JASDT Vol. (2) Is. (3) September 2024 121-134 121-134

Journal of Administrative Sciences and Digital Technology
https://hicmis.journals.ekb.eg/
Frequency: Quarterly
Print ISSN: 3009-6243 Online ISSN:3009-6936

EXHAUSTIVE ANALYSIS FOR DIFFERENT TYPES OF PARTIALLY ACCELERATED LIFE TESTS UNDER PROGRESSIVE CENSORING SCHEMES

Rashad M. EL-Sagheer

High Inst. for Compu. and Info. Systems, First Statement, New Cairo 11865, Cairo, Egypt.

ARTICLE INFO
Article history:
Received: 01/07/2024
Revised: 21/07/2024
Accepted: 26/08/2024

Key words:
Partially accelerated life tests; Constant-stress; Step-stress; Progressive-stress; Censoring schemes.

ABSTRACT

This paper offers a thorough examination of the partially accelerated life tests (PALT) within progressive censoring frameworks, with an emphasis on constant-stress, step-stress, and progressive-stress models. PALT methods play a vital role in assessing product reliability by exposing test units to different levels of stress to accelerate the occurrence of failures, all while keeping conditions relatable to actual usage. The constant-stress model uses a fixed stress level throughout the test duration, the step-stress model increases stress at specified times, and the progressive-stress model continuously varies the stress level. This research delves into the statistical approaches for modeling and estimating parameters within these stress frameworks and explores methods for managing progressively censored data. The findings provide valuable insights for enhancing test efficiency and improving predictions of product lifetime across various stress conditions, contributing to more accurate and reliable evaluations of product durability and performance in realistic environments.

INTRODUCTION

Accelerated life tests (ALT) are essential instruments in reliability engineering, designed to estimate the lifetime of products or components in a significantly shorter time by applying higher-than-usual stress levels. These tests help manufacturers predict failures, optimize designs, and ensure quality by mimicking the effects of long-term usage under extreme but controlled conditions. Different types of ALTs include **constant-stress, step-stress, and progressive-stress models**. In constant-stress ALTs, a fixed stress level is applied throughout the test, aiming to accelerate the failure process while maintaining a linear relationship with normal usage. Step-stress ALTs, in contrast, introduce different stress levels at predefined intervals, simulating real-world conditions where stress fluctuates over time. Progressive-stress ALTs further refine this approach by continuously increasing the stress level, providing more detailed insight into the relationship between stress and product lifespan. The application of appropriate censoring schemes, su ch as progressive censoring,

*Corresponding author: E-mail address: rashadmath27@gmail.com
https://doi.org/10.21608/hicmis.2025.332976.1003

allows for efficient data collection by removing certain units during the test, balancing cost and reliability. Several studies have contributed to the development of methodological approaches for ALTs under different types of censoring schemes including, **Zhang and Shi (2016)** focused on step-stress ALT under an adaptive progressive hybrid censoring scheme. The authors derived maximum likelihood estimators (MLEs) for the Weibull distribution parameters and acceleration factors. Also, they proposed an adaptive hybrid censoring mechanism to improve testing efficiency by balancing sample size reduction and information retention.

Li *et al.* (2018) developed Bayesian accelerated acceptance sampling plans for products following a lognormal lifetime distribution under Type-I censoring. They introduced a Bayesian framework to estimate the model parameters and design optimal sampling plans. **Han and Bai (2019)** concentrated on the MLE of model parameters for constant-stress and step-stress ALTs under progressive censoring. The authors provided closed-form solutions for parameter estimation and conducted simulation studies to evaluate the performance of their method. Their work highlighted the practicality of progressive censoring schemes in reducing test time while retaining useful information.

Rahman *et al.* (2020) developed statistical inference techniques for Burr Type X distribution under ALT with time-censored data. The authors employed a geometric process model to analyze the data and derive parameter estimates. Their method enhanced the accuracy of lifetime predictions for components subject to extreme conditions, with applications in fields such as telecommunications and electronics. **Hafez *et al.* (2020)** investigated Lindley distribution in the context of ALTs, focusing on applications and numerical simulation. They derived parameter estimates and validated their method using simulation studies. The proposed model is shown to be effective in reliability prediction, particularly for components with a non-monotonic hazard rate.

Ma *et al.* (2021) discussed the optimal design of hybrid accelerated tests using an inverse Gaussian process model. The authors proposed a design framework that accounts for both accelerated stresses and degradation processes, ensuring that test results are both accurate and time-efficient. Their model is validated through real-world data, demonstrating its practical applicability. **Kamal (2021)** examined parameter estimation for constant-stress ALTs with progressive censoring involving multiple stress levels. The study derived MLEs and confidence intervals for the model parameters.

Asadi *et al.* (2022) applied adaptive progressive hybrid censoring to the Gompertz distribution for ALT. The authors evaluated their model by analyzing data from virus-containing microdroplets. They derived MLEs for the distribution parameters and assessed the performance of their approach through simulation studies, demonstrating its effectiveness in reliability applications involving biological systems. **Zhuang *et al.* (2023)** analyzed progressive-stress ALT by introducing group effects into the data analysis. Their model captured the impact of different stress groups on the lifetime distribution, improving the accuracy of lifetime predictions. The study provided a detailed framework for practitioners dealing with heterogeneous stress environments.

