Peirce's Principle of Deductive Inference

"Essay on the Leading Principle"

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Abstract

Charles S. Peirce's leading principle serves as the logical rule that is followed to move from premises to conclusion, and without which no deductive inference could be formed. The leading principle does not add further information to the premises applied to it, they only determine the status of premises to ensure moving successfully to the conclusion. Therefore, the principle that organizes and governs the process of deductive inference itself. Peirce has distinguished two types of leading principles: the formal or logical principle, and the material or actual principle. The material (actual) leading principle resembles formal implication, whereas logical (formal) leading principle is a proposition of material implication. Peirce's concept of the leading principle, though significantly refined since his time, was a great advance over the concepts similar to it in his day. However, the lack of specificity is considered the main defect in Peirce's treatment of his concept of the leading principle

Keywords: Charles S. Peirce's, leading principle, deductive inference

Introduction

Charles S. Peirce has contributed many essays on the principle of deductive inference, or what he calls "leading principle". The present essay aims to analyze the "leading principle" through answering the following questions:

- what is the definition and nature of the leading principle
- What are the types of the leading principle

In an attempt to answer these questions, the researcher has depended basically on Peirce's essays, in addition to a number of writings for a group of brilliant writers who attempted to explain and interpret Peirce's leading principle.

What is the definition of the leading principle?

Peirce has started his paper "on the natural classification of arguments" by analyzing the basic elements that constitute the logical argument; He has indicated that "Argument" is a term that refers to a group of premises considered in a particular way. The "premise" refers exclusively to something that is introduced either in the form of an expression that can be communicated or in the form of a sign that can be imagined. It also exclusively refers to that part related (or supposed to be related) to the conclusion, but it doesn't refer to anything that is virtually contained in speech or thought (Peirce, 1867, p.261).

In the same context, Peirce treats the idea of "illation". First, he has presented a description of the general type of inference: $P \therefore C$ where $\therefore$ denotes illation, P denotes the premise or premises, and C denotes the conclusion, and illation is the inference or deduction that took place (Brady, 2000, p.56).
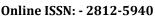
However, Peirce has add a very important and new point in that paper:

> **"Every inference involves the judgment that, if such propositions as the premises are true, then a proposition related to them, as the conclusion is, must be, or is likely to be, true. The principle implied in this judgment, respecting a genus of argument, is termed the leading principle of the argument" (Peirce, 1867, p.261).**

Inferences in logic are supposed to be drawn and subjected to criticism. As a result, we will require two forms: the form "$P \therefore C$" to express the argument, and the form $P_i -< C_i$ to express the truth of the leading principle (Peirce, 1880, p.18).

Any class of premises is denoted by Pi, whereas Ci denotes the corresponding conclusion. The copula is denoted by the symbol $-<$ which means that things in which the proposition of the class Pi is true are the things in which the corresponding proposition of the class Ci is true too (Peirce, 1880, p.18). Thus, the modal condition is expressed by this 'copula' which states that it is impossible to have P and not C (Bird, 1960, p.176).

$P_i -< C_i$ in modern notation is considered a truth-functional implication; whereas Peirce's "state" is a truth valuation. Astonishingly enough that Peirce's definition of $P_i -< C_i$ is close to the modern definition of the statement that is a systematic sequence of another one. However, Peirce, then, could not present a

complete precise definition of his "state", but now, this definition is presented by propositional or predicate truth values (Brady, 2000, p.56).

Thus, the leading principle serves as the logical rule that is followed to move from premises to conclusion, and without which no deductive inference could be formed. Peirce adds that this rule:

> **"Is logically good provided it would never (or in the case of a probable inference, seldom) lead from a true premise to a false conclusion" (Peirce, 1880, p.17).**

In other words, it's logically impossible in the deductive argument to reach false conclusions from true premises.

Thus, every proposition c related in a general way to true proposition p, is also true (Peirce, 1880, p.17).

So, the leading principle is the axiom that ensures the validity of inference without which the inference theory cannot be sustained as a certain science.

In logic, however, it is supposed that some inferences might be invalid, so there should be a form that signifies denying the leading premise; this form might be $P_i - \overline{<} C_i$ Note that adding a dash over any symbol means the negative of the symbol in our notation.

Consequently, this form $P_i -< C_i$ may have two meanings:

either, (1), it is impossible for the premise of the class P_i to be true,

or, (2), things in which P_i is true are the things in which C_i is true too.

On the other hand, this form $P_i - \overline{<} C_i$ may also have two meanings:

both, (1), the premise of the class P_i is possible,

and, (2), among the true possible cases of P_i, the corresponding C_i might not be true in only one case (Peirce, 1880, p.18).

