

## Numerical Investigation on Improving Over-Reinforced Concrete Beams Failure Mode

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### ABSTRACT

This numerical study presents a Finite Element Modeling (FEM) for over-reinforced concrete beams. There are various methods used to improve the flexural behavior of over-reinforced concrete beams, one of these methods is confinement in a compression zone. Which was used in this study using rectangular ties and steel wire meshes as internal jackets. Based on the verification of two previous studies, take one from them as a guide. The study included a parametric analysis of 10 beams, aiming to capture the effect of confinement, focusing on the rectangular ties with varied depths, steel wire meshes with varied depths, and lengths, and more than one layer of the jacket. Deflection ( $\Delta$ ), ductility (D), and loads were calculated to compare the beams. According to the results, the ductility increased by 37-66%, 6-25%, 6-22%, and 6-51%, respectively, of their values to beam B0 when the depth of rectangular ties, depth of steel wire mesh, length of steel wire mesh, and layers of wire mesh were changed. In addition to improving each specimen's deflection load and fracture patterns.

**Keywords:** Over-reinforced; steel wire mesh; rectangular ties; finite element; ductility.

### 1. Introduction

Sections in reinforced concrete design fall into one of three categories: over-reinforced, under-reinforced, or balanced. The proportion of steel reinforcement to concrete strength determines each categorization. A greater percentage of tensile reinforcement than what is necessary for equilibrium under maximum load circumstances is a sign of an over-reinforced section. In contrast to under-reinforced sections that show ductile behavior and give warning indications before failure, these sections often break due to the crushing of the concrete rather than the yielding of the steel when subjected to bending forces [1]. Low concrete quality and high reinforcing steel yield strength have a negative impact on the likelihood of brittle failure in a limited state [2].

Design codes, ACI 318 [3], Eurocode 2-2004 [4], and ECP 203-2020 [5] do not allow over-reinforced sections. When a beam is subjected to high loads or has a size constraint, it is imperative to employ an over-reinforced concrete beam. The code of practice permits the use of over-reinforced concrete beams under certain restrictions in these situations.

These criteria are used to forecast and design the flexural strength capacity of over-reinforced concrete

beams with an eye on safety. It may be possible to find cost-effective methods for increasing the ductility of over-reinforced concrete beams. The need for a better knowledge of how over-reinforced concrete behaves under different loadings, such as seismic and dynamic stresses, has been brought to light by recent studies.

Researchers around the world have performed experimental and numerical investigations to understand the structural behavior of over-reinforced concrete and improve failure mode. For example, external confinement or adding a layer in the compression zone is required to improve the brittle failure of over-reinforced concrete beams. External confinement involves employing steel plates [6] and other items to limit the compressive concrete. Externally gluing steel plates [7] or transverse fiber-reinforced polymer (FRP) [8] sheets or plates to the web faces of shear-deficient reinforced concrete beams is the primary method of strengthening these beams. Restricting the compression zone increases the ductility of over-reinforced normal-strength concrete (NSC) and high-strength concrete (HSC) beams [9].

Confined concrete is one of the key developments in

the field of modern structural engineering and has been applied mainly to increase the performance of reinforced concrete elements under load. Confinement, usually developed by transverse reinforcement using stirrups or jackets made from steel or FRP, significantly increases the compressive strength and ductility of concrete. Improvement takes place because confinement inhibits lateral expansion during compression, which avoids early failure and allows the material to resist greater deformations before cracking. In concrete, confinement has been found to offer a better stress-strain relationship compared to unconfined concrete; after reaching peak stress, strength decreases gradually, showing increased energy dissipation during seismic events. Thus, the confinement method and materials chosen play an important role in the performance of structures [10,11].

The structural behavior of over-reinforced concrete beams using confinement methods, using steel wire mesh and rectangular ties is examined numerically in this paper. A computational model was created and verified using experimental data that Atta et al. [12] had previously published. After that, a numerical study was conducted to evaluate several confinement strategies meant to improve these beams' flexural strength. Four beams were made using different stirrup depths, The first component to be evaluated was the depth of the rectangular ties. Three beams with varying depths were examined to determine the second element, which was the jacket's depth. The jacket's length was the third consideration; two beams had varying lengths. The number of layers in the jacket, three beams were constructed with different thicknesses, was the fourth consideration. To find improvements in the performance of the over-reinforced concrete beams, key performance indicators such as load at cracking, yield, and ultimate stages ( $P_{cr}$ ,  $P_y$ , and  $P_u$ ), as well as ( $D$ ), were examined and contrasted with a baseline B0 beam.

## 2. Research Significance

This study aims to improve the brittle failure behavior of over-reinforced concrete beams by enhancing ductility and the compressive capacity of the compression zone. Achieving a better balance between concrete compression and steel tension will improve structural performance and reduce the risk of sudden failure, contributing to safer and more efficient design.