Kateri and Nikolov (2024) proposed a novel parameter estimation method using the product of spacing for step-stress ALT, offering an alternative to MLE. Their approach is shown to be more robust under certain conditions, especially when dealing with small sample sizes. The authors validated their method through both simulations and real-world case studies. **Mahto *et al.* (2024)** developed Bayesian estimation and prediction methods for progressive stress ALT using a log-logistic model. The study presented a Bayesian framework to estimate model parameters and predict product lifetime more accurately.

Moala and Chagas (2024) introduced a maximal entropy prior approach for step-stress ALT to improve the parameter estimation process. Their method maximized the use of prior information, enhancing the efficiency of the estimation under complex stress scenarios. They validated their approach using simulation studies and industrial applications. **Bakouch *et al.* (2024)** studied Bayesian methods for step-stress ALT under the gamma distribution, introducing a useful re-parameterization technique. The authors applied their method to industrial data, demonstrating the practical benefits of Bayesian estimation in reliability analysis. Their approach provided improved predictive accuracy and insights into component behavior under stress.

Due to several practical challenges associated with conventional ALT methods include high cost and more time in addition to unrealistic stress conditions that do not align with actual operating environments, researchers have stirred from traditional ALT to partially accelerated life tests (PALT).

PALTs are a specialized form of ALT that combines normal and accelerated stress conditions to evaluate the reliability of products more efficiently. Unlike fully accelerated tests, PALTs alternate between normal and elevated stress levels, helping to maintain relevance to real-world usage while still accelerating the test. This approach reduces the cost and testing time without compromising accuracy. PALTs can be categorized into constant-stress and step-stress types. In constant-stress PALTs, a fixed high-stress level is applied for a predefined period. Step-stress PALTs, in contrast, gradually increase stress over time, providing deeper insights into product behavior under varying conditions. Both approaches can be enhanced with censoring methods, such as progressive or hybrid censoring, to manage incomplete data effectively.

The literature on PALTs encompasses diverse statistical methods and applications aimed at enhancing the estimation of product reliability under varying stress conditions. Several studies have made significant contributions in this field, including advancements in Bayesian estimation, expectation-maximization (EM) algorithms, hybrid censoring schemes, and reliability models for different lifetime distributions. These approaches provide practical solutions for handling incomplete data, multiple stress levels, and progressive censoring, making them highly relevant for industries seeking precise lifetime predictions and quality assurance under accelerated testing environments. Some notable contributions in this area are outlined as follows: **Ahmad *et al.* (2016)** focused on Bayesian estimation for the exponentiated Weibull distribution under PALTs. The authors introduced a framework to incorporate prior information into reliability estimates, enhancing prediction accuracy in cases with limited data. **Zheng and Fang (2017)** presented exact confidence limits for the acceleration factor in PALTs under constant-stress conditions with Type-I censoring. They developed precise statistical inferences, allowing practitioners to calculate reliable confidence intervals for the acceleration factor, which is crucial in reliability assessments.

Soliman *et al.* (2017) examined estimation procedures for the Chen distribution in step-stress PALTs with progressive Type-II censoring. The authors proposed new estimation methods to address varying stress levels, capturing how product reliability shifts under sequentially increasing stress conditions. **Kim *et al.* (2018)** discussed planning partially accelerated degradation tests with two stress variables to provide practical lifetime estimation. The study introduced experimental designs that improve predictive accuracy by using multiple stressors, offering insights for industries that require precise lifetime predictions under varied conditions.

Kumar *et al.* (2020) developed an optimal reliability acceptance sampling plan using PALT data. They provided a framework for manufacturers to determine the number of samples required to evaluate product reliability effectively, contributing to enhanced quality control practices. **EL-Sagheer *et al.* (2020)** investigated the Weibull-Gamma distribution within PALTs, focusing on statistical inferences. They offered extended flexibility in modeling complex failure patterns under accelerated conditions, making it relevant for real-world reliability problems. **Cheng (2020)** applied the EM algorithm to estimate parameters for the Burr XII distribution within PALTs. This approach facilitated parameter estimation for distributions with incomplete data, enabling more accurate reliability assessments.

Abdel-Hamid and Hashem (2021) introduced hybrid progressive censoring with time transformation functions to model the exponential distribution under PALTs. The proposed model enhanced data handling, making it suitable for environments where not all failure times are observed. **EL-Sagheer et al. (2022)** extended PALTs to stress-strength reliability models, analyzing the interaction between stress and component strength. The study provided insights into how the strength of a component responds under increasing stress, enhancing understanding of product durability. **Eliwa and Ahmed (2023)** utilized EM and Markov Chain Monte Carlo (MCMC) algorithms to analyze the Lomax distribution under progressive first-failure censoring. This research offered improved computational methods for estimating reliability metrics under complex censoring schemes.

Yousef *et al.* (2023) conducted parametric inference on the inverted Kumaraswamy distribution using progressive Type-II censoring under PALTs. The study expanded the applicability of PALT models by incorporating novel distributions, providing more options for reliability analysis. **Çetinkaya (2023)** explored the stress-strength reliability model for components tested under PALTs, focusing on their strength characteristics. The study introduced a model for a better understanding of component behavior under accelerated stress conditions, contributing to more reliable designs. **Lone (2023)** simulated the Fréchet distribution under multiple censored PALTs, offering tools to analyze failure data in scenarios with multiple censoring points. This study helped address complex censoring challenges in accelerated life tests, providing more robust reliability predictions.