From the above explanation, it can be concluded that:

● The leading principle of Peirce is a formal implication, this opinion is supported by the fact that Pi is not a preposition, it only refers to a category of different propositions, it is an expression that includes a variable when determined the expression becomes a proposition.

● Peirce has introduced two formulas for the leading principle; the affirmative formula that signifies the affirmative leading principle which is suitable for inferences, and the symbolical formula that signifies the negative leading principle which is suitable for criticizing and denying inferences.

Is it possible to use the leading principle as a premise for a new deductive inference?

The leading principle does not add further information to the premises applied to it, they only determine the status of premises to ensure moving successfully to the conclusion. Peirce says:

> **"Now shall the reader add this as a premise to the compound premise already adopted? He gains nothing by doing so. For he cannot reason at all without a monstrative sign of illation; and this sign is**

not really monstrative unless it makes clear the proposition here proposed to be abstractly stated. Nor could any use of that statement be made without using the truth which it expresses" (Peirce, 1894, p.35).

The leading principle cannot serve as a premise of inference. Although both are logical statements, yet they belong to different orders. As for the premise, it is the statement from which conclusions are drawn, it is formulated in the object-language within a logical system (Bird, 1960, p.175).

As for the leading principle, it is the rule statement according to which conclusions are drawn, it is formulated in the meta-language in the system. This rule is always directive as it states how to perform the inference in the logical system, and it is a matter of choice (Bird, 1960, p.175).

So, Peirce refers to the importance of distinguishing between the leading principle of an argument and its premises, knowing that both should be true for the argument to be valid.

Thus, Peirce means – by his leading principle – no more than what would now usually be referred to as a 'rule of inference' (Bird, 1960, p.175). Do not add further information to the premises applied to it, they only determine the status of premises to ensure moving successfully to the conclusion. Peirce says:

"This principle contains all that is necessary besides the premise P to justify the conclusion. (It will generally assert more than is necessary.) We may, therefore, construct a new argument which shall have for its premises the two propositions P and $P_i \prec C_i$ taken together, and for its conclusion, C. This argument, no doubt, las, like every other, its leading principle, because the inference is governed by some habit; but yet the substance of the leading principle must already be contained implicitly in the premises, because the proposition $P_i \prec C_i$ contains by hypothesis all that is requisite to justify the inference of C from P. Such a leading principle, which contains no fact not implied or observable in the premises, is termed a logical principle, and the argument it governs is termed a complete,[1] in contradistinction to an

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[1] Complete argument depends on logical principle, it is called empty or merely formal proposition, because it can add nothing to the premise of the argument it governs, though relevant (Peirce, 1880, p.18).

incomplete, argument, [2] **or enthymeme"**
(Peirce, 1880, p.18).

Peirce begins with the enthymeme as an example to explain what is said above:

Enoch was a man

∴ Enoch died

The leading principle here is "all men die". By adding this leading principle, we obtain the complete argument.

All men die

Enoch was a man

Enoch was to die

The leading principle this time is *nota notae*, by adding it as a premise we obtain:

Nota notae est nota rei ipsius

"Mortality is a mark of humanity, which is a mark of Enoch"

∴ Mortality is a mark of Enoch (Peirce, 1880, p.19).

Expanding this third stage of the argument further will result in laying down something already explained because it works according to the same principle; *nota notae*. Thus, reasserting its truth on the next level would be all that could be done.

Nota notae … is a true principle of inference

To say that mortality …, and that therefore Enoch …, is an inference drawn by this principle.

∴ This is a valid inference (Thompson, 1953, p.7).

However, the infinite regress resulting from that procedure could be avoided when it becomes obvious that the argument will be complete when its leading principle does not contain anything that might serve as a premises (Thompson, 1953, p.8).

It is noteworthy that only in case of enthymematic argument, or the first stage, the leading principle adds a factual element to the already introduced premise. In other words, saying "All men die" presents a factual assertion that differs from that in "Enoch was a man" so that it can be removed from the leading principle in order to be transferred to the premises (Thompson, 1953, p.8).

Hence, Peirce has reached an essential conclusion; any not superfluous can be omitted from the premises (for example; omitting "All men die" from the non-complete argument) without being thereby added to the leading principle, and nothing can be eliminated from the leading principle (for example; omitting "All men die" from the logical principle in the second stage of the argument) except by being expressed in the premises (Peirce, 1867, p.262).