## 3. The Considered Previous Experimental Studies

The first study was by Atta et al. [12] performed a flexural analysis on an over-reinforced concrete

beam, as demonstrated in Figure (1). The beam measured

1600 mm in length and had a simple supported span (L) 1400 mm with a cross-section 120 x 200 mm. It was tested with a four-point bending system. Two rebars of reinforcement steel with a diameter of 18 mm were used in the tension side and with 8 mm diameter were used as stirrups with a spacing of 100 mm. Table (1) lists the material parameters for VB1 (B0).

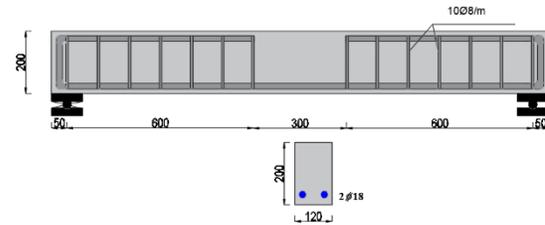


Figure 1- Details of -VB1 (B0) beam by Atta et al. [12]

The second study was by Ali et al. [13]. In this study, the effect of confinement in the compression zone was studied. This parameter under study is shown in specimen (VB2). The beams measured 3000 mm in length and had a simply supported span (L) of 2800 mm with a cross-section of 150 x 250 mm. They were tested with a four-point bending system. Six rebars of reinforcement steel with a diameter of 22 mm were used in tension side and 6 bars with 10 mm diameter were used as compression steel. The stirrups were 10 mm in diameter with a spacing of 100 mm and the stirrups in the compression zone with a diameter of 8 mm and spacing of 100 mm as shown in Figure (2). Table (1) lists the material parameters for all beam elements. The beam under study (VB2) behaves in a manner that is roughly comparable to the control beam. According to the result, VB2 outperformed CB in terms of flexure capacity, ultimate deflection, and energy absorption by 5%, 34%, and 50%, respectively. As may be observed, ductility has increased. There is a slight increase in strength, though. This indicates that the square confinement has little impact on increasing the flexural capacity. Furthermore, the goal of converting compression failure to tension failure was not met by the square confinement setup. This outcome is consistent with the square columns' typical low confinement efficiency. According to other studies, the effect of confinement with square ties is based on the compressive strength of concrete, reinforcement ratio, and spacing between ties [14].

Table 1- Materials properties of (VB1) B0 beam by Atta et al. [12]

Concrete	Tension steel			Stirrup steel		
Fcu (MPa)	Diameter (mm)	Fy (MPa)	Fu (MPa)	Diameter (mm)	Fy (MPa)	Fu (MPa)
30	18	450	580	8	250	360

Table 2. Materials properties of (VB2) beam for Ali et al. [13]

Concrete	Tension steel		Compression steel		Stirrup		Stirrup at compression zone	
Fcu (MPa)	Diameter (mm)	Fy (MPa)	Diameter (mm)	Fy (MPa)	Diameter (mm)	Fy (MPa)	Diameter (mm)	Fy (MPa)
25	22	460	10	480	10	480	8	360

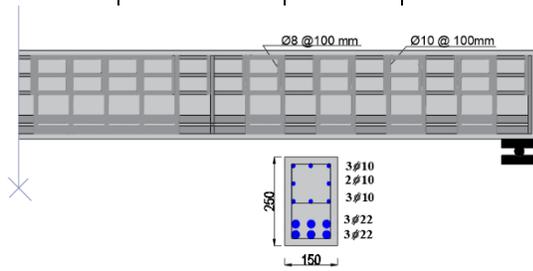


Figure 2- Details of -VB2 beam by Ali et al. [13]

#### 4. Preparation and Creation of a Numerical Model

The ABAQUS program was used to create a three-dimensional (FEM) [15]. Based on previously published experimental work on over-reinforced concrete specimens by Atta et al. [12] and Ali et al. [13], the first (VB1) and second (VB2) FEMs included geometric parameters and material properties. In terms of ultimate load, cracking, and failure mechanisms, the FEM results were verified by contrasting them with the earlier experimental data. Following successful validation, we can study parametric study on over-reinforced concrete beams using ABAQUS by one of these specimens.

##### 4.1 Modeling of Materials in ABAQUS

The concrete smeared cracking model and the concrete plastic damage (CDP) model are the two main methods for simulating concrete. For static stress, smeared cracking was employed [16], and for both static and dynamic stress, CDP was employed [17]. Compressive crushing and tensile cracking are the two concrete failure modes described by the CDP model. It is frequently used to simulate reinforced concrete nonlinear behavior. The CDP model

supplied by [17–24] was used in the current study to specify the behavior of the concrete material. Compressive, tensile, and plastic concrete behavior are all included in the ABAQUS CDP model. A set of parameters derived from the ABAQUS handbook is necessary for CDP modeling in order to accurately represent the mechanical behavior of concrete [15]. These characteristics include the ratio of the second stress invariant on the tensile to the compressive meridian ( $K_c$ ), the dilation angle ( $\psi$ ), the eccentricity ( $\epsilon$ ), the viscosity relaxation parameter ( $\mu$ ), and the biaxial to uniaxial compressive yield stresses ( $f_{bo}/f_{co}$ ). The default value of 0.68 produced satisfactory results, whereas the  $K_c$  value varied from 0.65 to 0.85. The ( $f_{bo}/f_{co}$ ) ratio varied between 1.10 and 1.16, with a value of 1.16 offering sufficient confirmation. [22]. The default eccentricity value of 0.1 was applied. As advised by Eurocode [4], numerical results for a range of viscosity values (0.00, 0.0002, 0.0004, 0.0006, 0.0008, and 0.001) demonstrated sensitivity to a zero value. A linear elastic-plastic model was used to model stirrups and reinforced steel bars.