Niu *et al.* (2023) examined PALTs with a bathtub-shaped lifetime distribution under progressive censoring, which models products with distinct early-life failures and wear-out periods. This work offered valuable insights into analyzing products with non-constant failure rates. **Rabie and Abd-Elrahman (2024)** provided statistical inference for the Weibull distribution using unified hybrid censoring within PALTs. The study enhanced estimation techniques, making it possible to derive more precise inferences for products subjected to complex stress conditions.

Constant-Stress Partially Accelerated Life Test

A Constant-Stress Partially Accelerated Life Test (CS-PALT) is a testing methodology used to assess product reliability by subjecting test units to elevated stress levels, but only for a fraction of the test duration. The goal is to shorten the time required to gather life data while maintaining the relevance of the results to real-world conditions. Unlike fully accelerated tests, where the entire test is conducted at high stress, CS-PALT involves both normal and elevated stress levels in sequence. This hybrid approach leverages the accelerated portion to reveal failure mechanisms faster, while the normal portion ensures that the data reflects typical operating conditions.

During a CS-PALT, a sample of the product is tested under constant elevated stress (e.g., higher temperature, increased voltage, or mechanical load) for a predefined period, followed by testing at normal stress. This sequence helps identify the impact of stress on failure behavior and provides more accurate predictions of the product’s lifespan under normal use. A key benefit of CS-PALT is the reduction in testing time compared to traditional life testing, without compromising the integrity of the results. It balances cost, time, and data relevance better than fully accelerated or purely standard tests.

In this case, the lifetimes of test items are modeled using a continuous probability distribution characterized by its cumulative distribution function $F(x)$, probability density function $f(x)$, reliability function $S(x)$, and hazard rate function $h_1(x)$. When the item is subjected to accelerated conditions, its hazard rate function becomes $h_2(x) = \mu h_1(x)$, where μ is an acceleration factor. This factor, $\mu > 1$, indicates the extent to which the stress increases the failure rate, speeding up the failure process relative to normal conditions. Several significant contributions in this field are summarized as follows.

EL-Sagheer (2018) developed inference techniques for CS-PALT based on progressive Type-II censoring. The work focused on deriving MLEs for parameters of the lifetime distributions under accelerated conditions. Additionally, confidence intervals and hypothesis testing procedures were provided to assess parameter estimates. **Mahmoud *et al.* (2018)** focused on estimating the parameters of the modified Weibull distribution under CS-PALT. The authors derived MLEs for model parameters and analyzed the effect of different acceleration factors on the estimation results. Nassr and Elharoun (2019) investigated parameter estimation for the exponentiated Weibull distribution under CS-PALT using multiple censoring schemes. The authors derived MLEs and constructed confidence intervals to estimate model parameters accurately.

Almarashi (2020) explored parameter estimation for the generalized half-logistic distribution under CS-PALT with progressive Type-II censoring. The study derived MLEs and provided confidence intervals for the parameters, focusing on enhancing estimation accuracy. **Shi *et al.* (2020)** addressed reliability estimation for hybrid systems tested under CS-PALT with progressively hybrid censoring. The authors proposed new algorithms for estimating system reliability and parameters. The study demonstrated the applicability of the methods through extensive simulations, emphasizing their effectiveness in dealing with complex hybrid systems and incomplete data. **Ahmadini *et al.* (2021)** discussed the classical and Bayesian estimation for the parameters of the Fréchet distribution under CS-PALT with Type-I censoring. The authors derived MLEs and constructed confidence intervals to improve reliability predictions.

Rabie (2021) introduced E-Bayesian estimation methods for the Burr-X distribution under CS-PALT using hybrid Type-I censoring. The paper compared E-Bayesian estimators with classical MLEs through Monte Carlo simulations, demonstrating the advantages of the Bayesian approach in terms of accuracy and robustness under accelerated life tests. **Nassar and Alam (2022)** investigated the modified Kies exponential distribution under CS-PALT using Type-II censoring. They derived estimators for the model parameters and provided confidence intervals, focusing on the practical applicability of the model in reliability analysis. **Lone and Panahi (2022)** explored parameter estimation for the Gompertz distribution using unified hybrid censoring under CS-PALT. The authors developed algorithms for estimating the model parameters efficiently. **Almalki *et al.* (2022)** proposed estimation techniques for the Kumaraswamy distribution under CS-PALT using adaptive Type-II progressive censoring. MLEs and Bayesian estimators were derived, and the performance of the two methods was compared.

Dey *et al.* (2022) focused on the Nadarajah-Haghighi distribution under CS-PALT with progressive Type-II censoring. The study developed inference methods for parameter estimation and reliability analysis. Simulation studies were conducted to verify the accuracy and stability of the proposed estimators under various stress levels. **Radwan and Alenazi (2023)** presented new parameter estimation techniques for the Chen distribution using complete data under CS-PALT. The authors derived both MLEs and Bayesian estimators, validating their performance through simulations. The research demonstrated the effectiveness of the methods in real-life reliability applications. **Nassar and Elshahhat (2023)** analyzed the inverse Weibull distribution under CS-PALT using adaptive progressive Type-I censoring. The authors proposed both classical and Bayesian estimation methods for parameter inference.

