Stability of Two Self-Gravitating Magneto -Dynamic Oscillating Fluids Interface

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Abstract

A compound on miscible fluid jet's magneto hydrodynamics (MHD) stability is discussed. For that model, which incorporates fluid inertia, capillary forces, and electromagnetic forces, a general eigenvalue relation is derived. Small axisymmetric disturbances are the only ones that cause the model to be capillary unstable, and the rest of the disturbances are stable. The attractive fields inside and outside to the gas-mantle fly have consistently a settling impact. The radii proportion of the concentric planes assumes a significant part in the (unsteadiness) security states and are (diminishing) expanding with expanding attractive field power as the outside span is a lot bigger than the inside range; under certain limitations of the radii proportion or more a specific worth of the attractive field the slim precariousness is overlooked and totally smothered and afterward dependability sets in. The last option result is checked logically and affirmed mathematically for the situation where the barrel shaped surface of the external stream is sited at endlessness.

Keywords: Magnetic Field, Oscillating and Self-Gravitating, Double-fluid.

1. Introduction

Chandrasekhar and Fermi [1] have been regarded as pioneers of establishing the principle of self-gravitating instability for a complete fluid jet enclosed in a gravitationally low-inertia medium. This can be   derived using the normal mode analysis, which is originally   attributed to Chandrasekhar [2]. Such a complete analysis is related to the influence of surface tension whether acting separately or combined with other factors. In our present work, we are going to study the hydrodynamic stability on a fluid cylinder caused by various acting forces. Meanwhile, several studies related to this in this field of stability theory are quite relevant.

o   Moreover, it is worth mentioning that Chandrasekhar [2], investigated the effects of a constant magnetic field on the gravitational instability of a liquid jet for small axisymmetric perturbations. Such a type of studying the self-gravitating instability of a liquid jet is inevitable especially by applying the method of presenting solenoidal vector in a sense of existing on poloidal and toroidal quantities. Also, Radwan [3] has produced several extensions for it as well as the number of other models that incorporate additional electromagnetic or electrodynamic forces [11-13]. Now, in our context, we are going to examine the effect of the magneto gravitational stability for flowing, coaxial fluid cylinders that are magnetised, with twice disrupted interface. This phenomenon may be intriguing for applying geological drilling operations on the earth's crust. Such a study may be utilised within internal gas cylinder flowing through cylindrical oil which will be discussed in our future work.

2. The underlying Problem

The fluid is assumed to be incompressible, non-viscous, and non-dissipative of primality coefficient. We consider a fluid cylinder with a uniform cross-section of (radius R_0). The fluid contains a homogeneous axial magnetic field that surrounds the fluid jet and moves little.

$$H_0^{(i)} = (0,0,H_0) \tag{1}$$

Additionally, the transversely varying electric field is permeating the nearby self-gravitating tenuous medium.

$$H_0^{(e)} = (0,0,\alpha H_0) \tag{2}$$

The fluid is thought to be flowing with an oscillating velocity where H_0 the magnetic field's intensity is and is a parameter.

$$\underline{u_0} = (0,0,U \cos\cos \Omega t) \tag{3}$$

The fluid's oscillation frequency at time zero is Ω. U is the amplitude of velocity u_0.

The fluid cylinder's axis coincides with the z-axis, and the components of $H_0^{(i)}$, $H_0^{(e)}$ and u_0 are taken into consideration along the cylinder coordinates (r, φ, z). The fluid is subject to the combined effects of self-gravitating, magneto dynamic, and pressure gradient forces.
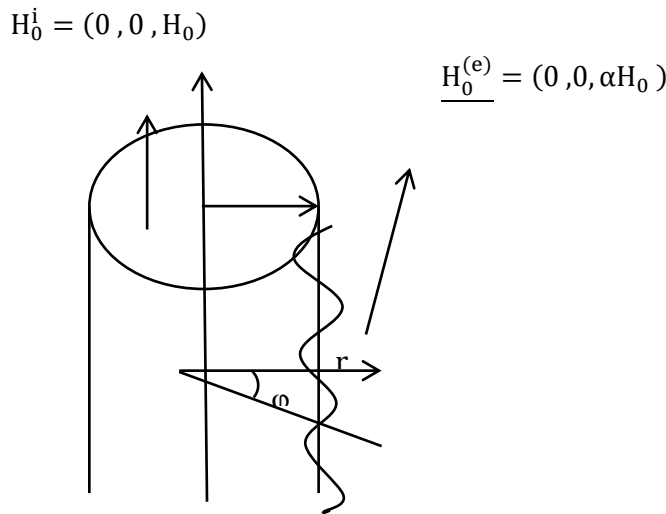
Shown in Fig.1.