##### 4.2 Model Set-up

The concrete volume of beams is modeled using the element C3D8R. Thus, the C3D8R designation can be interpreted as follows: continuous stress/displacement is represented by the first letter, three-dimensional elements by the second and third letters, eight-node brick by the fourth letter, and reduced integration by the last letter. The solid element can crush under compression and crack under stress. Eight nodes with three degrees of freedom, translation in the nodal x, y, and z axes define the element. Steel bars and stirrups are

represented by two-node truss elements (T3D2). The Y and X directions were set to zero in order to represent the roller support. The beam was unable to translate in the Z direction due to the anti-symmetry boundary condition. The embedded element constraint in ABAQUS was used to create the assumed perfect interaction between the type of concrete and reinforcing steel bars/stirrups. The mesh size was taken at 20 mm while by trying 10 mm it took a long time to analyze and with 30 mm the accuracy decreased. The model setup is depicted in Figure (3) and Figure (4-a). The positioning of the stirrups and reinforcing steel bars is shown in Figure (3-b) and (4-b), respectively.

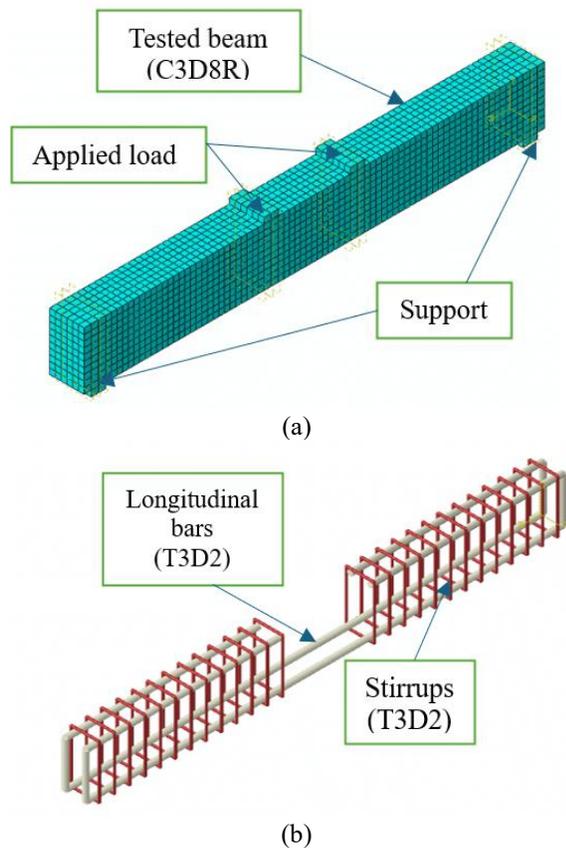


Figure 3- Model set-up of Atta et al. [12]: (a) Concrete elements type, loading, and boundary condition, and (b) Types of steel elements.