Alomani et al. (2024) developed estimation techniques for the power half-logistic distribution under CS-PALT. They provided robust methods for parameter estimation using both complete and censored data. Simulation results highlighted the effectiveness of the proposed models in practical applications involving lifetime prediction. **Fathi *et al.* (2024)** studied the Weibull inverted exponential distribution under CS-PALT with progressive first-failure censoring. The authors developed statistical models for parameter estimation and reliability analysis.

Alrashidi *et al.* (2024) investigated the exponentiated gamma distribution under CS-PALT with unified hybrid censoring. The authors derived MLEs and confidence intervals for the parameters, demonstrating the effectiveness of the proposed estimation techniques through simulation experiments. **Zhang *et al.* (2024)** provided inference methods for the generalized inverted exponential distribution under CS-PALT using progressive Type-II censored samples. The authors developed algorithms for parameter estimation and validated the models through simulation studies, highlighting their reliability in accelerated life testing scenarios.

Step-Stress Partially Accelerated Life Test

A Step-Stress Partially Accelerated Life Test (SS-PALT) is a reliability testing method used to estimate the lifespan of products under varying stress conditions while reducing the testing time. In SS-PALT, the product is exposed to gradually increasing levels of stress (e.g., temperature, voltage, pressure) in steps rather than applying a single high stress level continuously. This approach partially accelerates the failure process by increasing the stress level in stages, but it still allows for some time at each stress level, capturing the product's performance under more realistic conditions. The goal of SS-PALT is to gather data on the product's life under these controlled, accelerated conditions, which is then used to model and predict its expected performance under normal operating conditions. By leveraging these insights, manufacturers can make informed decisions on product design, quality assurance, and warranty estimates. In this case we can describe the model as follows:

- **Setup**: Start the life test by placing n identical and independent items under testing, where each item's life follows a known continuous probability distribution.
- **Initial testing conditions**: Each item begins operation under normal conditions. If an item surpasses a specified time τ without failure, it is then subjected to an increased stress level.
- **Test termination criterion**: The test concludes upon the occurrence of the $m-th$ failure, where m is a pre-set number and $m \leq n$, in the case of using censored data.
- **Lifetime model application**: Apply the tampered random variable model by Degroot and Goel (1979). In this model, the life of an item X under the SS-PALT can be defined as:

$$X = \begin{cases} T & if\ T \leq \tau, \\ \tau + \dfrac{1}{\mu}(T-\tau) & if\ T > \tau, \end{cases}$$

where T represents the life of the items under normal conditions, τ is the time of stress application, and $\mu > 1$ is the acceleration factor.

This type of PALT has been shaped by numerous noteworthy contributions, which are highlighted as follows: **Ismail (2016)**: introduced a statistical approach for analyzing data from the SS-PALT model with an adaptive Type-I progressively hybrid censoring scheme for Weibull-distributed lifetimes. The study addressed the challenge of testing reliability in situations where products experience increased stress over time. This approach allows for precise inferences about product lifespans under escalating stress levels, providing an innovative model for lifetime prediction and enhancing the reliability of testing in engineering applications.

Nassar *et al.* (2017) explored the Burr Type-XII distribution in the context of SS-PALT with Type-I and adaptive Type-II progressively hybrid censoring. Their research focused on developing a statistical model that captures failure behavior under progressively increasing stress conditions. **Kamal *et al.* (2021)** delved into statistical inference under SS-PALT for adaptive Type-II progressive hybrid censored data. They proposed a model for predicting product life under changing stress conditions, providing a robust framework for engineers to estimate failure probability accurately. **Lone *et al.* (2021)** addressed statistical inference for SS-PALT models using adaptive Type-I progressively hybrid censored data. The researchers analyzed methods for accurately estimating product reliability under step-stress conditions, contributing significant advancements to statistical methods in ALT.

Kamal (2022) presented parameter estimation techniques for PALT in hybrid systems when failure causes are unknown. The work offered a model that handles complex, censored data to improve reliability assessments in uncertain testing environments, furthering the understanding of hybrid life testing applications. **Rahman *et al.* (2023)** provided a comprehensive analysis of statistical inferences under SS-PALT, incorporating multiple censoring approaches. Using simulated and real-life engineering data, this article demonstrated advanced modeling techniques for product reliability and lifespan predictions, offering engineers robust methods for data analysis under accelerated stress conditions.

Kim *et al.* (2023) discussed the design of a partially accelerated degradation test with two stress variables under a step-stress loading scheme. They provided a novel approach to designing tests that accurately simulate product degradation under real-world conditions. **Tian and Gui (2024)** examined inference and expected test duration in SS-PALT when complementary risks and incomplete data were present. Their work highlighted the challenges of risk analysis under stress-acceleration conditions and offered new methodologies for handling incomplete data. **Alam *et al.* (2024)** presented an analysis of SS-PALT under adaptive Type-II progressive hybrid censoring for the inverse Lomax distribution. The study provided innovative techniques for estimating product life in cases where failure data is censored; enriching the methodology for analyzing accelerated life tests with complex distributions. **Aljohani (2024)** focused on the Lomax distribution in ALTs, particularly on the estimation of $P(X > Y)$ under step-stress conditions.

