algorithm

3.1.1. Algorithm procedure

The Grover algorithm was presented by Grover in 1996 with the goal of quickly locating a particular target element within a set of N unsorted elements. The Grover algorithm uses quantum computing's parallel processing power to find the target item with a probability of $1 / 2$ with an average of only N queries, compared to classical computers' average of N^2 queries. This increases its probability of finding the target item close to 1. The Grover algorithm is still vital when N is high, even if it only offers a quadratic speedup. In order to lower the probability amplitude of non-target items and enhance the probability amplitude of the target item, the technique first creates an amplitude superposition of unitary transformation. It then applies the Grover quantum process repeatedly. The probability of locating the target element approaches 1 in the best-case scenario. Fig. 2 describes the Grover quantum search algorithm's implementation processes. The following stages include applying the algorithm:

1. Put $n + 1$ qubits into a quantum register, including that the last qubit is in state $|1\rangle$ and the first n qubits are in state $|0\rangle$.
2. Superpose every state in both $|0\rangle$ and $|1\rangle$ by using the Hadamard gate in parallel to every qubit.
3. Implement the oracle that gives the match amplitudes' phase shift.
4. mplement Diffusion Operator G for the first n qubits. The target state's amplitudes are increased while the

amplitudes of other states are decreased by this operator.

5. Measure the first n qubits to determine the final state of the system.

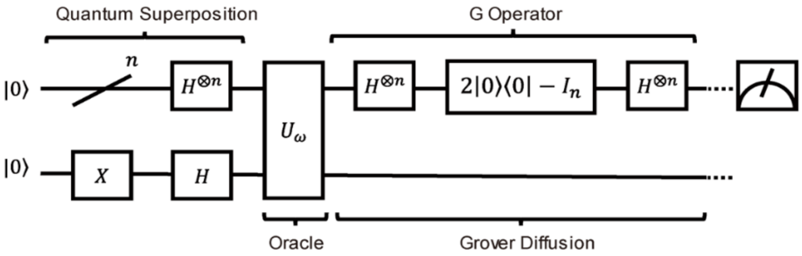


Figure 2: Quantum circuit representation of Grover algorithm.

3.1.2. Experimental Implementation and Analysis

Here's an example implementation of the Grover algorithm for a 5-qubit database search in Figure 3. Implementation of the Boolean function and Grover operator, which are used in Figure 3, is as follows in Figure 4 and Figure 5. The Grover algorithm is applied for three iterations, which is enough to find the target element with high probability. The results are obtained by simulating the quantum circuit using the Qiskit Aer backend for the Qasm simulator, as shown in Figures 6 and 7.

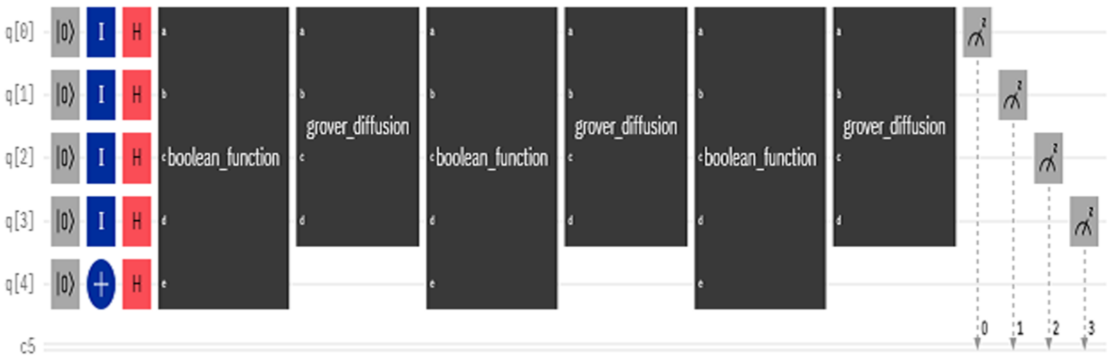


Figure 3: Implementation of the Grover algorithm for a 5-qubit database search.

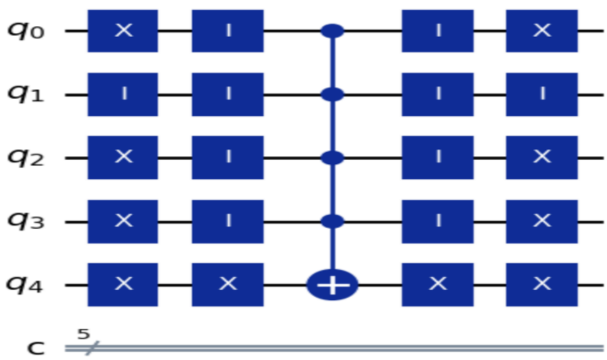


Figure 4: Implementation of Boolean function for searching the target element '10010' in a 5-qubit database.

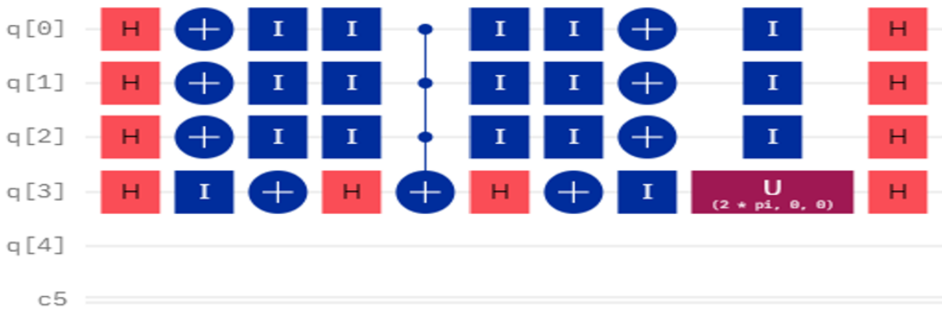


Figure 5: Implementation of Grover diffusion on 4 qubits.

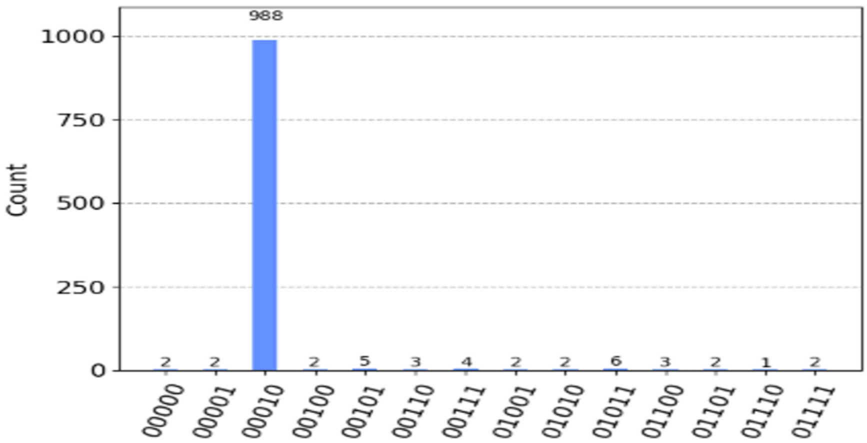


Figure 6: The results using the Qiskit Aer backend for the qasm simulator.

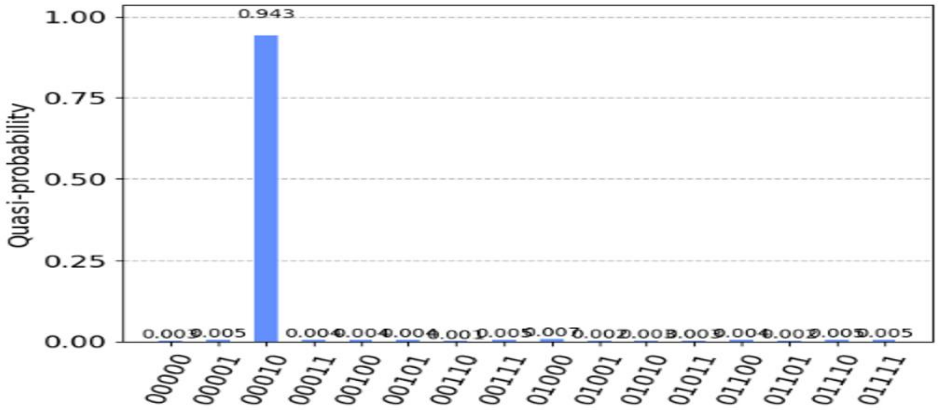


Figure 7: The probability of using the Qiskit Aer backend for the qasm simulator.

3.2. Younes et al. algorithm

3.2.1. Algorithm procedure

Younes et al. algorithm applies partial diffusion, which only affects some of the qubits in the system rather than all of them. The Younes et al. algorithm has been proven to have various benefits over Grover's algorithm, including quicker convergence to the goal state and increased resilience against noise and errors. Moreover, the Younes et al. algorithm may be used for a larger variety of search problems with non-uniform distributions and those with many target states. Fig. 8 describes the Younes et quantum search algorithm's implementation processes. The following stages include applying the algorithm:

1. Put $n + 1$ qubits into a quantum register, including that the last qubit is in state $|0\rangle$ and the first n qubits are in state $|0\rangle$.
2. Superpose every state in both $|0\rangle$ and $|1\rangle$ by using the Hadamard gate in parallel to the first n qubits.
3. mplement the oracle that makes the match amplitudes marked.

4. Apply partial diffusion operator on $n+1$ qubits. The amplitudes of the target states are entangled.
5. For the purpose of increasing the probability of finding the target element, steps 3 and 4 have to be done a specified number of times.
6. Measure the first n qubits to determine the final state of the system.

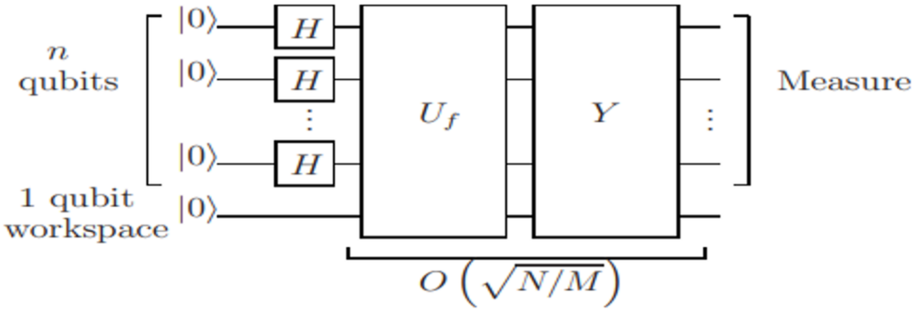


Figure 8: Quantum circuit representation of Younes et al. algorithm.

3.2.2. Experimental implementation and analysis

Here's an example implementation of the Younes et al. algorithm for a 5-qubit database search in Figure 9. The implementation of the partial diffusion operator, which is used in Figure 9, is shown in Figure 10. The Younes et al. algorithm is applied for four iterations, which is enough to find the target element with high probability. The results obtained by simulating the quantum circuit using the Qiskit Aer backend for the Qasm simulator are shown in Figures 11 and 12.