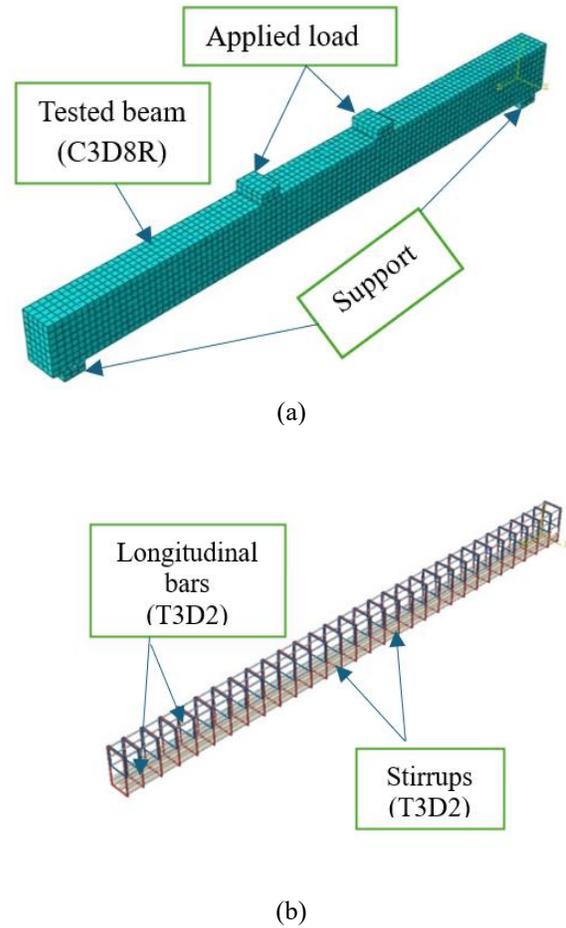


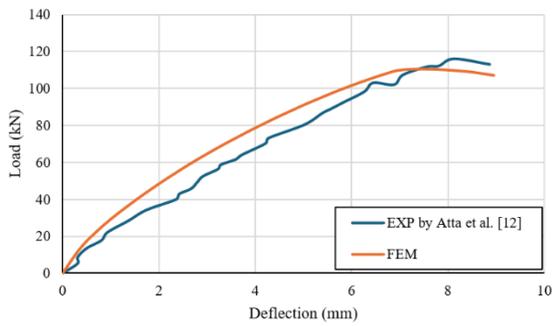
Figure 4- Model set-up of Ali et al. [13]: (a) Concrete elements type, loading, and boundary condition, and (b) Types of steel elements.

#### 4.3 FEM Validation

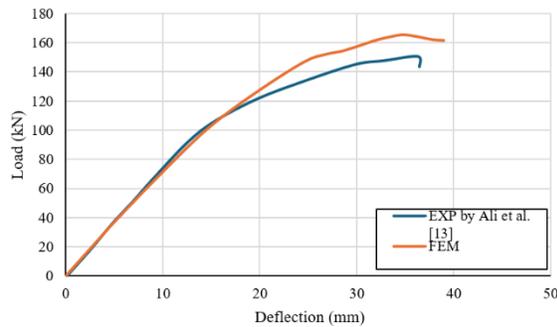
Overall, it is found that the FEM and the experimental of both Atta et al. [12], and Ali et al.[13] findings match quite well as shown in Tables (3) and (4). The FEM load-deflection for both the experimental and FEM results are displayed in Figure (5-a) and Figure (5-b), respectively. Atta et al.[12] found that the FEM accurately predicted experimental data, with a 9% variation, and Ali et al. with 7%. Figure (6) displays the crack patterns that were derived from the FEM and compared to those found in experimental observations. Considering differences in the elastic and plastic stages, the FEM results demonstrated a good agreement with the experimental data. This numerical model successfully simulates its geometric and reinforcement techniques.

Table 3- Load and deflection of experimental and FEM for Atta et al. [12]

Model	Pu (kN)	$\Delta u$ (mm)
EXP	115	8.18
FEM	110.4	7.4
Accuracy	95.6%	91%



(a)



(b)

Figure 5- Comparison between deflection-load curves and the current FEM at the mid-span (a) for Atta et al. [12] and (b) for Ali et al. [13].

#### 4.4. Parametric study

The parametric analysis was carried out using the model created based on the experimental work done by Atta et al. [12]. In this study, ten models were created and divided into four groups to examine the impact of various parameters. i.e., groups A to D. The details of the parametric study are found in Table (5). The study's limitations are displayed in the table.

Table 4- Load and deflection of experimental and FEM for Ali et al. [13]

Model	Pu (kN)	$\Delta u$ (mm)
EXP	153	36.20
FEM	164	37
Accuracy	107%	102%

The initial group (A)'s goal was to study the effect of the depth of rectangular ties with the same diameter of 8 mm and spacing of 50 mm in the compression zone. The names were used to build four models B1, B2, B3, and B4 when different depths were employed 65 (0.325d), 90 (0.45d), 120 (0.6d), and 170 (0.85d) mm, respectively as shown in Figure (7-a). In group (B), there are B5, B6, and B7. Each model has a different depth from steel welded wire mesh (WWM) with open size (15 x 15 mm); 65 (.325d), 90 (0.45d), and 100 (0.5d) mm, respectively as shown in Figure (7-b). Group (C) has two models B6, and B8 with different lengths (300 mm) as the distance between loads and length of the beam (1570 mm) in WWM, respectively as shown in Figure (7-c). The last group (D) has three models to study the effect of the number of layers (one, two, and three layers) of WWM (B6, B9, and B10). All beams have the same dimensions and reinforcement as shown in Figure (7). The model set up is displayed in Figure (8).

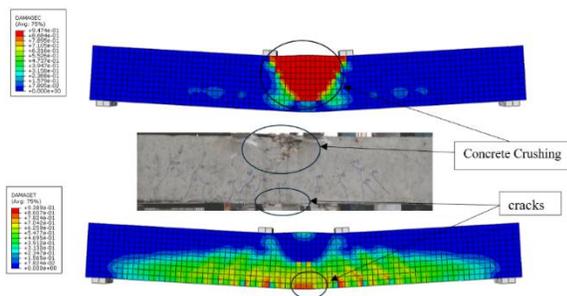


Figure 6- Crack pattern and failure mode

Table 5. Parametric study details and limitations

Group	Sample ID	WWM		Stirrups in compression Depth of stirrup
		Depth	Length	
Group A	B1			65 (0.325d)
	B2			90 (0.45d)
	B3			120 (0.6d)
	B4			170 (0.85d)
Group B	B5	65 (0.325d)	300	
	B6	90 (0.45d)	300	
	B7	100 (0.5d)	300	
Group C	B6	90 (0.45d)	300	
	B8	90 (0.45d)	1570	
Group D	B6	One layer		
	B9	Two-layer		
	B10	Three-layer		