Progressive-Stress Partially Accelerated Life Test

A Progressive-Stress Partially Accelerated Life Test (PS-PALT) is an advanced reliability testing technique designed to evaluate product life by incrementally increasing the stress levels applied to test items. This gradual elevation of stress, typically across predefined stages, allows researchers to accelerate the occurrence of failures without subjecting items to abrupt, potentially unrealistic levels of stress. PS-PALT is particularly useful in situations where testing under normal conditions would be too lengthy or cost-prohibitive. It provides valuable data on failure mechanisms and life distribution, which can then be extrapolated to predict product behavior under normal conditions. This approach often uses censoring schemes to manage items that survive past certain stress levels or specific time points, making it ideal for situations requiring limited test time or resources. Progressive stress testing is applicable across various industries, including electronics, automotive, and materials engineering, where understanding life under gradual stress variation is crucial. Figure 1 depicts stages of increasing stress on items. Based on the exposure model outlined in Section 3, we can adjust the model to address this situation as follows:

$$X = \begin{cases} T & if\ 0 < T \leq \tau, \\ \tau + \dfrac{1}{\mu_1}[T-\tau] & if\ \tau < T \leq 2\tau, \\ \vdots & \\ (k-1)\tau + \dfrac{1}{\mu_{(k-1)}}[T-(k-1)\tau] & if\ (k-1)\tau < T \leq \infty, \end{cases}$$

where k is the total number of stages. Several important contributions are highlighted in this type of PALT including: **Ismail and Al-Babtain (2015)** explored the methodology of PALT under progressive stress conditions. They focused on how different stress levels can affect product lifespan and reliability. The authors provided a theoretical framework and practical considerations for implementing these tests, emphasizing the importance of understanding the effects of gradual stress increments on product failure.

Liu *et al.* (2017) addressed the challenges of reliability analysis when dealing with masked data in adaptive PALT. The authors proposed a statistical approach that incorporates progressive removal strategies to improve the accuracy of reliability estimates. By analyzing data under adaptive stress conditions, the paper highlighted the significance of understanding the underlying distribution of failure times, ultimately providing a framework that can be applied to real-world testing scenarios where data may be incomplete or obscured.

EL-Sagheer and Khder (2021) investigated the estimation processes in K-stage SS-PALT, specifically focusing on the generalized Pareto distribution under progressive type-I censoring. Their research offered a comprehensive methodology for estimating parameters within this framework, taking into account the complexities introduced by progressive censoring. **Ismail (2022)** examined the implementation of PS-PALT for the inverse Weibull failure model, emphasizing parametric inference techniques. The research outlined how the inverse Weibull model can be effectively utilized to characterize failure times under progressively applied stress, offering insights into the underlying mechanisms of product degradation.

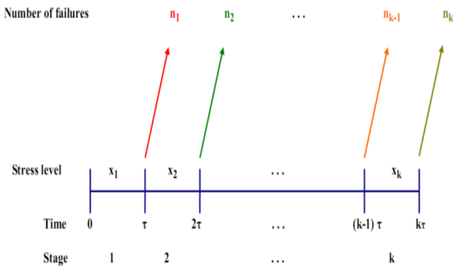

Figure 1. k-stage of PS- PALT.

Comparative for The PALT Types

Table 1. A comparative table summarizing the advantages, limitations, and applications of PALT models

PALT Model	Advantages	Limitations	Applications
Constant-Stress	-Simple design and analysis -Well-established statistical models	-Time-consuming for highly reliable products -May require a long test duration	-Basic life data analysis -Products with known failure distributions
Step-Stress	-Reduces test duration -More efficient than constant-stress	-Complex data analysis -Potential non-linearity between stress levels	-When failure mechanisms are stress-dependent -Preliminary reliability testing
Progressive-Stress	-Accelerates failure detection - Useful for identifying failure thresholds	-Not ideal for estimating life under normal conditions -Potential for overstress	-Screening tests -Identifying weak design points or materials

Model Limitations and Future Directions

In this section, an expanded look is specifically at the limitations of each PALT model, along with proposed directions for future research to guide improvements and further studies:

Table 2. The limitations of each PALT model along with proposed directions for future research

Highlights	Constant-Stress	**Step-Stress**	Progressive-Stress
limitations	-Time-Intensive: Requires long durations to obtain meaningful failure data, especially for highly reliable products. -Inefficient Use of Resources: May require many test units and test time without guaranteeing failure occurrence. -Lack of Flexibility: Does not adapt to changing product performance or external factors during testing.	-Modeling Complexity**:** Requires accurate modeling of stress-life relationships, which can be nonlinear and product-specific. -Assumption Sensitivity**:** Relies on assumptions about cumulative damage and stress transitions that may not hold true universally. -Potential for Bias: Failure behavior at higher stress may not extrapolate reliably to normal use conditions.	-Overstress Risk: A Gradual increase in stress may induce artificial or unrealistic failure modes. -Poor Fit for Life Estimation: Not suitable for estimating life under nominal conditions due to continuous stress changes. -High Variability: Failure thresholds may vary significantly across units, complicating data interpretation
proposed directions for future research	-Hybrid Models: Develop models that combine constant-stress with adaptive stress strategies for improved efficiency. -Predictive Analytics**:** Use machine learning to estimate failure time based on partial life data. -Real-Time Monitoring Integration: Incorporate IoT sensors and real-time data to inform stress adjustments dynamically.	-Bayesian Approaches: Explore Bayesian statistical frameworks for more flexible and robust inference under uncertainty. -Multicores Integration: Study multi-factorial step-stress models where more than one stressor is varied (e.g., temperature and humidity).	-Threshold Modeling: Develop refined models to predict and analyze individual failure thresholds. -Adaptive Algorithms: Investigate adaptive progressive-stress methods that respond to unit performance in real-time.