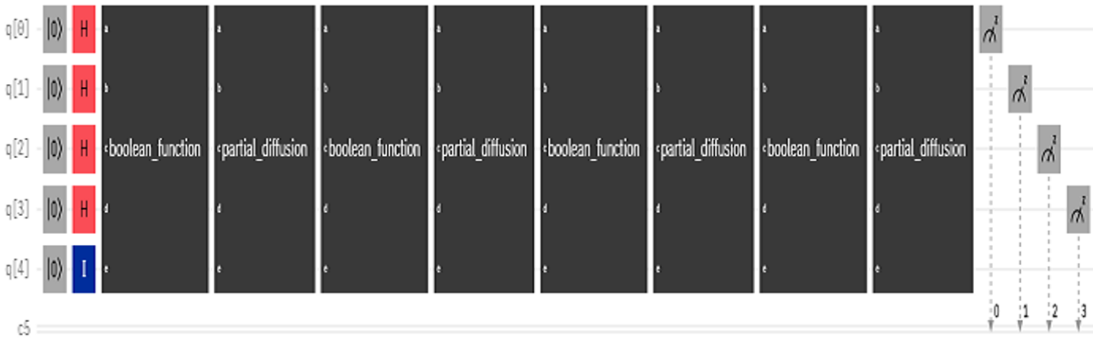


Figure 9: Implementation of the Younes et al. algorithm for a 5-qubit database search

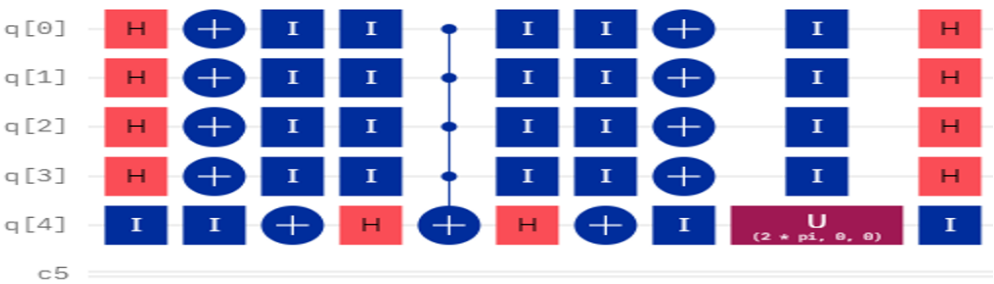


Figure 10: Implementation of Partial diffusion on 5 qubits.

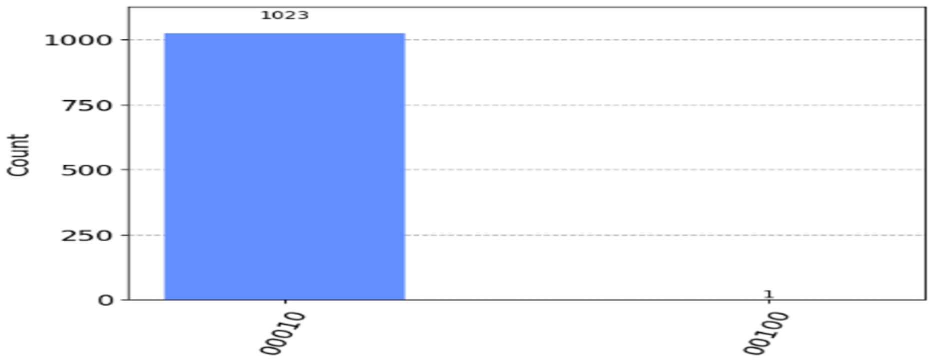


Figure 11: The results using the Qiskit Aer backend for the qasm simulator.

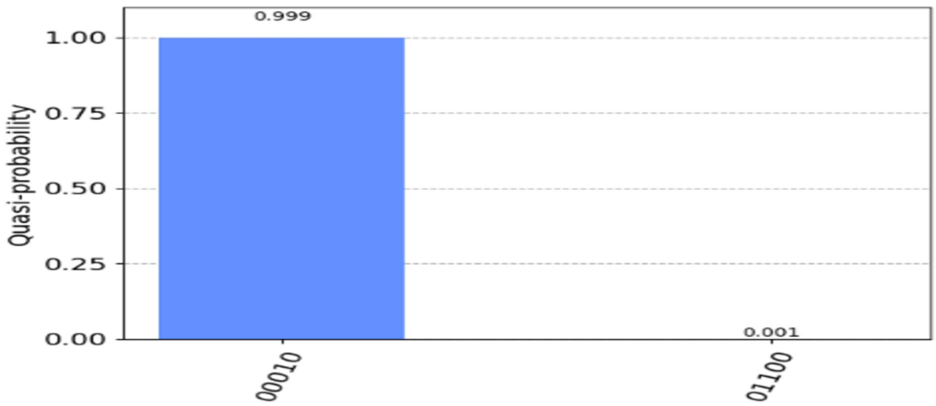


Figure 12: The probability of using the Qiskit Aer backend for the qasm simulator.

3.3. Arima algorithm

3.3.1. Algorithm procedure

When the dataset's initial amplitude distribution is uniform (complete superposition), Grover's technique works well[8], but when the dataset's initial amplitude distribution is non-uniform (incomplete superposition), it is not always successful, so Ventura proposed an algorithm that is just as comprehensive as Grover's. When the stored data is big in complete superposition or little in incomplete superposition, respectively, Ventura's algorithm work well[12]. An algorithm to cover a different subset of instances is proposed by the Arima algorithm[13]. The Arima algorithm consists of the following steps:

1. Create an incomplete superposition for the system's initial state, such that the dataset's initial amplitude distribution is non uniform.
2. The steps 3, 4, 5, and 6 are iterated in order to enhance the probability of finding the desired element.
3. Apply the operator to represent the state that is being searched.
4. Apply the Grover diffusion operator.
5. Apply the operator to represent all of the data that has been stored.
6. Apply the Grover diffusion operator.
7. Analyze the system.

4. THE PROPOSED ALGORITHMS

In this section, we will propose algorithms that focus on testing the current viability of available IBM quantum computers and Qiskit using the Elgendy et al. algorithm[31], which proposes quantum search algorithms for set operations, such as true intersection, false intersection, difference, and union. The main contributions to this paper include a new 5-qubit implementation of Elgendy et al. and test results showing the current capabilities of quantum computers.

4.1. Set operation: True Intersection

4.1.1. Quantum Algorithm

The procedure of the Elgendy et al. algorithm to find the true intersection between two Boolean functions f_1 and f_2 is as follows:

1. Apply the Younes et al. algorithm to evaluate f_1 on $n+1$ qubits register. This involves initializing the register to the state$|0\rangle$, applying the Hadamard gate to the first n qubits, and iterating through a series of steps that involve applying the oracle operator $I_{f_1^T}$ to evaluate f_1, applying partial diffusion operator D_p to the register, and observing the auxiliary qubit. If the outcome is 1, apply Z followed by H to the auxiliary qubit.
2. Apply the Arima algorithm to find the true intersection between f_1 and f_2. This involves applying phase oracles $I_{f_1^T}$ and $I_{f_2^T}$ to the register, iterating through a series of steps that involve applying $I_{f_2^T}$, G, $I_{f_1^T}$ and G to the register, and observing the system to find the true intersection between the two Boolean functions.

4.1.2. Implementation

we can take as an example $n=4$, the possible number of items of the Boolean functions equals N, where $N=16$. Assume that the number of the stored elements in f_1 and f_2 equals m, where $m=8$. Suppose that f_1 evaluates to true for each pattern in the set$\{|0\rangle, |2\rangle, |3\rangle, |6\rangle, |9\rangle, |10\rangle, |13\rangle, |14\rangle\}$, and f_2 evaluates to true for each pattern in the set $\{|0\rangle, |1\rangle, |4\rangle, |5\rangle, |7\rangle, |8\rangle, |11\rangle, |12\rangle\}$. The complete implementation of the algorithm, including the oracle function, can be found in Figure 13. Implementation of f_1 which is used in Figure 13 is as follows in Figure 14, and implementation of f_2 which is used in Figure 13 is as follows in Figure 15.

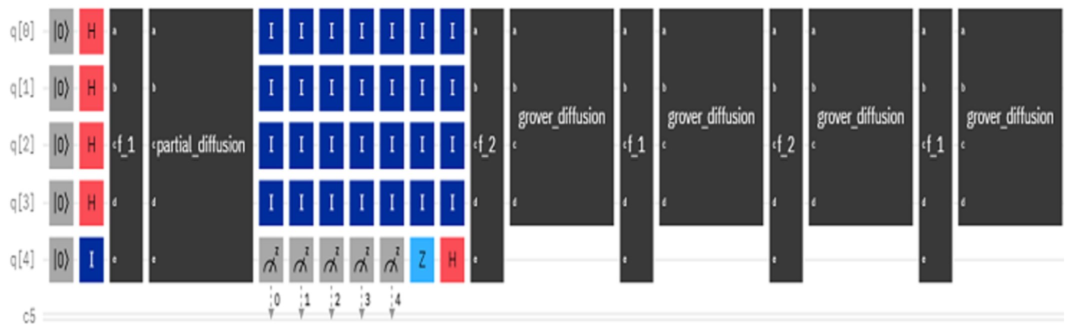


Figure 13: Finding the True Intersection of two Boolean functions f_1 and f_2.

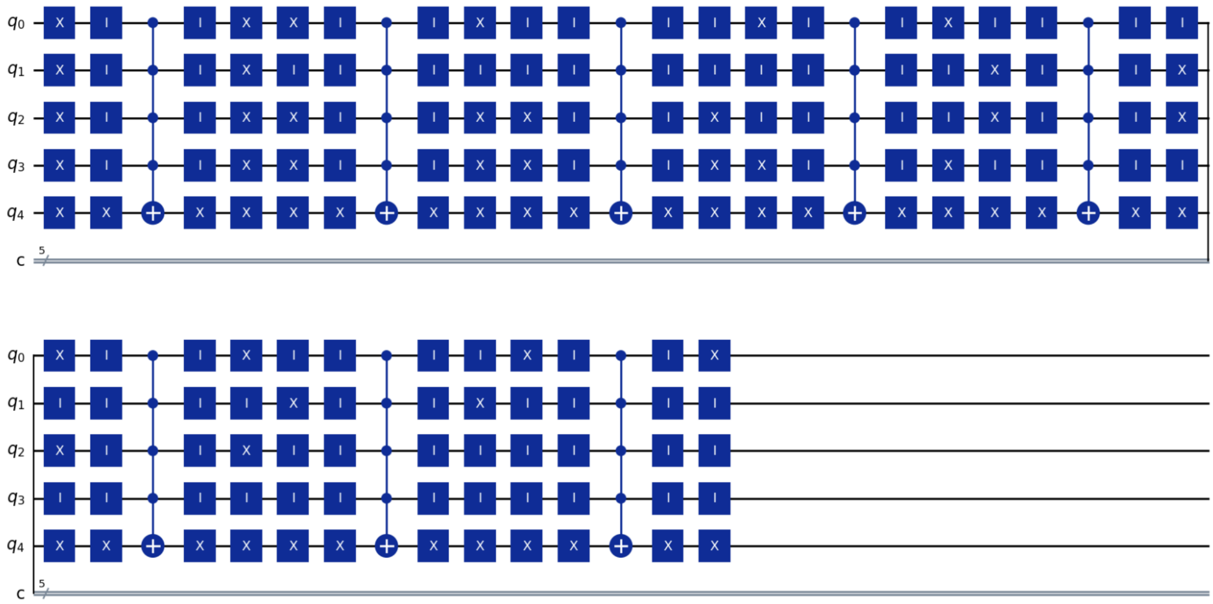


Figure 14: Implementation of f_1 which marks 00000, 00010, 00011, 00110, 01001, 01010, 01101, and 01110.