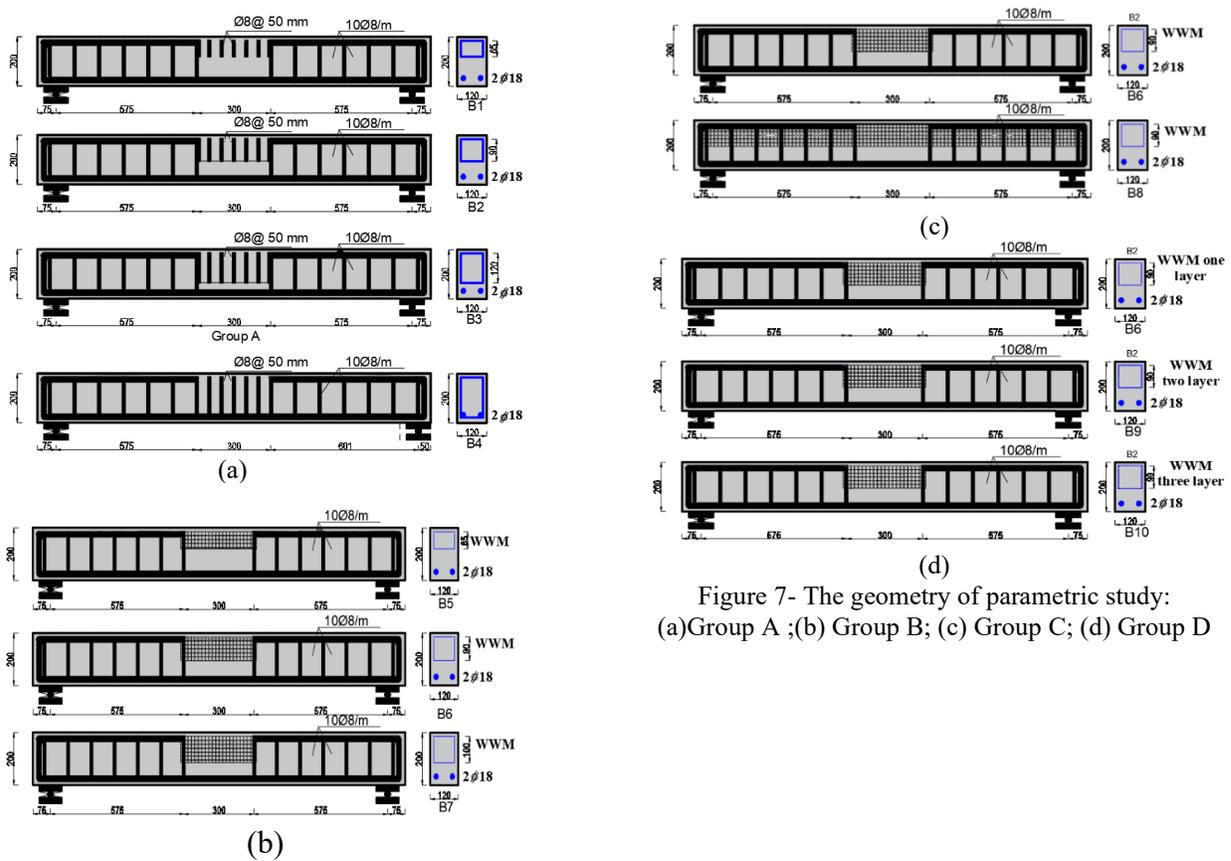
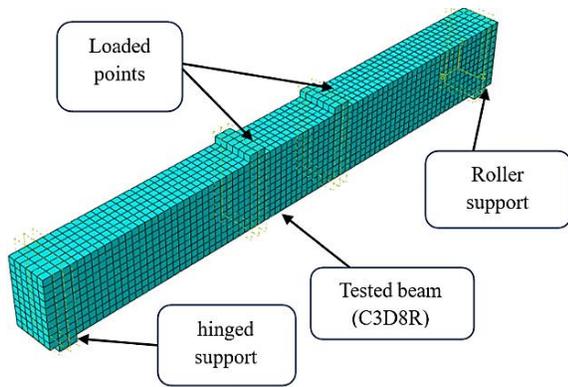
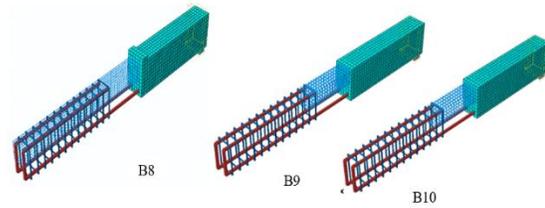


Figure 7- The geometry of parametric study: (a)Group A ;(b) Group B; (c) Group C; (d) Group D