Yet, by adding the leading principle of the new stage of the argument, namely – the principle of *nota notae* – to the premises to form the third stage, no factual

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[2] the incomplete argument is considered a type of syllogism but with an unstated premise, and it is used in a specific type of argument in which the conclusion is not derived from the stated hypothesis only but from the additional unstated premises as well, which are prejudices in most cases (Brady, 2000, p.56).

elements will be transferred this time. Thus, it is noticed that the leading principle corresponds to the logical principle which is the aspect of argument that differs from what is presented in the premises, and is precisely critical for inference (Thompson, 1953, p.8).

From this analysis, it is concluded that if two facts are linked together through this relation "if the former is true, then the latter is necessarily or probably true", then this relation represents a determinate fact, therefore, the complete argument has two premises at least because its leading principle contains no matter of fact (Peirce, 1867, p.262).

Again, if the leading principle of a complete argument does not contain a matter of fact, it will be necessary to lay down more than two premises (Thompson, 1953, p.8).

The conclusion may be considered a substitute for one of its premises provided that the other premise justifies that substitution.[3]

So, whatever is required to justify that substitution would be the only relevant thing to the other premises (Peirce, 1867, p. 263).

What are the types of the leading principle?

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[3] Peirce refers to the preposition may take the following form, in one way at least; S is P, which means that things that are classified under S, or the total subject have the characteristics attributed to things that are classified under P, or total predicate. So, the significance of the term depends on whether it is a subject or a predicate, namely, it has two powers.

The subject, or the breadth of a term, includes all the objects to which it is applied, whereas the predicate, or the depth of the term, includes the characteristics attributed to all the objects to which it is applied. It is necessary not to confuse the breadth and depth with logical extension and comprehension. When substituting a proposition for another, it is important to substitute every term for another term. For such substitution to be valid; the first term should represent what the second term represents. Thus, the possible substitution would be:

(1) Substituting a term that signifies the function of the subject with another term whose breadth is contained in the first, former, one,
(2) Substituting a term that satisfies the function of a predicate with another term whose depth is contained in the first, or former, one.

So, if a term appears as a subject in either premise and doesn't appear as a subject in the conclusion, the other premise should signify that the breadth of that term includes the breadth of its substitute in conclusion. This also signifies that the objects of the latter term carry the characteristics of the former term. As for the eliminated term, if it doesn't satisfy the function of predicate, it does so in the other; but if it does satisfy the function of predicate in either premise, the other premise would signify that its depth includes the depth of its substitute in the conclusion. In other words, the characteristics of the latter term belong to the objects of the former term, namely, it must satisfy the function of a subject in the other premise. This can be formulated as follows:

M is P

S is M

(then) S is P

This means that the terms of the syllogistic argument may satisfy the function of a subject or predicate, yet the argument cannot be grammatically expressed in the same way (Peirce, 1867, pp. 264 – 265).

As mentioned earlier, Peirce has distinguished two types of leading principles: the formal or logical principle, and the material or actual principle (Peirce, 1902, p.1).

Peirce has adopted this distinction in Baldwin's paper and he has presented it in his own way and terms saying:

> **"Any leading principle whose truth is implied in the premises of every inference which it governs is called a "logical" (or, less appropriately, a formal); leading principle; while a leading principle whose truth is not implied in the premises of every inference which it governs is called a "factual" (or material); leading principle" (Peirce, 1902, pp.1 – 2).**

It might be said that the ordinary logical principles of inferences are logical leading principles, although "All men are mortal" (in the above example) is a material leading principle (Buchler, 1966, p.194).

The major premise of the previous syllogism affirms that if the thing is a man, then it is also mortal. This reflects a generalization of the world as it is in reality. In other words, this is a rule that can be verified through uniformities of nature, and contingent conditions. Accordingly, Peirce has called that generalization "material leading principle", a deduction would be incomplete and its conclusion would lose its formal necessity if it doesn't contain a material leading principle. Let us consider the following argument: "Enoch is a man, therefore Enoch is mortal". The conclusion in this argument could be denied without contradicting what has been contained explicitly in the premise. However, adding the material leading principle "all men are mortal" will necessitate the conclusion and, consequently, completes the argument. Actually, there is another leading principle here, though the argument is complete, the term logical leading principle – as Peirce may take to mean – is the principle which:

> **"Must be supposed true in order to sustain the logical validity of any argument" (Peirce, 1880, p.19).**

In the syllogism under consideration, the logical leading principle cited by Peirce is:

> **"The predicate of the predicate is the predicate of the subject" (Peirce, 1902, p.183).**

He also says about the truth status of such principles:

> **"Since it can never be requisite that a fact stated should also be implied in order to justify, a conclusion, every logical principle considered as a proposition will be found to be quite empty. Considered as regulating the procedure of inference, it is determinate; but considered as expressing truth, it is**

nothing" (Peirce, 1867, p. 263).