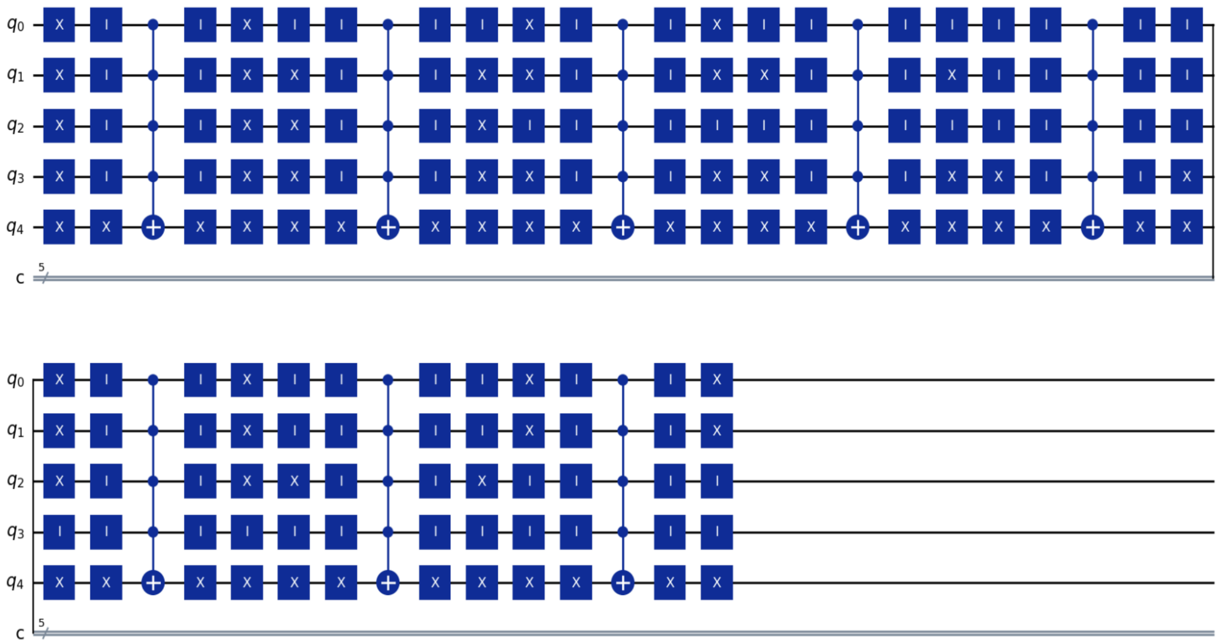


Figure 15: Implementation of f_2 which marks 00000, 00001, 00100, 00101, 00111, 01000, 01011, and 01100.

4.2. Set operation: False Intersection

4.2.1. Quantum Algorithm

The procedure of the Elgendy et al. algorithm to find the false intersection between two Boolean functions $f_1 \oplus 1$ and $f_2 \oplus 1$ is as follows:

1. Apply the Younes et al. algorithm to evaluate f_1 on an $n+1$ qubit register. This involves initializing the register to the state$|0\rangle$, applying the Hadamard gate to the first n qubits, and iterating through a series of steps that involve applying the oracle operator $I_{f_1^T}$ to evaluate f_1, applying X on the auxiliary qubit,

applying partial diffusion D_p to the register, and observing the auxiliary qubit. If the outcome is 1, apply Z followed by H to the auxiliary qubit. The result is a superposition of states that satisfy $f_1 \oplus 1$.

2. apply the Arima algorithm to find the false intersection between f_1 and f_2. This involves applying phase oracles $I_{f_1^T}$ and $I_{f_2^T}$ to the register, iterating through a series of steps that involve applying $I_{f_2^T}$, $(I_n \otimes X)$, G, $I_{f_1^T}$ and G to the register, and observing the system to find the False intersection between the two Boolean functions.

4.4.2. Implementation

we can take as an example $n = 4$, the possible number of items of the Boolean functions equals N, where $N = 16$. Assume that the number of the stored elements in f_1 and f_2 equals m, where $m = 8$. Suppose that f_1 evaluates to true for each pattern in the set$\{|0\rangle, |2\rangle, |3\rangle, |6\rangle, |9\rangle, |10\rangle, |13\rangle, |14\rangle\}$, and f_2 evaluates to true for each pattern in the set $\{|0\rangle, |1\rangle, |4\rangle, |5\rangle, |7\rangle, |8\rangle, |11\rangle, |12\rangle\}$. The complete implementation of the algorithm, including the oracle function, can be found in Figure 16.

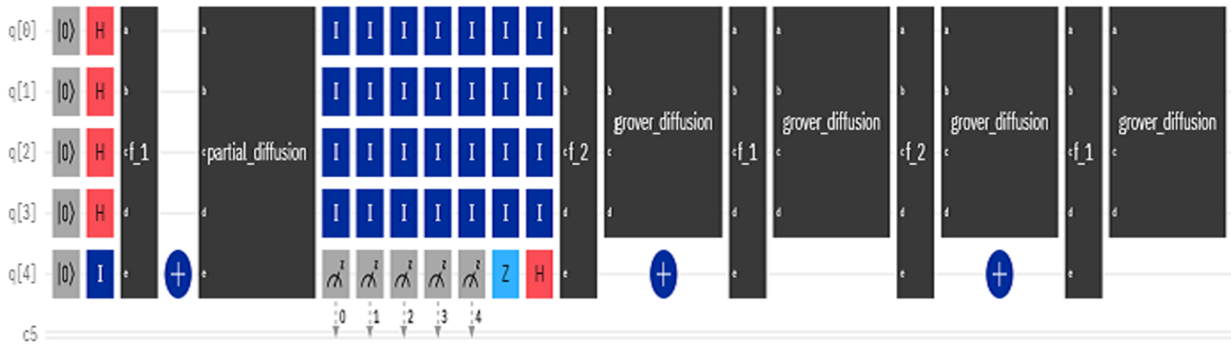


Figure 16: Finding the False Intersection of two Boolean functions f_1 and f_2.

4.3. Set operation: Difference

4.3.1. Quantum Algorithm

The procedure of the Elgendy et al. algorithm to find the difference between two Boolean functions f_1 and $f_2 \oplus 1$ is as follows:

1. Apply the Younes et al. algorithm to evaluate f_1 on an $n + 1$ qubit register. This involves initializing the register to the state $|0\rangle$, applying the Hadamard gate to the first n qubits, and iterating through a series of steps that involve applying the oracle operator $I_{f_1^T}$ to evaluate f_1, applying partial diffusion D_p to the register, and observing the auxiliary qubit. If the outcome is 1, apply Z followed by H to the auxiliary qubit. The result is a superposition of states that satisfy f_1.
2. Apply the Arima algorithm to find the difference between f_1 and f_2. This involves applying phase oracles $I_{f_1^T}$ and $I_{f_s^T}$ to the register, iterating through a series of steps that involve applying $I_{f_s^T}$, $(I_n \otimes X)$, G, $I_{f_1^T}$ and G to the register, and observing the system to find the difference between the two Boolean functions.

4.3.2. Implementation

we can take as an example $n = 4$, the possible number of items of the Boolean functions equals N, where $N = 16$. Assume that the number of the stored elements in f_1 and f_2 equals m, where $m = 8$. Suppose that f_1 evaluates to true for each pattern in the set $\{|0\rangle, |2\rangle, |3\rangle, |6\rangle, |9\rangle, |10\rangle, |13\rangle, |14\rangle\}$, and f_s evaluates to true for each pattern in the set $\{|0\rangle, |1\rangle, |4\rangle, |5\rangle, |7\rangle, |8\rangle, |11\rangle, |12\rangle\}$. The complete implementation of the algorithm, including the oracle function, can be found in Figure 17. Implementation of f_s which is used in Figure 17 is as follows in Figure 18.

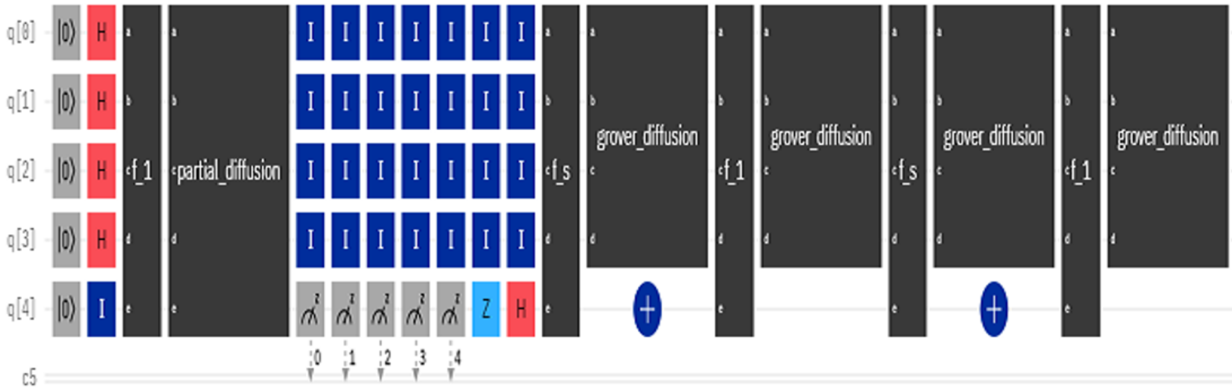


Figure 17: Finding the Difference between the two Boolean functions f_1 and f_s.

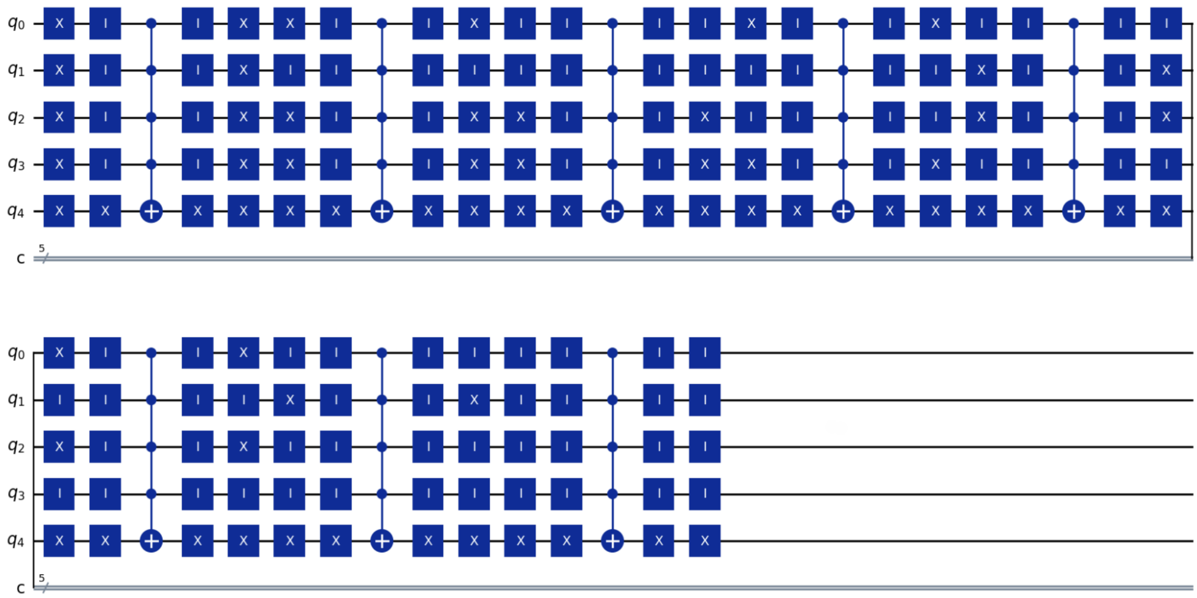


Figure 18: Implementation of f_s which marks 00000, 00010, 00011, 00110, 01001, 01010, 01101, and 01111.