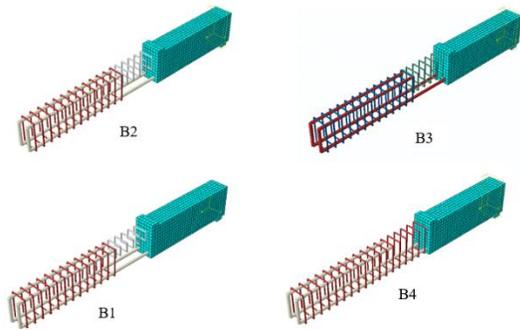
(a)



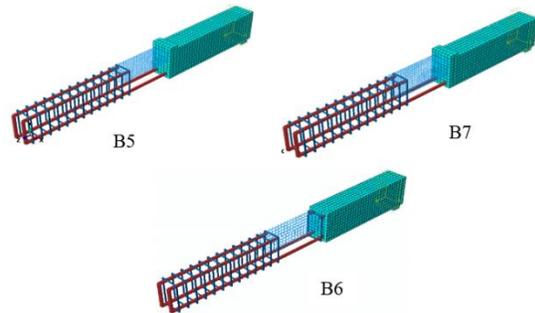
(d)

(e)

Figure 8- Model set-up for all groups: (a) Types of elements, loading, and boundary condition, (b) Group A, (c) Group B, (d) Group C, and (e) Group D.



(b)



(c)

#### 4.4.1. The effect of the depth of rectangular ties

The equivalent load values for the cracking, yielding, and ultimate stages ( $P_{cr}$ ,  $P_y$ , and  $P_u$ ) are displayed in Table (6), along with the same values but in deflection ( $\Delta_{cr}$ ,  $\Delta_y$ , and  $\Delta_u$ ) and the ductility is the proportion between the deflection at the yield load ( $\Delta_y$ ) and the maximum load ( $\Delta_u$ ). Figure (9) (a, b, c, d) shows the computed load-deflection curves for the investigated beams for groups GA, GB, GC, and GD, respectively. Capturing the impact of increasing the depth of the confined region utilizing ties on over-reinforced concrete beams is one of the primary goals of this parametric study. The effect of four different depths of ties is investigated in this section. Four specimens with the same geometrical details and reinforcement are considered to evaluate the influence of increasing depth. These depths were: 65 mm, 90 mm, 120mm, and 170 mm for beams B1, B2, B3, and B4, respectively. The results of these FEMs were compared with a main beam (B0). The numerical results of the FEMs for the four beams are shown in Table (6). Using ties improves ductility by 66, 52, 50, and 37% compared to B0 for B1, B2, B3, and B4, respectively. Also, the ultimate load increase is shown in Table (6). From Figure (9-a), it is noticed by increasing depth, the improvement decreases with bigger depth after peak load. The performance of fracture loads, ultimate loads, deflection, and (D) was improved by the presence of ties to moderate levels; the average values (AVG) of these values fell between 1.12 and 1.41. Figure (10-a) displayed the fracture pattern in the numerical ties under study. This group is the best of the rest groups.

**4.4.2. The effect of increasing the depth of steel wire mesh**

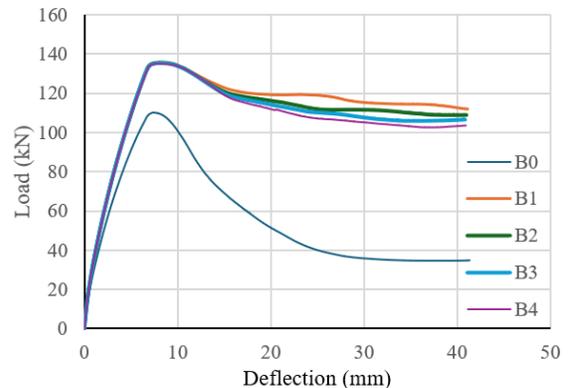
The effect of the depth of wire mesh on the flexural behavior of over-reinforced concrete beams was of great importance. Different beams were divided into three beams B5 with 65 mm depth, B6 with a 90 mm depth, and B7 with 100 mm depth. The results of these FEMs were compared with a main beam (B0) to investigate the effect of depth on the listed beams. The numerical results of FEMs for the three beams are shown in Table (6). By increasing depth, the ductility increased by about 6 and 25% than estimated from B0 for B6 and B7, respectively. The increase of the ultimate load of B5, B6, and B6 is almost the same for B0, about 12%. From Figure (9-b), it is noticed by increasing depth, the improvement increases with bigger depth after peak load. The depth improved the performance of crack loads, ultimate loads, and (D) to high levels, with mean values (AVG) ranging from 1.08-1.1, a standard deviation (SD) from 0.06-0.14, and a coefficient of variation (CoV) from 0.05-0.13 as shown in Table (6).