Practical Significance and Real-World Impact

The results of this study carry significant practical implications for industries and researchers involved in reliability engineering, quality assurance, and product lifecycle management. By providing a comprehensive analysis of different types of PALTs under progressive censoring schemes, the study offers practitioners a flexible and realistic framework to assess product reliability more efficiently and cost-effectively.

In real-world scenarios such as electronics, automotive, aerospace, and materials testing fully accelerated life tests often impose stress levels that may not reflect actual usage, leading to inaccurate lifetime predictions. In contrast, PALTs maintain a balance between accelerated and normal conditions, preserving the relevance of the data while reducing the overall testing time and cost.

The comparative analysis of constant-stress, step-stress, and progressive-stress models allows manufacturers to tailor their testing strategies to match specific product characteristics and operational environments. For example:

- **Constant-stress PALTs** are suitable for components with stable failure mechanisms, enabling quick estimations with minimal complexity.
- **Step-stress PALTs** help capture behavior under variable operating conditions, making them ideal for products exposed to fluctuating workloads.
- **Progressive-stress PALTs** are beneficial in revealing gradual degradation patterns, often required in predictive maintenance planning.

The statistical methodologies and estimation techniques discussed in this paper can be directly applied to optimize test planning, resource allocation, and failure risk assessment in industrial applications. Moreover, the integration of progressive censoring improves test efficiency by minimizing unnecessary test durations without compromising the quality of inference.

Finally, the research supports data-driven decision-making in product development, allowing engineers and reliability analysts to identify weaknesses early in the design process and to develop more robust and durable products. The findings thus contribute to enhancing consumer safety, reducing warranty costs, and improving product reputation in competitive markets.

Conclusion

This study has provided an in-depth analysis of PALTs using progressive censoring schemes, covering constant-stress, step-stress, and progressive-stress models. By exploring these different stress models and their impact on reliability estimation, the study highlights the advantages and limitations of each approach in accelerating failure observations while ensuring test conditions reflect real-world use. The methodologies and statistical techniques discussed here enhance the accuracy of lifetime predictions and offer practical guidance on improving test efficiency and managing progressively censored data. Overall, these insights contribute to the advancement of reliability testing and the effective assessment of product durability under varied stress scenarios.

REFERENCES

Abdel-Hamid, A.H., & Hashem, A.F. (2021). Inference for the exponential distribution under generalized progressively hybrid censored data from partially accelerated life tests with a time transformation function. Math., 9(13): 1510.

Ahmad, A.E.B.A., Soliman, A.A., & Yousef, M.M. (2016). Bayesian estimation of exponentiated Weibull distribution under partially acceleration life tests. Bulletin of the Malaysian Mathematical Sci. Society, 39, 227-244.

Ahmadini, A.A.H., Mashwani, W.K., Sherwani, R.A.K., Alshqaq, S.S., Jamal, F., Miftahuddin, M. & Al-Marzouki, S. (2021). Estimation of constant stress partially accelerated life test for Fréchet distribution with Type-I censoring. Mathematical Problems in Engineering, 2021(1): 9957944.

Alam, I., Kamal, M., Rahman, A., & Nayabuddin. (2024). A study on step stress partially accelerated life test under adaptive type-II progressive hybrid censoring for inverse Lomax distribution. International Journal of Reliability and Safety, 18(1): 1-18.

Aljohani, H.M. (2024). Estimation for the P (X> Y) of Lomax distribution under accelerated life tests. Heliyon, 10(3): e25802.

Almalki, S.J., Farghal, A.W.A., Rastogi, M.K., & Abd-Elmougod, G.A. (2022). Partially constant-stress accelerated life tests model for parameters estimation of Kumaraswamy distribution under adaptive Type-II progressive censoring. Alexandria Engineering Journal, 61(7): 5133-5143.

Almarashi, A.M. (2020). Parameters estimation for constant-stress partially accelerated life tests of generalized half-logistic distribution based on progressive type-II censoring. REVSTAT-Statistical Journal, 18(4): 437-452.

Alomani, G.A., Hassan, A.S., Al-Omari, A.I., & Almetwally, E.M. (2024). Different estimation techniques and data analysis for constant-partially accelerated life tests for power half-logistic distribution. Scientific Reports, 14(1), 20865.

Alrashidi, A., Rabie, A., Mahmoud, A.A., Nasr, S.G., Mustafa, M.S., Al Mutairi, A. & Hossain, M.M. (2024). Exponentiated gamma constant-stress partially accelerated life tests with unified hybrid censored data: Statistical inferences. Alex. Engineering J., 88, 268-275.

Asadi, S., Panahi, H., Swarup, C., & Lone, S.A. (2022). Inference on adaptive progressive hybrid censored accelerated life test for Gompertz distribution and its evaluation for virus-containing micro droplets data. Alex. Engineering J., 61(12), 10071-10084.

Bakouch, H.S., Moala, F.A., Alghamdi, S., & Albalawi, O. (2024). Bayesian methods for step-stress accelerated test under gamma distribution with a useful reparametrization and an industrial data application. Math., 12(17): 2747.

Çetinkaya, Ç. (2023). The stress-strength reliability model with component strength under partially accelerated life test. Communications in Statistics-Simulation and Computation, 52(10): 4665-4684.