4.4. Set operation: Union

4.4.1. Quantum Algorithm

The procedure of the Elgendy et al. algorithm to find the union between two Boolean functions f_1 and f_2 is as follows:

1. The result of the Boolean function $f = (f_1 \oplus 1) \cap (f_2 \oplus 1)$ is found using the algorithm presented in Section 4.2.
2. The Younes et al. algorithm is applied to find the complement of f, i.e., $f \oplus 1$. The algorithm initializes the register by applying the Hadamard gate H on the first n qubits in parallel. Then, the algorithm iterates the following steps: applying the oracle operator I_f on $n+1$ qubits, applying X on the auxiliary qubit, and applying the partial diffusion D_p on the $n+1$ qubits.
3. The system is observed to determine the probability of $f \oplus 1$.

4.4.2. Implementation

we can take as an example $n = 4$, the possible number of items of the Boolean functions equals N, where $N = 16$. Assume that the number of the stored elements in f_1 and f_2 equals m, where $m = 8$. Suppose that f_1 evaluates to true for each pattern in the set $\{|0\rangle, |2\rangle, |3\rangle, |6\rangle, |9\rangle, |10\rangle, |13\rangle, |14\rangle\}$, and f_2 evaluates to true for each pattern in the set $\{|0\rangle, |1\rangle, |4\rangle, |5\rangle, |7\rangle, |8\rangle, |11\rangle, |12\rangle\}$. The complete implementation of the algorithm, including the oracle function, can be found in Figure 16 and Figure 19.

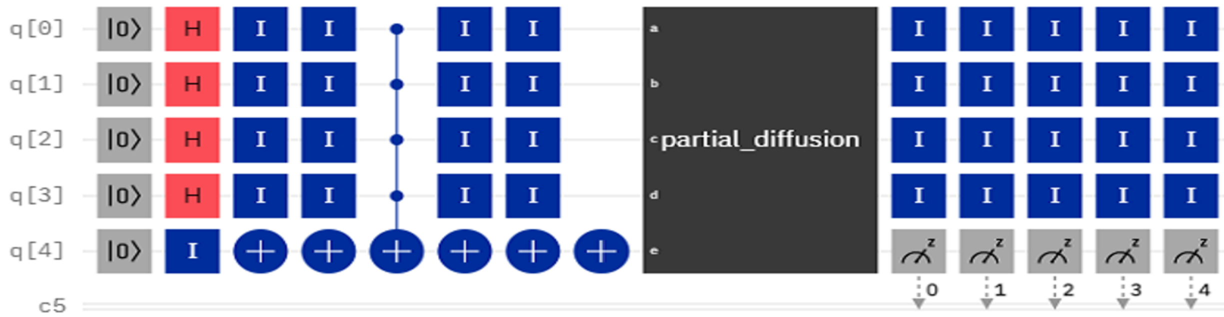


Figure 19: Finding the Union between the two Boolean functions f_1 and f_2.

5. Evaluation on IBM's Quantum Computers

5.1. Set operation: True intersection

Run the quantum circuit in Figure 13 on a statevector simulator backend, then the superposition of the intersection of f_1 and f_2 is as in Figure 20. Probability of the true intersection of f_1 and f_2 is as in Figure 21.

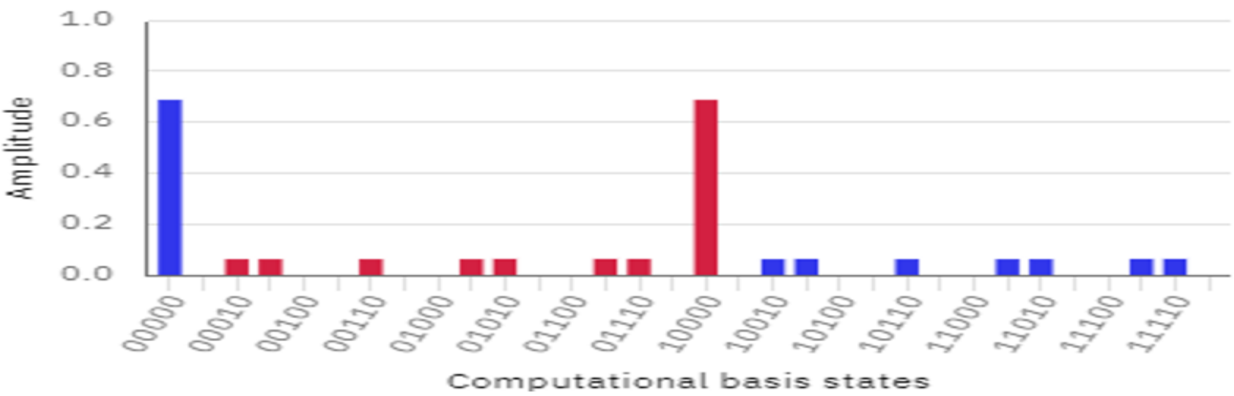


Figure 20: Superposition of the intersection of f_1 and f_2.

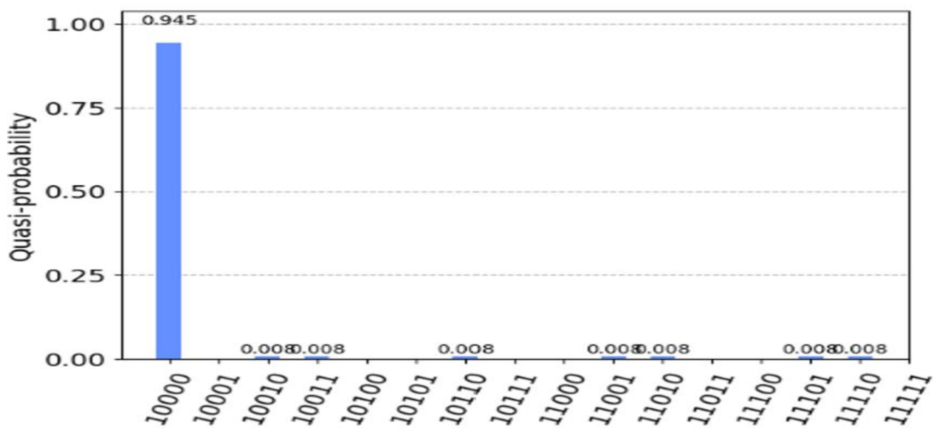
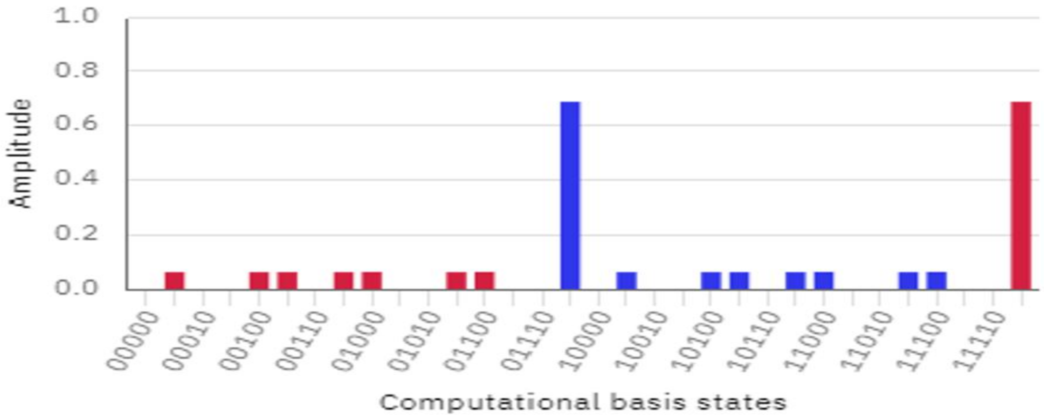


Figure 21: Probability of the intersection of f_1 and f_2.

5.2. Set operation: False intersection

Run the quantum circuit in Figure 16 on a statevector simulator backend, then the superposition of the intersection of $f_1 \oplus 1$ and $f_2 \oplus 1$ is as in Figure 22. Probability of the intersection of $f_1 \oplus 1$ and $f_2 \oplus 1$ is as in Figure 23.

Figure 22: Superposition of the intersection of $f_1 \oplus 1$ and $f_2 \oplus 1$.



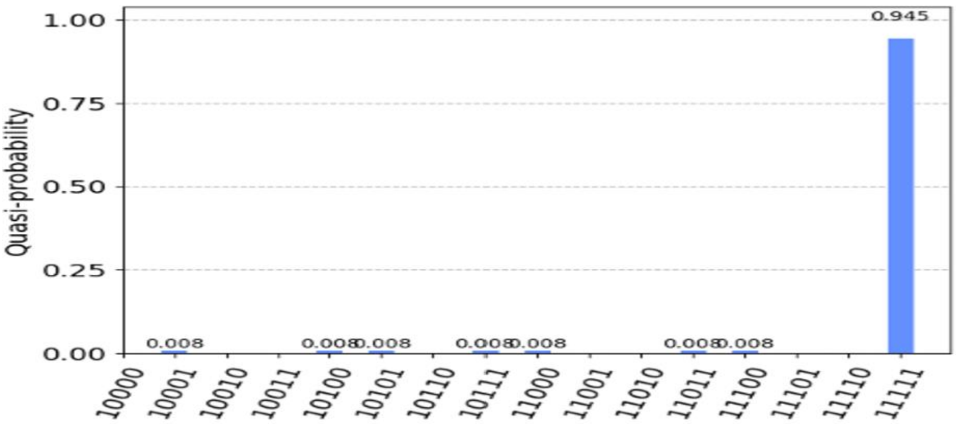
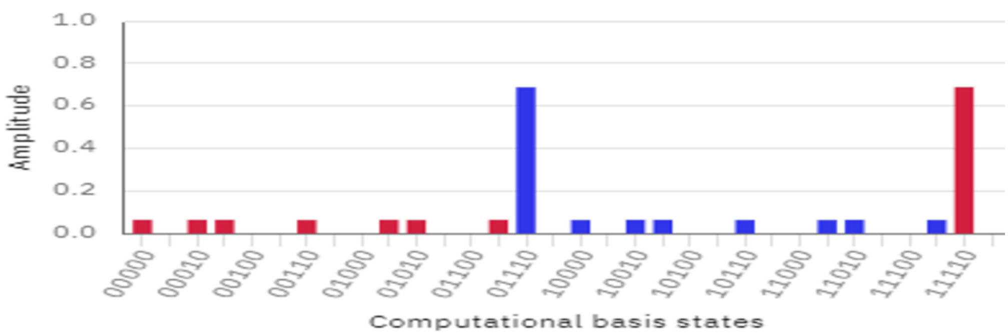


Figure 23: Probability of the intersection of $f_1 \oplus 1$ and $f_2 \oplus 1$.