**4.4.3. The effect of increasing the length of steel wire mesh**

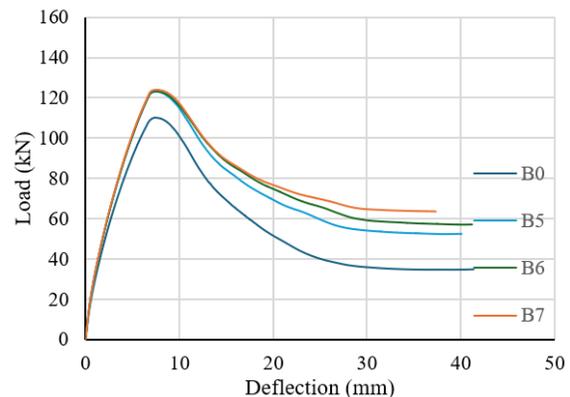
The group studied the effect of the length on the flexural behavior of over-reinforced concrete beams. As shown in Figure (9), the beams were divided into two specimens, B6 and B8, each measuring 300 mm and 1570 mm. To evaluate the impact of length, these FEM findings were contrasted with the main beam (B0). The beams' numerical findings are shown in Table (6). The results indicate that increasing the length of the wire mesh does not lead to a significant improvement in the performance of over-reinforced concrete beams. Wire mesh generally enhances the structural capacity and crack resistance of concrete beams. Increasing the length from 300 mm to 1570 mm did not have a significant effect on load but ductility increased by 16.3% than using 300 mm and the load increased by 12 and 13 % compared to B0 for B6 and B8, respectively as shown in Table (6). This group ranked last because it did not adequately enhance the failure mode of over-reinforced concrete beams. The group numerical study's crack patterns, shown in Figure (10-c), showed vertical cracks with a moderate number of irregularly spaced cracks under bending. Length improved the performance of crack loads, ultimate loads, and (D) to high levels, with mean values (AVG) ranging from 1.07-1.1, a standard deviation (SD) from 0.06-0.12, and a coefficient of variation (CoV) from 0.05-0.11 as shown in Table (6).

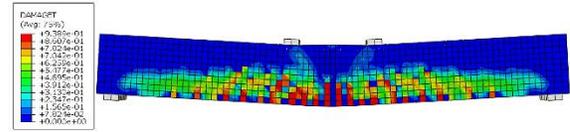
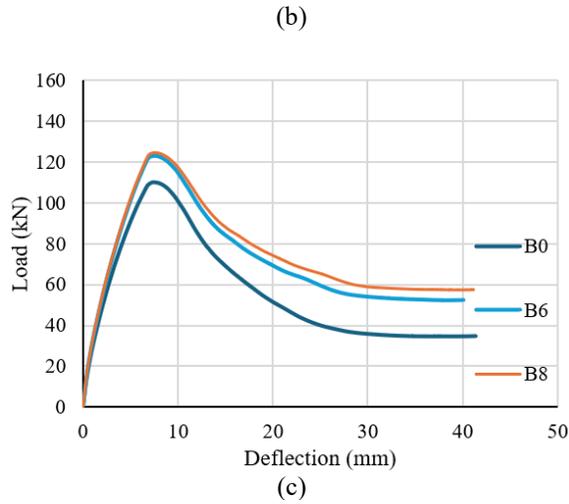
**4.4.4. Effect of number of layers in steel wire mesh**

The effect of the number of layers of wire mesh on the flexural behavior of over-reinforced concrete beams was of great importance. The beams were divided into three categories: B6, B7, and B8. They each had one, two, and three layers of steel wire mesh. To investigate the number of layers effect, these FEM results were contrasted with the main beam (B0). The beams' numerical findings are displayed in Table (6). Application of more than one layer improved the behavior as shown in Figure (9). The ultimate load improved by about 12 to 27%. The ductility improved by about 6 to 51%. From Figure (9-d), it is noticed how the number of layers affects the behavior of the specimen, the improvement increases with more than one layer after peak load. The number of layers increased the yield loads, crack loads, and (D) to high levels by more than one layer with mean values (AVG) ranging from 1.14-1.21, a standard deviation (SD) from 0.11-0.23, and a coefficient of variation (CoV) from 0.1-0.19, as shown in Table (6). Using more than one layer of steel wire mesh improves crack pattern as shown in Figure (10). This group ranked as a second group after using ties but if it will be used more than two layers or three to change the failure mode to ductile failure instead of brittle failure.

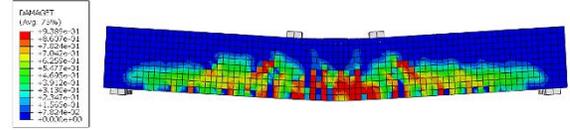


(a)





(c)



(d)

Figure 10- Crack patterns: (a) Group A, (b) Group B, (c) Group C, and (d) Group D.