Moreover, Peirce speaks of the truth status of these principles saying that they are to be evaluated according to their efficiency in taking us from the premises to true conclusions and not according to their truth or falsity.

Peirce confirms that if these logical principles lack factual reference, then, so would be the deductions governed by them in their essence.

The case or rule introduced in the premises of inference or deductive argument is simply a supposition made by the reasoner, and the fact that is related to whether such supposition corresponds to anything in reality, will not influence the quality of deduction he has drawn. as Peirce describes this point:

> **"In deduction, or necessary reasoning, we set out from a hypothetical state of things which we define in certain abstracted respects. Among the characters to which we pay no attention in this mode of argument is whether or not the hypothesis of our premises conforms more or less to the state of things in the outward world. We consider this hypothetical state of things and are led to conclude that, however it may be with the universe in other respects, wherever and whenever the hypothesis may be realized, something else not explicitly supposed in that hypothesis will be true invariably. Our inference is valid if and only if there really is such a relation between the state of things supposed in the premises and the state of things stated in the conclusion. Whether this really be so or not is a question of reality, and has nothing at all to do with how we may be inclined to think" (Peirce, 1903, p.212).**

In more economical way, we can say: (1) the validity criterion: the argument's validity is determined by whether its leading principle is 'true' or not, but not by *Gefuhl*; (2) the difference between probable and necessary arguments: the probable argument is governed by material leading principle whereas the necessary argument is governed by logical leading principle (Buchler, 1966, p.194).

As for the material leading principle, it does not serve as a universal synthetic statement as Peirce always says, however, it acts as a propositional function whose values are ascertained statistically for that true proportion. It is necessary to differentiate between the leading principle of an argument and its premises. For an argument to be valid, both its leading principle and premises must be true (Buchler, 1966, p.195).

Let us suppose that Peirce discusses an implication that may be a basis for an influence rule, then in Saying that the truth of factual leading principle ' is not implied in premises', Peirce is referring to the logical truth, and noting that the implication may not hold in some terms even with the same form and disposition. As for the formal or logical leading principle, it is considered "a merely formal or empty proposition", namely, a tautology or logical law. When Peirce says that the truth of this leading principle "is implied in the premises of every inference governing them", he means an implication of a form that "holds for all terms that keep the same form" in the medieval logician language (Bird, 1960, pp.177 - 178).

From the division presented by Peirce, it is better to consider his leading principle as an implication that forms the basis for an inference rule. Thus, if it reflects logical truth or tautology, it represents his logical leading principle, but if it doesn't, then it represents his material or factual leading principle which can be completed -if valid- by adding another premise to form tautology. That factual leading principle is what the medieval logicians used to call "Topical maxims" (Bird, 1960, p.178).

We should not conclude from the previous discussion that there is resemblance between the logical (formal) leading principle and formal implication, or that there is resemblance between the material (actual) leading principle and material implication. Although Peirce has identified the two types of implication mentioned above and distinguished between them precisely, yet, the material (actual) leading principle for him means "the proposition that adds something to the premises of inference to justify its validity", and consequently, the leading principle is considered essential for the premises of inference. However, the logical (formal) leading principle does not add anything to the premises of the deductive inference, so, it is not essential as a premise in deductive inference.

In conclusion, we can say that material (actual) leading principle resembles formal implication, whereas logical (formal) leading principle is a proposition of material implication.

Conclusion

It is now clear that the leading principle is the rule that ensures the validity of deductive inference in the field of quantitative thought, without such rule, quantitative inferences may be formulated but they will lack what ensures their validity, namely, the principle that organizes and governs the process of deductive inference itself.

Peirce's leading principle reflects the recognition of the difference between the deductive system and its governing inferential rules, and the dependence of the system's character on the character of its rules. Peirce's concept of the leading principle, though significantly refined since his time, was a great advance over the concepts similar to it in his day (which were employed mainly by mathematicians).

The equivalent of this concept is the transformation rule as it is understood today. However, the lack of specificity is considered the main defect in Peirce's treatment of his concept of the leading principle. He has not presented enough examples of this leading principle to the extent that we are uncertain how they

would be stated in his view. In arguments that have material leading principle, particularly, it is uncertain how to state the leading principle of an induction. Peirce's accounts of the probable inference types help us conjecture a statement of their rules, yet, the instructive examples may illustrate these accounts more clearly.

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