5.3. Set operation: Difference

Run the quantum circuit in Figure 17 on a statevector simulator backend, then the superposition of the difference between f_1 and $f_s \oplus 1$ is as in Figure 24. Probability of the difference between f_1 and $f_s \oplus 1$ is as in Figure 25.

Figure 24: Superposition of the difference between f_1 and $f_s \oplus 1$.



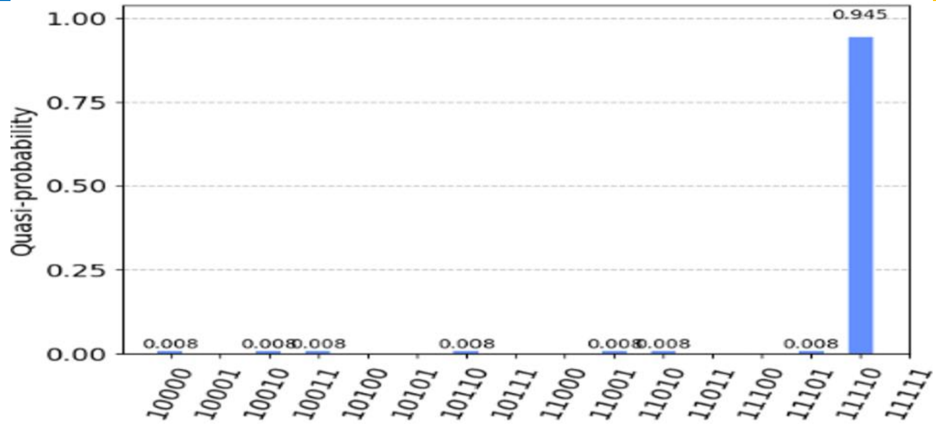


Figure 25: Probability of the difference between f_1 and $f_s \oplus 1$.

5.4. Set operation: Union

Run the quantum circuit in Figure 19 on a statevector simulator backend, then the superposition of the intersection of f_1 and f_2 is as in Figure 26. Probability of the difference between f_1 and f_2 is as in Figure 27.

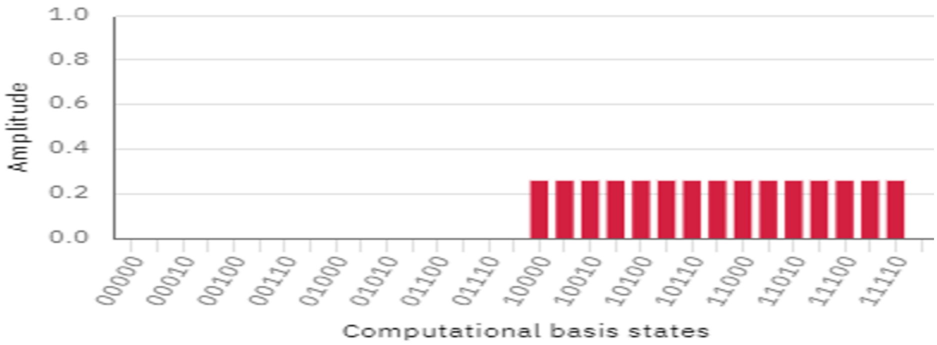


Figure 26: Superposition of the union between f_1 and f_2.

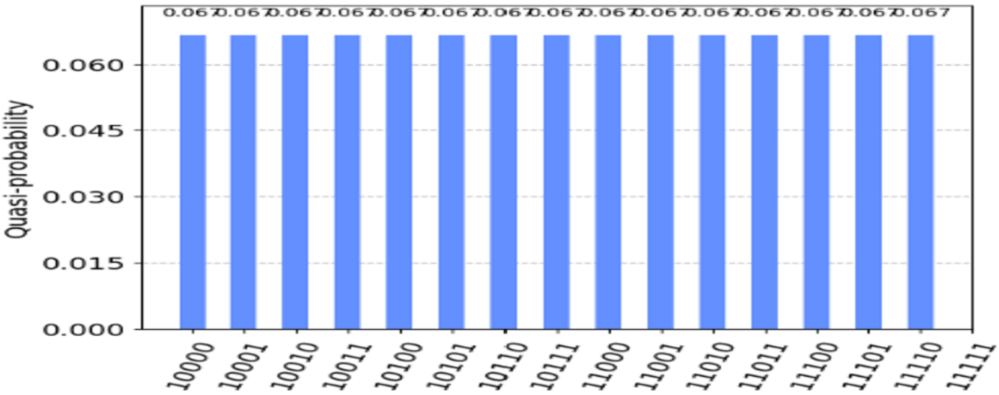


Figure 27: Probability of the union between f_1 and f_2.

6. Evaluation by mathematical calculations

6.1. Set operation: True intersection

1. We apply the Younes et al. algorithm[11] with the following stages:

Prepare a register:

$$|\psi_0\rangle = |0\rangle^{\otimes 4} \otimes |0\rangle = |00000\rangle. \quad (5)$$

Initialize the register:

$$|\psi_1\rangle = \left(H^{\otimes 4} \otimes I\right)|\psi_0\rangle = \left(H^{\otimes 4} \otimes I\right)|00000\rangle$$
$$= \frac{1}{4}(|00000\rangle + |00010\rangle + |00100\rangle + |00110\rangle + |01000\rangle$$
$$+|01010\rangle + |01100\rangle + |01110\rangle + |10000\rangle + |10010\rangle$$
$$+|10100\rangle + |10110\rangle + |11000\rangle + |11010\rangle + |11100\rangle$$
$$+|11110\rangle). \quad (6)$$

Apply the oracle operator $I_{f_1^T}$:

$$|\psi_2\rangle = I_{f_1^T}|\psi_1\rangle = I_{f_1^T}\left(H^{\otimes 4} \otimes I\right)|00000\rangle$$
$$= \frac{1}{4}(|00001\rangle + |00010\rangle + |00101\rangle + |00111\rangle + |01000\rangle$$
$$+|01010\rangle + |01101\rangle + |01110\rangle + |10000\rangle + |10011\rangle$$
$$+|10101\rangle + |10110\rangle + |11000\rangle + |11011\rangle + |11101\rangle$$
$$+|11110\rangle). \quad (7)$$

Apply partial diffusion D_p:

$$|\psi_3\rangle = D_p|\psi_2\rangle = D_p \ I_{f_1^T}\left(H^{\otimes 4} \otimes I\right)|00000\rangle$$
$$= \frac{1}{4}(|0000\rangle + |0010\rangle + |0011\rangle + |0110\rangle + |1001\rangle$$
$$+|1010\rangle + |1101\rangle + |1110\rangle) \otimes |0\rangle$$
$$-\frac{1}{4}(|0000\rangle + |0010\rangle + |0011\rangle + |0110\rangle + |1001\rangle$$
$$+|1010\rangle + |1101\rangle + |1110\rangle) \otimes |1\rangle. \quad (8)$$

Measure the auxiliary qubit:

$$|\psi_{4i}\rangle = \frac{-\sqrt{2}}{4}(|0000\rangle + |0010\rangle + |0011\rangle + |0110\rangle$$
$$+|1001\rangle + |1010\rangle + |1101\rangle + |1110\rangle) \otimes |1\rangle. \quad (9)$$

Apply Z on the auxiliary qubit:

$$|\psi_{4ii}\rangle = (I^{\otimes 4} \otimes Z)|\psi_{4i}\rangle$$
$$= \frac{\sqrt{2}}{4}(|0000\rangle + |0010\rangle + |0011\rangle + |0110\rangle$$
$$+|1001\rangle + |1010\rangle + |1101\rangle + |1110\rangle) \otimes |1\rangle. \quad (10)$$

Apply H on the auxiliary qubit:

$$|\psi_{4iii}\rangle = (I^{\otimes 4} \otimes H)|\psi_{4ii}\rangle = (I^{\otimes 4} \otimes H) \otimes (I^{\otimes 4} \otimes Z)|\psi_{4i}\rangle$$
$$= \frac{1}{4}(|0000\rangle + |0010\rangle + |0011\rangle + |0110\rangle + |1001\rangle$$
$$+|1010\rangle + |1101\rangle + |1110\rangle) \otimes |0\rangle$$
$$-\frac{1}{4}(|0000\rangle + |0010\rangle + |0011\rangle + |0110\rangle + |1001\rangle$$
$$+|1010\rangle + |1101\rangle + |1110\rangle) \otimes |1\rangle. \quad (11)$$

2. Apply the Arima algorithm[13]

Apply the oracle operator $I_{f_2^T}$:

$$|\psi_{5i}\rangle = \ I_{f_2^T}|\psi_{4iii}\rangle$$
$$= \frac{1}{4}(-|0000\rangle + |0010\rangle + |0011\rangle + |0110\rangle + |1001\rangle$$
$$+|1010\rangle + |1101\rangle + |1110\rangle) \otimes |0\rangle$$
$$-\frac{1}{4}(-|0000\rangle + |0010\rangle + |0011\rangle + |0110\rangle + |1001\rangle$$
$$+|1010\rangle + |1101\rangle + |1110\rangle) \otimes |1\rangle. \quad (12)$$

Apply the Grover operator G:

$$|\psi_{5ii}\rangle = G|\psi_{5i}\rangle$$
$$= \frac{1}{16}(7|0000\rangle + 3|0001\rangle - |0010\rangle - |0011\rangle + 3|0100\rangle$$
$$+3|0101\rangle - |0110\rangle + 3|0111\rangle + 3|1000\rangle - |1001\rangle - |1010\rangle$$
$$+3|1011\rangle + 3|1100\rangle - |1101\rangle - |1110\rangle + 3|1111\rangle) \otimes |0\rangle$$
$$+\frac{1}{16}(-7|0000\rangle - 3|0001\rangle + |0010\rangle + |0011\rangle - 3|0100\rangle$$
$$-3|0101\rangle + |0110\rangle - 3|0111\rangle - 3|1000\rangle + |1001\rangle + |1010\rangle$$
$$-3|1011\rangle - 3|1100\rangle + |1101\rangle + |1110\rangle - 3|1111\rangle) \otimes |1\rangle. \quad (13)$$