Cheng, Y.F. (2020). EM algorithm for estimating the Burr XII parameters in partially accelerated life tests. Reliability: Theory & Applications, 15(4): 86-101.

DeGroot, M.H., & Goel, P.K. (1979). Bayesian estimation and optimal designs in partially accelerated life testing. Naval research logistics quarterly, 26(2): 223-235.

Dey, S., Wang, L., & Nassar, M. (2022). Inference on Nadarajah–Haghighi distribution with constant stress partially accelerated life tests under progressive type-II censoring. Journal of Applied Statistics, 49(11): 2891-2912.

Eliwa, M.S., & Ahmed, E.A. (2023). Reliability analysis of constant partially accelerated life tests under progressive first failure type-II censored data from Lomax model: EM and MCMC algorithms. AIMS Math, 8(1): 29-60.

EL-Sagheer, R.M. (2018). Inferences in constant-partially accelerated life tests based on progressive type-II censoring. Bulletin of the Malaysian Math. Sci. Society, 41: 609-626.

EL-Sagheer, R.M., & Khder, M.A. (2021). Estimation in K-stage step-stress partially accelerated life tests for generalized pareto distribution with progressive type-I censoring. Applied Math. and Info. Sci., 15(3): 299-305.

EL-Sagheer, R.M., Mahmoud, M.A., & Mansour, M.M. (2020). Inferences for weibull-gamma distribution in presence of partially accelerated life test. J. Statistics Appl. & Probab., 9(1): 93-107.

EL-Sagheer, R.M., Tolba, A.H., Jawa, T.M., & Sayed-Ahmed, N. (2022). Inferences for stress-strength reliability model in the presence of partially accelerated life test to its strength variable. Computational Intelligence and Neuroscience, 2022(1): 4710536.

Fathi, A., Farghal, A.W.A., & Soliman, A.A. (2024). Inference on Weibull inverted exponential distribution under progressive first-failure censoring with constant-stress partially accelerated life test. Statistical Papers, 1-33.

Hafez, E.H., Riad, F.H., Mubarak, S.A., & Mohamed, M.S. (2020). Study on Lindley distribution accelerated life tests: Application and numerical simulation. Symmetry, 12(12): 2080.

Han, D., Bai, T. (2019). On the maximum likelihood estimation for progressively censored lifetimes from constant-stress and step-stress accelerated tests. Electronic J. Appl. Stat. Analysis, 12(2): 392-404.

Ismail, A.A. (2016). Statistical inference for a step-stress partially-accelerated life test model with an adaptive Type-I progressively hybrid censored data from Weibull distribution. Statistical Papers, 57(2): 271-301.

Ismail, A.A. (2022). Progressive stress accelerated life test for inverse Weibull failure model: A parametric inference. J. King Saud Univ.-Sci., 34(4): 101994.

Ismail, A.A., & Al-Babtain, A.A. (2015). On Studying partially accelerated life tests under progressive stress. J. Testing and Evaluation, 43(4): 897-905.

Kamal, M. (2021). Parameter estimation for progressive censored data under accelerated life test with k levels of constant stress. Reliability: Theory & Applications, 16(3 (63)): 149-159.

Kamal, M. (2022). Parameter Estimation Based on Censored Data under Partially Accelerated Life Testing forHybrid Systems due to Unknown Failure Causes. CMES-Computer Modeling in Engineering & Sci., 130(3): 1239- 1269.

Kamal, M., Rahman, A., Zarrin, S., & Kausar, H. (2021). Statistical inference under step stress partially accelerated life testing for adaptive type-II progressive hybrid censored data. J. Reliability and Statistical Studies, 585-614.

Kateri, M., & Nikolov, N.I. (2024). Product of spacings estimation in step-stress accelerated life testing: An alternative to maximum likelihood. IEEE Transactions on Reliability, 73(3): 1433-1445.

Kim, S., Kim, Y., & Sung, S. (2023). Practical design of partially accelerated degradation test with two stress variables under step-stress loading scheme. Microelectronics Reliability, 150, 115109.

Kim, S.K., Lim, H., Sung, S.I., & Kim, Y.S. (2018). Planning of partially accelerated degradation test with two stress variables for practical lifetime estimation. Appl. Sci., *8*(11): 2162.

Kumar, M., Bajeel, P.N., & Ramyamol, P.C. (2020). Optimal design of reliability acceptance sampling plan based on data obtained from partially accelerated life test. Inter. J. Reliability, Quality and Safety Engineering, 27(02): 2040002.

Li, X., Chen, W., Sun, F., Liao, H., Kang, R. (2018). Bayesian accelerated acceptance sampling plans for a lognormal lifetime distribution under Type-I censoring. Reliability Engineering & System Safety, 171(3): 78-86.

Liu, B., Shi, Y., Cai, J., & Wang, R. (2017). Reliability analysis of masked data in adaptive step-stress partially accelerated lifetime tests with progressive removal. Communications in Statistics-Theory and Methods, 46(12): 6174-6191.

Lone, S.A. (2023). Simulating fréchet distribution under multiply censored partially accelerated life testing. Annals of Data Sci., 10(6): 1447-1458.

Lone, S.A., & Panahi, H. (2022). Estimation procedures for partially accelerated life test model based on unified hybrid censored sample from the Gompertz distribution. Eksploatacja i Niezawodność, 24(3): 427-436.