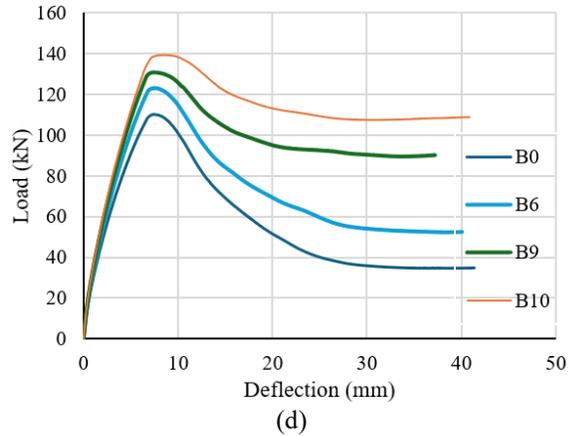
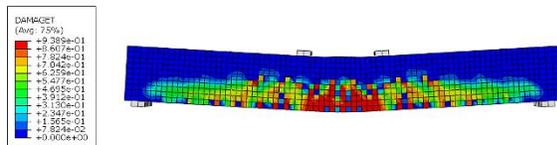
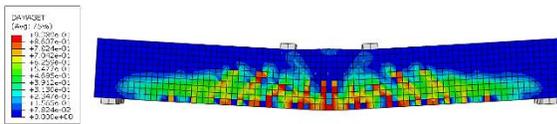


Figure 9- Deflection-load curves obtained Numerically: (a) Group A, (b) Group B, (c) Group C, and (d) Group D.



(a)



(b)

Table 6- Numerical results of the parametric study

Group	Sample ID	Yield stage			Cracking stage			Ultimate stage			Ductility	
		Py	Δy	Py/PyB0	Pcr	Δcr	Pcr/PcrB0	P	Δ	PU/PUB0	D	D/DB0
Group (A)	B0	100	5.5	1	30	1.06	1	110	7.4	1	1.34	1
	B1	124	6.05	1.24	36	1.1	1.2	136	13.5	1.236	2.23	1.66
	B2	124.8	6.12	1.25	36	1.06	1.2	136.2	12.5	1.24	2.04	1.522
	B3	123.09	6.06	1.23	36.1	1.1	1.2	136.5	12.24	1.24	2.01	1.5
	B4	128.9	6.5	1.29	30	0.8	1	137	11.9	1.25	1.83	1.37
AVG				1.2			1.12			1.2		1.41
SD				0.12			0.11			0.11		0.25
CoV				0.1			0.1			0.1		0.18
Group (B)	B0	100	5.5	1	30	1.06	1	110	7.4	1	1.34	1
	B5	110.7	5.8	1.11	30	.99	1	123.2	7.8	1.12	1.34	1
	B6	110.6	5.8	1.1	33.69	1.06	1.09	123.01	8.2	1.118	1.42	1.06
	B7	112	6	1.12	39	1.3	1.3	124	10	1.127	1.67	1.25
AVG				1.08			1.10			1.09		1.08
SD				0.06			0.14			0.06		0.12
CoV				0.05			0.13			0.06		0.11
Group (C)	B0	100	5.5	1	30	1.06	1	110	7.4	1	1.34	1
	B6	110.6	5.8	1.1	33.69	1.06	1.09	123.01	8.2	1.12	1.42	1.06
	B8	109.85	5.64	1.1	34.07	1.06	1.13	124.6	9.3	1.13	1.64	1.223
AVG				1.07			1.07			1.08		1.1
SD				0.06			0.07			0.07		0.12
CoV				0.05			0.06			0.07		0.11
Group (D)	B0	100	5.5	1	30	1.06	1	110	7.4	1	1.34	1
	B6	110.6	5.8	1.1	33.69	1.06	1.123	123.01	8.2	1.12	1.42	1.06
	B9	123	6.2	1.23	38.8	1.18	1.29	130.7	9.4	1.18	1.52	1.134
	B10	128	6.14	1.28	42.85	1.29	1.43	139.3	12.5	1.27	2.035	1.51
AVG				1.15			1.21			1.14		1.18
SD				0.13			0.19			0.11		0.23
CoV				0.11			0.16			0.1		0.19

### 5. Conclusions

The numerical modeling and analysis of over-reinforced concrete beams (ORC) with various characteristics are presented in this work. FEMs were used to investigate the bending-subject beams. By choosing a B0 beam that had undergone experimental testing in earlier research and comparing the FEM

findings with those derived from the experimental work, the correctness of the FEM was previously confirmed [12]. The effects of steel wire mesh and ties on the structural performance of the modeled beams, including depth and multiple layers, were further investigated in a parameter study. The main conclusions of this study are summarized as follows:

1. Using ties with different depths increases the ultimate load by 23.6- 25% compared to B0 and ductility by 37- 66% compared to B0. This technique has a good effect on the failure mode of over-reinforced concrete.
2. Using more than one layer of steel wire mesh in the compression zone has been demonstrated to significantly improve the performance of over-reinforced concrete. By distributing the reinforcement over multiple layers, the section can better resist compressive stress, enhance ductility by about 6 to 51%, and improve the ultimate load-carrying capacity of the concrete beam by about 12 to 27% compared to B0.
3. It has been demonstrated that using wire mesh of varying lengths for concrete reinforcement does not substantially alter the structural performance after a certain point. Overall wire mesh generally enhances the structural capacity and crack resistance of concrete beams, ductility increased by 16.3% than using 300 mm, and the load increased by 12, and 13 % compared to B0 for B6 and B8, respectively.
4. By increasing the depth of the jacket in the compression zone, the ductility increased by about 6 and 25% than estimated from B0 for B6 and B7, respectively. It was concluded that the increased depth of the jacket did not cause a noticeable change in the behavior of the beams.
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