Apply the oracle operator $I_{f_1^T}$:

$$|\psi_{6i}\rangle = I_{f_1^T}|\psi_{5ii}\rangle$$
$$= \frac{1}{16}(-7|0000\rangle + 3|0001\rangle + |0010\rangle + |0011\rangle + 3|0100\rangle$$
$$+3|0101\rangle + |0110\rangle + 3|0111\rangle + 3|1000\rangle + |1001\rangle + |1010\rangle$$
$$+3|1011\rangle + 3|1100\rangle + |1101\rangle + |1110\rangle + 3|1111\rangle) \otimes |0\rangle$$
$$+\frac{1}{16}(7|0000\rangle - 3|0001\rangle - |0010\rangle - |0011\rangle - 3|0100\rangle$$
$$-3|0101\rangle - |0110\rangle - 3|0111\rangle - 3|1000\rangle - |1001\rangle - |1010\rangle$$
$$-3|1011\rangle - 3|1100\rangle - |1101\rangle - |1110\rangle - 3|1111\rangle) \otimes |1\rangle. \quad (14)$$

Apply the Grover operator G:

$$|\psi_{6ii}\rangle = G|\psi_{6i}\rangle$$
$$= \frac{1}{8}(5|0000\rangle + |0010\rangle + |0011\rangle + |0110\rangle + |1001\rangle$$
$$+|1010\rangle + |1101\rangle + |1110\rangle) \otimes |0\rangle$$
$$-\frac{1}{8}(5|0000\rangle + |0010\rangle + |0011\rangle + |0110\rangle + |1001\rangle$$
$$+|1010\rangle + |1101\rangle + |1110\rangle) \otimes |1\rangle. \quad (15)$$

Apply the oracle operator $I_{f_2^T}$:

$$|\psi_{6iii}\rangle = I_{f_2^T}|\psi_{6ii}\rangle$$
$$= \frac{1}{8}(-5|0000\rangle + |0010\rangle + |0011\rangle + |0110\rangle + |1001\rangle$$
$$+|1010\rangle + |1101\rangle + |1110\rangle) \otimes |0\rangle$$
$$-\frac{1}{8}(-5|0000\rangle + |0010\rangle + |0011\rangle + |0110\rangle + |1001\rangle$$
$$+|1010\rangle + |1101\rangle + |1110\rangle) \otimes |1\rangle. \quad (16)$$

Apply the Grover operator G:

$$|\psi_{6iv}\rangle = G|\psi_{6iii}\rangle$$
$$= \frac{1}{32}(21|0000\rangle + |0001\rangle - 3|0010\rangle - 3|0011\rangle + |0100\rangle$$
$$+|0101\rangle - 3|0110\rangle + |0111\rangle + |1000\rangle - 3|1001\rangle - 3|1010\rangle$$
$$+|1011\rangle + |1100\rangle - 3|1101\rangle - 3|1110\rangle + |1111\rangle) \otimes |0\rangle$$
$$+\frac{1}{32}(-21|0000\rangle - |0001\rangle + 3|0010\rangle + 3|0011\rangle - |0100\rangle$$
$$-|0101\rangle + 3|0110\rangle - |0111\rangle - |1000\rangle + 3|1001\rangle + 3|1010\rangle$$
$$-|1011\rangle - |1100\rangle + 3|1101\rangle + 3|1110\rangle - |1111\rangle) \otimes |1\rangle. \quad (17)$$

Apply the oracle operator $I_{f_1^T}$:

$$|\psi_{6v}\rangle = I_{f_1^T}|\psi_{6iv}\rangle$$
$$= \frac{1}{32}(-21|0000\rangle + |0001\rangle + 3|0010\rangle + 3|0011\rangle + |0100\rangle$$
$$+|0101\rangle + 3|0110\rangle + |0111\rangle + |1000\rangle + 3|1001\rangle + 3|1010\rangle$$
$$+|1011\rangle + |1100\rangle + 3|1101\rangle + 3|1110\rangle + |1111\rangle) \otimes |0\rangle$$
$$+\frac{1}{32}(21|0000\rangle - |0001\rangle - 3|0010\rangle - 3|0011\rangle - |0100\rangle$$
$$-|0101\rangle - 3|0110\rangle - |0111\rangle - |1000\rangle - 3|1001\rangle - 3|1010\rangle$$
$$-|1011\rangle - |1100\rangle - 3|1101\rangle - 3|1110\rangle - |1111\rangle) \otimes |1\rangle. \quad (18)$$

Apply the Grover operator G:

$$|\psi_{6ii}\rangle = G|\psi_{6i}\rangle$$
$$= \frac{1}{16}(11|0000\rangle - |0010\rangle - |0011\rangle - |0110\rangle - |1001\rangle$$
$$-|1010\rangle - |1101\rangle - |1110\rangle) \otimes |0\rangle$$
$$-\frac{1}{16}(11|0000\rangle - |0010\rangle - |0011\rangle - |0110\rangle - |1001\rangle$$
$$-|1010\rangle - |1101\rangle - |1110\rangle) \otimes |1\rangle. \quad (19)$$

Thus, the common area where the functions f_1 and f_2 both evaluate to True can be obtained with a probability of 0.945 by measuring $|0\rangle$. This probability is derived from the expression $2 * (11/16)^2$.

6.2. Set operation: False intersection

1. We apply the Younes et al. algorithm[11] with the following stages:

Prepare a register:

$$|\psi_0\rangle = |0\rangle^{\otimes 4} \otimes |0\rangle = |00000\rangle. \quad (20)$$

Initialize the register:

$$\begin{aligned}|\psi_1\rangle &= \left(H^{\otimes 4} \otimes I\right)|\psi_0\rangle = \left(H^{\otimes 4} \otimes I\right)|00000\rangle \\ &= \tfrac{1}{4}(|00000\rangle + |00010\rangle + |00100\rangle + |00110\rangle + |01000\rangle \\ &\quad +|01010\rangle + |01100\rangle + |01110\rangle + |10000\rangle + |10010\rangle \\ &\quad +|10100\rangle + |10110\rangle + |11000\rangle + |11010\rangle + |11100\rangle \\ &\quad +|11110\rangle).\end{aligned} \quad (21)$$

Apply the oracle operator $I_{f_1^T}$:

$$\begin{aligned}|\psi_2\rangle &= I_{f_1^T}|\psi_1\rangle = I_{f_1^T}\left(H^{\otimes 4} \otimes I\right)|00000\rangle \\ &= \tfrac{1}{4}(|00001\rangle + |00010\rangle + |00101\rangle + |00111\rangle + |01000\rangle \\ &\quad +|01010\rangle + |01101\rangle + |01110\rangle + |10000\rangle + |10011\rangle \\ &\quad +|10101\rangle + |10110\rangle + |11000\rangle + |11011\rangle + |11101\rangle \\ &\quad +|11110\rangle).\end{aligned} \quad (22)$$

Apply X on the auxiliary qubit:

$$\begin{aligned}|\psi_3\rangle &= (I^{\otimes 4} \otimes X)|\psi_2\rangle \\ &= \tfrac{1}{4}(|00000\rangle + |00011\rangle + |00100\rangle + |00110\rangle + |01001\rangle \\ &\quad +|01011\rangle + |01100\rangle + |01111\rangle + |10001\rangle + |10010\rangle \\ &\quad +|10100\rangle + |10111\rangle + |11001\rangle + |11010\rangle + |11100\rangle \\ &\quad +|11111\rangle).\end{aligned} \quad (23)$$

Apply partial diffusion D_p:

$$\begin{aligned}|\psi_4\rangle &= D_p|\psi_3\rangle \\ &= \tfrac{1}{4}(|00010\rangle - |00011\rangle + |01000\rangle - |01001\rangle + |01010\rangle \\ &\quad -01011\rangle + |01110\rangle - |01111\rangle + |10000\rangle - |10001\rangle \\ &\quad +|10110\rangle - |10111\rangle + |11000\rangle - |11001\rangle + |11110\rangle \\ &\quad -|11111\rangle).\end{aligned} \quad (24)$$

Measure the auxiliary qubit:

$$\begin{aligned}|\psi_{5i}\rangle &= \tfrac{-\sqrt{2}}{4}(|0001\rangle + |0100\rangle + |0101\rangle + |0111\rangle + |1000\rangle \\ &\quad +|1011\rangle + |1100\rangle + |1111\rangle) \otimes |1\rangle\end{aligned} \quad (25)$$

Apply Z on the auxiliary qubit.

$$\begin{aligned}|\psi_{4ii}\rangle &= (I^{\otimes 4} \otimes Z)|\psi_{4i}\rangle \\ &= \tfrac{\sqrt{2}}{4}(|0001\rangle + |0100\rangle + |0101\rangle + |0111\rangle + |1000\rangle \\ &\quad +|1011\rangle + |1100\rangle + |1111\rangle) \otimes |1\rangle\end{aligned} \quad (26)$$

Apply H on the auxiliary qubit.

$$\begin{aligned}|\psi_{5iii}\rangle &= \left(I^{\otimes 4} \otimes H\right)|\psi_{5ii}\rangle \\ &= \tfrac{1}{4}(|00010\rangle - |00011\rangle + |01000\rangle - |01001\rangle + |01010\rangle \\ &\quad -01011\rangle + |01110\rangle - |01111\rangle + |10000\rangle - |10001\rangle \\ &\quad +|10110\rangle - |10111\rangle + |11000\rangle - |11001\rangle + |11110\rangle \\ &\quad -|11111\rangle).\end{aligned} \quad (27)$$

2. Apply the Arima algorithm[13]

Apply the oracle operator $I_{f_2^T}$:

$$\begin{aligned}|\psi_6\rangle &= I_{f_2^T}|\psi_{5iii}\rangle \\ &= \tfrac{-1}{4}(|00010\rangle - |00011\rangle + |01000\rangle - |01001\rangle + |01010\rangle \\ &\quad -01011\rangle + |01110\rangle - |01111\rangle + |10000\rangle - |10001\rangle \\ &\quad +|10110\rangle - |10111\rangle + |11000\rangle - |11001\rangle - |11110\rangle\end{aligned}$$

$+|11111\rangle).$ (28)

Apply X on the auxiliary qubit:

$$|\psi_7\rangle = \left(I^{\otimes 4} \otimes X\right)|\psi_6\rangle$$
$$= \frac{1}{4}(|00010\rangle - |00011\rangle + |01000\rangle - |01001\rangle + |01010\rangle$$
$$-01011\rangle + |01110\rangle - |01111\rangle + |10000\rangle - |10001\rangle$$
$$+|10110\rangle - |10111\rangle + |11000\rangle - |11001\rangle - |11110\rangle$$
$$+|11111\rangle). \quad (29)$$

Apply the Grover operator G:

$$|\psi_8\rangle = G|\psi_7\rangle$$
$$= \frac{1}{16}(3|0000\rangle - |0001\rangle + 3|0010\rangle + 3|0011\rangle - |0100\rangle$$
$$-|0101\rangle + 3|0110\rangle - |0111\rangle - |1000\rangle + 3|1001\rangle$$
$$+3|1010\rangle - |1011\rangle - |1100\rangle + 3|1101\rangle + 3|1110\rangle$$
$$+7|1111\rangle) \otimes |0\rangle$$
$$+\frac{1}{16}(-3|0000\rangle + |0001\rangle - 3|0010\rangle - 3|0011\rangle + |0100\rangle$$
$$+|0101\rangle - 3|0110\rangle + |0111\rangle + |1000\rangle - 3|1001\rangle$$
$$-3|1010\rangle + |1011\rangle + |1100\rangle - 3|1101\rangle - 3|1110\rangle$$
$$-7|1111\rangle) \otimes |1\rangle. \quad (30)$$