Lone, S.A., Rahman, A., & Tarray, T.A. (2021). Inference for step-stress partially accelerated life test model with an adaptive type-I progressively hybrid censored data. J. Modern Appl. Stat. Methods, 19(1): 13.

Ma, Z., Liao, H., Ji, H., Wang, S., Yin, F. (2021). Optimal design of hybrid accelerated test based on the inverse Gaussian process model. Reliability Engineering & System Safety, 210(2): 107509.

Mahmoud, M.A.W., EL-Sagheer, R. M., & Abou-Senna, A.M. (2018). Estimating the modified weibull parameters in presence of constant-stress partially accelerated life testing. Journal of Statistical Theory and Applications, 17(2): 242-260.

Mahto, A.K., Tripathi, Y.M., Dey, S., Alsaedi, B.S., Alhelali, M.H., Alghamdi, F.M. & Alshawarbeh, E. (2024). Bayesian estimation and prediction under progressive-stress accelerated life test for a log-logistic model. Alex. Eng. J., 101, 330-342.

Moala, F.A., & Chagas, K.D. (2024). Maximal entropy prior for the simple step-stress accelerated test. Quality and Reliability Engineering International, 40(7): 3934-3964.

Nassar, M., & Alam, F.M.A. (2022). Analysis of modified Kies exponential distribution with constant stress partially accelerated life tests under type-II censoring. Mathematics, 10(5): 819.

Nassr, S.G., & Elharoun, N.M. (2019). Inference for exponentiated Weibull distribution under constant stress partially accelerated life tests with multiple censored. Communications for Statistical Applications and Methods, 26(2): 131-148.

Nassar, M., & Elshahhat, A. (2023). Statistical analysis of inverse Weibull constant-stress partially accelerated life tests with adaptive progressively type I censored data. Mathematics, 11(2): 370.

Nassar, M., Nassr, S.G., Dey, S. (2017). Analysis of Burr Type-XII distribution under step-stress partially accelerated life tests with Type-I and adaptive Type-II progressively hybrid censoring schemes. Annals of Data Sci., 4(2): 227-248.

Niu, Y., Wang, L., Tripathi, Y.M., & Liu, J. (2023). Inference for partially accelerated life test from a Bathtub-Shaped lifetime distribution with progressive censoring. Axioms, 12(5): 417.

Rabie, A. (2021). E-Bayesian estimation for a constant-stress partially accelerated life test based on Burr-X Type-I hybrid censored data. J. Statistics and Manag. Sys., 24(8): 1649-1667.

Rabie, A., & Abd-Elrahman, A.M. (2024). Statistical Inference Using Partially Accelerated Life Test Model for the Weibull Population Mean Unified Hybrid Censored Data. Inter. J. Reliability, Quality and Safety Engineering, 31(03): 2450014.

Radwan, H.M.M., & Alenazi, A. (2023). Different estimation techniques for constant-partially accelerated life tests of chen distribution using complete data. Sci. Reports, 13(1): 15600.

Rahman, A., Sindhu, T.N., Lone, S.A., Kamal, M. (2020). Statistical inference for Burr Type X distribution using geometric process in accelerated life testing design for time censored data. Pakistan J. Stat. and Operation Research, 16(3): 577-586.

Rahman, A., Kamal, M., Khan, S., Khan, M.F., Mustafa, M.S., Hussam, E. & Al Mutairi, A. (2023). Statistical inferences under step stress partially accelerated life testing based on multiple censoring approaches using simulated and real-life engineering data. Sci. Reports, 13(1): 12452.

Shi, X., Lu, P., & Shi, Y. (2020). Reliability estimation for hybrid system under constant-stress partially accelerated life test with progressively hybrid censoring. Recent Patents on Engineering, 14(1): 82-94.

Soliman, A.A., Abd-Elmougod, G.A., & Al-Sobhi, M.M. (2017). Estimation in step-stress partially accelerated life tests for the Chen distribution using progressive Type-II censoring. Applied Math. & Info. Sci., 11(1): 325-332.

Tian, Y., & Gui, W. (2024). Inference and expected total test time for step-stress life test in the presence of complementary risks and incomplete data. Computational Statistics, 39(2): 1023-1060.

Yousef, M.M., Alsultan, R., & Nassr, S. G. (2023). Parametric inference on partially accelerated life testing for the inverted Kumaraswamy distribution based on Type-II progressive censoring data. Mathematical Biosciences and Engineering, 20(2): 1647-1694.

Zhang, C., Shi, Y. (2016). Estimation of the extended Weibull parameters and acceleration factors in the step-stress accelerated life tests under an adaptive progressively hybrid censoring data. J. Stat. Computation and Simulation, 86(16): 3303-3314.

Zhang, H., Wu, J., & Gui, W. (2024). Inferences on the generalized inverted exponential distribution in constant stress partially accelerated life tests using generally progressively Type-II censored samples. Appl. Sci., 14(14): 6050.

Zheng, D., & Fang, X. (2017). Exact confidence limits for the acceleration factor under constant-stress partially accelerated life tests with type-I censoring. IEEE Transactions on Reliability, 67(1): 92-104.

Zhuang, L., Xu, A., Wang, B., Xue, Y., & Zhang, S. (2023). Data analysis of progressive-stress accelerated life tests with group effects. Quality Technol. & Quantitative Manag., 20(6): 763-783.