Apply the oracle operator $I_{f_1^T}$:

$$|\psi_{9i}\rangle = I_{f_1^T}|\psi_8\rangle$$
$$= \frac{1}{16}(-3|0000\rangle - |0001\rangle - 3|0010\rangle - 3|0011\rangle - |0100\rangle$$
$$-|0101\rangle - 3|0110\rangle - |0111\rangle - |1000\rangle - 3|1001\rangle$$
$$-3|1010\rangle - |1011\rangle - |1100\rangle - 3|1101\rangle - 3|1110\rangle$$
$$+7|1111\rangle) \otimes |0\rangle$$
$$+\frac{1}{16}(3|0000\rangle + |0001\rangle + 3|0010\rangle + 3|0011\rangle + |0100\rangle$$
$$+|0101\rangle + 3|0110\rangle + |0111\rangle + |1000\rangle + 3|1001\rangle$$
$$+3|1010\rangle + |1011\rangle + |1100\rangle + 3|1101\rangle + 3|1110\rangle$$
$$-7|1111\rangle) \otimes |1\rangle. \quad (31)$$

Apply the Grover operator G:

$$|\psi_{9ii}\rangle = G|\psi_{9i}\rangle$$
$$= \frac{1}{8}(-|00010\rangle + |00011\rangle - |01000\rangle + |01001\rangle - |01010\rangle$$
$$+01011\rangle - |01110\rangle + |01111\rangle - |10000\rangle + |10001\rangle$$
$$-|10110\rangle + |10111\rangle - |11000\rangle + |11001\rangle - 5|11110\rangle$$
$$+5|11111\rangle). \quad (32)$$

Apply the oracle operator $I_{f_2^T}$:

$$|\psi_{9iii}\rangle = I_{f_2^T}|\psi_{9ii}\rangle$$
$$= \frac{1}{8}(|00010\rangle - |00011\rangle + |01000\rangle - |01001\rangle + |01010\rangle$$
$$-01011\rangle + |01110\rangle - |01111\rangle + |10000\rangle - |10001\rangle$$
$$+|10110\rangle - |10111\rangle + |11000\rangle - |11001\rangle - 5|11110\rangle$$
$$+5|11111\rangle). \quad (33)$$

Apply X on the auxiliary qubit:

$$|\psi_{9iv}\rangle = \left(I^{\otimes 4} \otimes X\right)|\psi_{9iii}\rangle$$
$$= \frac{1}{8}(-|00010\rangle + |00011\rangle - |01000\rangle + |01001\rangle - |01010\rangle$$
$$+01011\rangle - |01110\rangle + |01111\rangle - |10000\rangle + |10001\rangle$$
$$-|10110\rangle + |10111\rangle - |11000\rangle + |11001\rangle + 5|11110\rangle$$
$$-5|11111\rangle). \quad (34)$$

Apply the Grover operator G:

$$|\psi_{9v}\rangle = G|\psi_{9iv}\rangle$$
$$= \frac{1}{32}(-|0000\rangle + 3|0001\rangle - |0010\rangle - |0011\rangle + 3|0100\rangle$$
$$+3|0101\rangle - |0110\rangle + 3|0111\rangle + 3|1000\rangle - |1001\rangle$$
$$-|1010\rangle + 3|1011\rangle + 3|1100\rangle - |1101\rangle - |1110\rangle$$

$$-21|1111\rangle) \otimes |0\rangle$$
$$+\frac{1}{32}(|0000\rangle - 3|0001\rangle + |0010\rangle + |0011\rangle - 3|0100\rangle$$
$$-3|0101\rangle + |0110\rangle - 3|0111\rangle - 3|1000\rangle + |1001\rangle$$
$$+|1010\rangle - 3|1011\rangle - 3|1100\rangle + |1101\rangle + |1110\rangle$$
$$+21|1111\rangle) \otimes |1\rangle. \tag{35}$$

Apply the oracle operator $I_{f_1^T}$:

$$|\psi_{9vi}\rangle = I_{f_1^T}|\psi_{9v}\rangle$$
$$= \frac{1}{32}(|0000\rangle + 3|0001\rangle + |0010\rangle + |0011\rangle + 3|0100\rangle$$
$$+3|0101\rangle + |0110\rangle + 3|0111\rangle + 3|1000\rangle + |1001\rangle$$
$$+|1010\rangle + 3|1011\rangle + 3|1100\rangle + |1101\rangle + |1110\rangle$$
$$-21|1111\rangle) \otimes |0\rangle$$
$$+\frac{1}{32}(-|0000\rangle - 3|0001\rangle - |0010\rangle - |0011\rangle - 3|0100\rangle$$
$$-3|0101\rangle - |0110\rangle - 3|0111\rangle - 3|1000\rangle - |1001\rangle$$
$$-|1010\rangle - 3|1011\rangle - 3|1100\rangle - |1101\rangle - |1110\rangle$$
$$+21|1111\rangle) \otimes |1\rangle. \tag{36}$$

Apply the Grover operator G:

$$|\psi_{9vii}\rangle = G|\psi_{9vi}\rangle$$
$$= \frac{1}{16}(-|00010\rangle + |00011\rangle - |01000\rangle + |01001\rangle - |01010\rangle$$
$$+01011\rangle - |01110\rangle + |01111\rangle - |10000\rangle + |10001\rangle$$
$$-|10110\rangle + |10111\rangle - |11000\rangle + |11001\rangle + 11|11110\rangle$$
$$-11|11111\rangle). \tag{37}$$

Thus, the common area where the functions f_1 and f_2 both evaluate to False can be obtained with a probability of 0.945 by measuring $|15\rangle$. This probability is derived from the expression $2 * (11/16)^2$.

6.3. Set operation: Difference

1. We apply the Younes et al. algorithm[11] with the following stages:

Prepare a register:

$$|\psi_0\rangle = |0\rangle^{\otimes 4} \otimes |0\rangle = |00000\rangle. \tag{38}$$

Initialize the register:

$$|\psi_1\rangle = (H^{\otimes 4} \otimes I)|\psi_0\rangle = (H^{\otimes 4} \otimes I)|00000\rangle$$
$$= \frac{1}{4}(|00000\rangle + |00010\rangle + |00100\rangle + |00110\rangle + |01000\rangle$$
$$+|01010\rangle + |01100\rangle + |01110\rangle + |10000\rangle + |10010\rangle$$
$$+|10100\rangle + |10110\rangle + |11000\rangle + |11010\rangle + |11100\rangle$$
$$+|11110\rangle). \tag{39}$$

Apply the oracle operator $I_{f_1^T}$:

$$|\psi_2\rangle = I_{f_1^T}|\psi_1\rangle = I_{f_1^T}(H^{\otimes 4} \otimes I)|00000\rangle$$
$$= \frac{1}{4}(|00001\rangle + |00010\rangle + |00101\rangle + |00111\rangle + |01000\rangle$$
$$+|01010\rangle + |01101\rangle + |01110\rangle + |10000\rangle + |10011\rangle$$
$$+|10101\rangle + |10110\rangle + |11000\rangle + |11011\rangle + |11101\rangle$$
$$+|11110\rangle). \tag{40}$$

Apply partial diffusion D_p:

$$|\psi_3\rangle = D_p|\psi_2\rangle = D_p\ I_{f_1^T}(H^{\otimes 4} \otimes I)|00000\rangle$$
$$= \frac{1}{4}(|0000\rangle + |0010\rangle + |0011\rangle + |0110\rangle + |1001\rangle$$
$$+|1010\rangle + |1101\rangle + |1110\rangle) \otimes |0\rangle$$
$$-\frac{1}{4}(|0000\rangle + |0010\rangle + |0011\rangle + |0110\rangle + |1001\rangle$$
$$+|1010\rangle + |1101\rangle + |1110\rangle) \otimes |1\rangle. \tag{41}$$

Measure the auxiliary qubit:

$$|\psi_{4i}\rangle = \frac{-\sqrt{2}}{4}(|0000\rangle + |0010\rangle + |0011\rangle + |0110\rangle$$

$$+|1001\rangle + |1010\rangle + |1101\rangle + |1110\rangle) \otimes |1\rangle. \quad (42)$$

Apply Z on the auxiliary qubit:

$$|\psi_{4ii}\rangle = (I^{\otimes 4} \otimes Z)|\psi_{4i}\rangle$$
$$= \frac{\sqrt{2}}{4}(|0000\rangle + |0010\rangle + |0011\rangle + |0110\rangle$$
$$+|1001\rangle + |1010\rangle + |1101\rangle + |1110\rangle) \otimes |1\rangle. \quad (43)$$

Apply H on the auxiliary qubit:

$$|\psi_{4iii}\rangle = (I^{\otimes 4} \otimes H)|\psi_{4ii}\rangle = (I^{\otimes 4} \otimes H) \otimes (I^{\otimes 4} \otimes Z)|\psi_{4i}\rangle$$
$$= \frac{1}{4}(|0000\rangle + |0010\rangle + |0011\rangle + |0110\rangle + |1001\rangle$$
$$+|1010\rangle + |1101\rangle + |1110\rangle) \otimes |0\rangle$$
$$-\frac{1}{4}(|0000\rangle + |0010\rangle + |0011\rangle + |0110\rangle + |1001\rangle$$
$$+|1010\rangle + |1101\rangle + |1110\rangle) \otimes |1\rangle. \quad (44)$$

2. Apply the Arima algorithm[13]

Apply the oracle operator $I_{f_S^T}$:

$$|\psi_{5i}\rangle = I_{f_S^T}|\psi_{4iii}\rangle$$
$$= \frac{-1}{4}(|0000\rangle + |0010\rangle + |0011\rangle + |0110\rangle + |1001\rangle$$
$$+|1010\rangle + |1101\rangle - |1110\rangle) \otimes |0\rangle$$
$$+\frac{1}{4}(|0000\rangle + |0010\rangle + |0011\rangle + |0110\rangle + |1001\rangle$$
$$+|1010\rangle + |1101\rangle - |1110\rangle) \otimes |1\rangle. \quad (45)$$

Apply X on the auxiliary qubit:

$$|\psi_6\rangle = \left(I^{\otimes 4} \otimes X\right)|\psi_5\rangle$$
$$= \frac{1}{4}(|0000\rangle + |0010\rangle + |0011\rangle + |0110\rangle + |1001\rangle$$
$$+|1010\rangle + |1101\rangle - |1110\rangle) \otimes |0\rangle$$
$$+\frac{-1}{4}(|0000\rangle + |0010\rangle + |0011\rangle + |0110\rangle + |1001\rangle$$
$$+|1010\rangle + |1101\rangle - |1110\rangle) \otimes |1\rangle. \quad (46)$$

Apply the Grover operator G:

$$|\psi_7\rangle = G|\psi_6\rangle$$
$$= \frac{1}{16}(-|0000\rangle + 3|0001\rangle - |0010\rangle - |0011\rangle + 3|0100\rangle$$
$$+3|0101\rangle - |0110\rangle + 3|0111\rangle + 3|1000\rangle - |1001\rangle$$
$$-|1010\rangle + 3|1011\rangle + 3|1100\rangle - |1101\rangle + 7|1110\rangle$$
$$+3|1111\rangle) \otimes |0\rangle$$
$$+\frac{1}{16}(|0000\rangle - 3|0001\rangle + |0010\rangle + |0011\rangle - 3|0100\rangle$$
$$-3|0101\rangle + |0110\rangle - 3|0111\rangle - 3|1000\rangle + |1001\rangle$$
$$+|1010\rangle - 3|1011\rangle - 3|1100\rangle + |1101\rangle - 7|1110\rangle$$
$$-3|1111\rangle) \otimes |1\rangle. \quad (47)$$

Apply the oracle operator $I_{f_1^T}$:

$$|\psi_{8i}\rangle = I_{f_1^T}|\psi_7\rangle$$
$$= \frac{1}{16}(|0000\rangle + 3|0001\rangle + |0010\rangle + |0011\rangle + 3|0100\rangle$$
$$+3|0101\rangle + |0110\rangle + 3|0111\rangle + 3|1000\rangle + |1001\rangle$$
$$+|1010\rangle + 3|1011\rangle + 3|1100\rangle + |1101\rangle - 7|1110\rangle$$
$$+3|1111\rangle) \otimes |0\rangle$$
$$+\frac{-1}{16}(|0000\rangle + 3|0001\rangle + |0010\rangle + |0011\rangle + 3|0100\rangle$$
$$+3|0101\rangle + |0110\rangle + 3|0111\rangle + 3|1000\rangle + |1001\rangle$$
$$+|1010\rangle + 3|1011\rangle + 3|1100\rangle + |1101\rangle - 7|1110\rangle$$
$$+3|1111\rangle) \otimes |1\rangle. \quad (48)$$

Apply the Grover operator G:

$$|\psi_{8ii}\rangle = G|\psi_{8i}\rangle = \frac{1}{8}(|0000\rangle + |0010\rangle + |0011\rangle + |0110\rangle + |1001\rangle$$
$$+|1010\rangle + |1101\rangle + 5|1110\rangle) \otimes |0\rangle$$

$$-\frac{1}{8}(|0000\rangle + |0010\rangle + |0011\rangle + |0110\rangle + |1001\rangle$$
$$+|1010\rangle + |1101\rangle + 5|1110\rangle) \otimes |1\rangle. \quad (49)$$

Apply the oracle operator $I_{f_S^T}$:

$$|\psi_{8iii}\rangle = I_{f_S^T}|\psi_{8ii}\rangle = \frac{-1}{8}(|0000\rangle + |0010\rangle + |0011\rangle + |0110\rangle + |1001\rangle$$
$$+|1010\rangle + |1101\rangle + 5|1110\rangle) \otimes |0\rangle$$
$$+\frac{1}{8}(|0000\rangle + |0010\rangle + |0011\rangle + |0110\rangle + |1001\rangle$$
$$+|1010\rangle + |1101\rangle + 5|1110\rangle) \otimes |1\rangle. \quad (50)$$

Apply X on the auxiliary qubit:

$$|\psi_{8iv}\rangle = \left(I^{\otimes 4} \otimes X\right)|\psi_{8iii}\rangle$$
$$= \frac{1}{8}(|0000\rangle + |0010\rangle + |0011\rangle + |0110\rangle + |1001\rangle$$
$$+|1010\rangle + |1101\rangle - 5|1110\rangle) \otimes |0\rangle$$
$$+\frac{-1}{8}(|0000\rangle + |0010\rangle + |0011\rangle + |0110\rangle + |1001\rangle$$
$$+|1010\rangle + |1101\rangle - 5|1110\rangle) \otimes |1\rangle. \quad (51)$$

Apply the Grover operator G:

$$|\psi_{8v}\rangle = G|\psi_{8iv}\rangle$$
$$= \frac{1}{32}(-3|0000\rangle + |0001\rangle - 3|0010\rangle - 3|0011\rangle + |0100\rangle$$
$$+|0101\rangle - 3|0110\rangle + |0111\rangle + |1000\rangle - 3|1001\rangle$$
$$-3|1010\rangle + |1011\rangle + |1100\rangle - 3|1101\rangle + 21|1110\rangle$$
$$+|1111\rangle) \otimes |0\rangle$$
$$+\frac{1}{32}(3|0000\rangle - |0001\rangle + 3|0010\rangle + 3|0011\rangle - |0100\rangle$$
$$-|0101\rangle + 3|0110\rangle - |0111\rangle - |1000\rangle + 3|1001\rangle$$
$$+3|1010\rangle - |1011\rangle - |1100\rangle + 3|1101\rangle - 21|1110\rangle$$
$$-|1111\rangle) \otimes |1\rangle. \quad (52)$$

Apply the oracle operator$I_{f_1^T}$:

$$|\psi_{8vi}\rangle = I_{f_1^T}|\psi_{8v}\rangle$$
$$= \frac{1}{32}(3|0000\rangle + |0001\rangle + 3|0010\rangle + 3|0011\rangle + |0100\rangle$$
$$+|0101\rangle + 3|0110\rangle + |0111\rangle + |1000\rangle + 3|1001\rangle$$
$$+3|1010\rangle + |1011\rangle + |1100\rangle + 3|1101\rangle - 21|1110\rangle$$
$$+|1111\rangle) \otimes |0\rangle$$
$$+\frac{-1}{32}(3|0000\rangle + |0001\rangle + 3|0010\rangle + 3|0011\rangle + |0100\rangle$$
$$+|0101\rangle + 3|0110\rangle + |0111\rangle + |1000\rangle + 3|1001\rangle$$
$$+3|1010\rangle + |1011\rangle + |1100\rangle + 3|1101\rangle - 21|1110\rangle$$
$$+|1111\rangle) \otimes |1\rangle. \quad (53)$$

Apply the Grover operator G:

$$|\psi_{8vii}\rangle = G|\psi_{8vi}\rangle = \frac{-1}{16}(|0000\rangle + |0010\rangle + |0011\rangle + |0110\rangle + |1001\rangle$$
$$+|1010\rangle + |1101\rangle - 11|1110\rangle) \otimes |0\rangle$$
$$+\frac{1}{16}(|0000\rangle + |0010\rangle + |0011\rangle + |0110\rangle + |1001\rangle$$
$$+|1010\rangle + |1101\rangle - 11|1110\rangle) \otimes |1\rangle. \quad (54)$$

Thus, the common area where the functions f_1 evaluates to True and f_2 evaluates to False can be obtained with a probability of 0.945 by measuring $|14\rangle$. This probability is derived from the expression $2 * (11/16)^2$.

6.4. Set operation: Union

1. We apply the Younes et al. algorithm[11] with the following stages Prepare a register:

$$|\psi_0\rangle = |0\rangle^{\otimes 4} \otimes |0\rangle = |00000\rangle. \quad (55)$$

Initialize the register:

$$|\psi_1\rangle = \left(H^{\otimes 4} \otimes I\right)|\psi_0\rangle = \left(H^{\otimes 4} \otimes I\right)|00000\rangle$$

$$= \frac{1}{4}(|00000\rangle + |00010\rangle + |00100\rangle + |00110\rangle + |01000\rangle + |01010\rangle + |01100\rangle + |01110\rangle + |10000\rangle + |10010\rangle + |10100\rangle + |10110\rangle + |11000\rangle + |11010\rangle + |11100\rangle + |11110\rangle). \quad (56)$$

Apply the oracle operator $I_{f_1^T}$:

$$|\psi_2\rangle = I_{f_1^T}|\psi_1\rangle = I_{f_1^T}\left(H^{\otimes 4} \otimes I\right)|00000\rangle$$
$$= \frac{1}{4}(|00000\rangle + |00010\rangle + |00100\rangle + |00110\rangle + |01000\rangle + |01010\rangle + |01100\rangle + |01110\rangle + |10000\rangle + |10010\rangle + |10100\rangle + |10110\rangle + |11000\rangle + |11010\rangle + |11100\rangle + |11111\rangle). \quad (57)$$

Apply X on the auxiliary qubit:

$$|\psi_3\rangle = (I^{\otimes 4} \otimes X)|\psi_2\rangle$$
$$= \frac{1}{4}(|00001\rangle + |00011\rangle + |00101\rangle + |00111\rangle + |01001\rangle + |01011\rangle + |01101\rangle + |01111\rangle + |10001\rangle + |10011\rangle + |10101\rangle + |10111\rangle + |11001\rangle + |11011\rangle + |11101\rangle + |11110\rangle). \quad (58)$$

Apply partial diffusion D_p:

$$|\psi_4\rangle = D_p|\psi_3\rangle = D_p\ I_{f_1^T}\left(H^{\otimes 4} \otimes I\right)|00000\rangle$$
$$= \frac{1}{32}(|0000\rangle + |0001\rangle + |0010\rangle + |0011\rangle + |0100\rangle + |0101\rangle + |0110\rangle + |0111\rangle + |1000\rangle + |1001\rangle + |1010\rangle + |1011\rangle + |1100\rangle - |1101\rangle + |1110\rangle - 7|1111\rangle) \otimes |0\rangle - \frac{8}{32}(|0000\rangle + |0001\rangle + |0010\rangle + |0011\rangle + |0100\rangle + |0101\rangle + |0110\rangle + |0111\rangle + |1000\rangle + |1001\rangle + |1010\rangle + |1011\rangle + |1100\rangle + |1101\rangle + |1110\rangle) \otimes |1\rangle. \quad (59)$$

Measure the auxiliary qubit:

$$|\psi_3\rangle = \frac{-1}{\sqrt{14}}(|00001\rangle + |00011\rangle + |00101\rangle + |00111\rangle + |01001\rangle + |01011\rangle + |01101\rangle + |01111\rangle + |10001\rangle + |10011\rangle + |10101\rangle + |10111\rangle + |11001\rangle + |11011\rangle + |11101\rangle). \quad (60)$$

Thus, the common area where the functions either f_1 evaluates to True or f_2 evaluates to True can be obtained with a probability of 0.071 by measuring $|0\rangle$, $|1\rangle$, $|2\rangle$, $|3\rangle$, $|4\rangle$, $|5\rangle$, $|6\rangle$, $|7\rangle$, $|8\rangle$, $|9\rangle$, $|10\rangle$, $|11\rangle$, $|12\rangle$, $|13\rangle$, or $|14\rangle$. This probability is derived from the expression $(-1/\sqrt{14})^2$.

7. CONCLUSION

Quantum computing is a fascinating and rapidly emerging field that has the potential to change many fields of science, such as machine learning, cryptography, pattern recognition, and database querying and industry, such as IBM, Google, Microsoft, D-Wave Systems, and Xanadu Quantum Technologies. In this paper, we have illustrated the uses of quantum computing in set operations and Boolean function matching. We have implemented the Elgendy et al. algorithm, which represents quantum algorithms for set operations, such as true intersection, false intersection, difference, and union on IBM Q quantum devices, and examined the results.

The potential applications of the quantum algorithms established in this study are enormous, having important consequences for many areas; for example, in biology, these algorithms may be used to evaluate massive amounts of genetic material and uncover patterns that may be difficult to detect with traditional computers[34]; Furthermore, in security, the capacity of quantum computers to break existing encryption techniques might be utilized to produce more secure communication networks[35].

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