Fig.1.Self-gravitation magneto dynamic cylindrical Fluid sketch is the basis for the stability of the present model.

$$\rho \left[\frac{\partial u}{\partial t} + \left(\underline{u}.\nabla\right)\underline{u}\right] = -\nabla P + \rho \nabla V + \mu(\nabla \wedge H) \wedge H \tag{4}$$

$$\nabla.\underline{u} = 0 \tag{5}$$

$$\frac{\partial H}{\partial t} = \nabla \wedge \left(\underline{u} \wedge \underline{H}\right) \tag{6}$$

$$\nabla.\underline{H} = 0 \tag{7}$$

$$\nabla^2 V^i = -4\pi\rho^i G \tag{8}$$

$$\nabla \cdot \underline{H}^{(e)} = 0 \tag{9}$$

$$\nabla \wedge \underline{H}^{(e)} = 0 \tag{10}$$

$$\nabla^2 V^e = -4\pi\rho^e G \tag{11}$$

Where
$$\underline{N}_s = \frac{\nabla f(r,\varphi,z;t)}{|\nabla f(r,\varphi,z;t)|} \tag{12}$$

The variables u, p, T, and Ns stand in for the fluid's velocity vector, kinematic pressure, surface tension coefficient, normal to the fluid interface as a unit vector.
Where

$$F(r,\varphi,z;t) = 0 \tag{13}$$

3. State of equilibrium

Equation (4) can be written as

$$[\rho[\frac{\partial u}{\partial t} + (\underline{u}.\nabla)\underline{u}]]^{i.e} = -\nabla\Pi^{i.e} \tag{14}$$

Where

$$\Pi^{i.e} = [p + \rho V + \frac{\mu}{2}(H_0.H_0)]^{i.e} \tag{15}$$

Where π stands for total magneto hydrodynamic pressure. The basic Equations (4) - (15) are resolved by applying the boundary condition to Equations (1) through (3) in their unperturbed states. At $r = R_0$ we get

$$\Pi_0 = p_0 - \rho V_0 + \frac{\mu}{2}(\underline{H}_0.\underline{H}_0) = const. \tag{16}$$

But the balance of the pressure $p_0 = \Pi_0 + \rho V_0 - \frac{\mu}{2}(\underline{H}_0.\underline{H}_0)$

The equilibrium's self-gravitating potentials V_0 and $V_0^{(e)}$ satisfy

$$\nabla^2 V_0^{(i)} = -4\pi G\rho \tag{17}$$

$$\nabla^2 V_0^{(e)} = -4\pi G\rho \tag{18}$$

The solutions of equations (17), (18)

$$V_0 = -\pi\rho G r^2 + c_1 \tag{19}$$

$$V_0^{(e)} = -\pi\rho G r^2 + c_2 \tag{20}$$

Where the integration constants c_1, c_2 and c_3 must be determined in conjunction with the boundary conditions. $c_1 = 0$

$$c_2 = \pi G R_0^2 [\rho^e - \rho^i] \tag{21}$$

Therefore

$$V_0 = -\pi G \rho^i r^2 \tag{22}$$

$$V_0^{(e)} = -\pi G \rho^e r^2 - 2\pi G R_0^2 (\rho^e - \rho^i) \left[\ln\ln \left(\frac{r}{R_0} \right) - \frac{1}{2} \right] \tag{23}$$

By balancing the pressure over the boundary surface, $r = R_0$ rating, the fluid pressure P_0 in the equilibrium state is established.

$$p_0^i = -\pi G \rho^i \left[\rho^i (r^2 - R_0^2) + \rho^e R_0^2 \right] + \frac{\mu}{2} H_0^2 \tag{24}$$

$$p_0^e = -\pi G \rho^e \left[\rho^e r^2 - 2R_0^2 (\rho^i - \rho^e) \left[\ln\ln \left(\frac{r}{R_0} \right) - \frac{1}{2} \right] \right] + \frac{\mu}{2} \alpha H_0^2 \tag{25}$$

4. Perturbed State

It is possible to construct any dimensionally scale Q(r, φ, z; t) as for small departures from the equilibrium state:

$$Q(r, \varphi, z; t) = Q_0(r) + \varepsilon(t) Q_1(r, \varphi, z) + \cdots \tag{26}$$

Where

$$Q_1 = \varepsilon_0 q_1(r) expexp \left(\sigma t + i(kz + m\varphi) \right) \tag{27}$$

The modified form of the cylindrical interface's formula is provided by

$$r = R_0 + R_1 + \cdots \tag{28}$$

With

$$R_1 = \varepsilon(t) expexp \left(i(kz + m\varphi) \right) \tag{29}$$

Where

$$\varepsilon(t) = \varepsilon_0 expexp \left(\sigma t \right)$$

The height of the surface wave measured from the un-perterbuted state. From eq. (26) and (29) in the basic equations (4) - (14), the pertinent perturbation equations are given by

$$[\rho \left[\frac{\partial u}{\partial t} + (\underline{u}_0 . \nabla) \underline{u}_1 \right] - \mu (\underline{H}_0 . \nabla) \underline{H}_1]^i = -\nabla \Pi_1^i \tag{30}$$

Where

$$[\Pi_1]^i = [p_1 - \rho V_1 + \mu (H_0 . H_1)]^i \tag{31}$$

$$\nabla . \underline{u}_1^i = 0 \tag{32}$$

$$[\frac{\partial H_1}{\partial t}]^i = [(\underline{H}_0 . \nabla)\underline{u}_1 - (\underline{u}_0 . \nabla)\underline{H}_1]^i \tag{33}$$

$$\nabla . H_1^i = 0 \tag{34}$$

$$\nabla^2 V_1^i = 0 \tag{35}$$

A system similar to (30) - (35) may be produced for the outside of the self-gravitating dielectric fluid cylinder. For such a perturbed quantity $Q(r, \varphi, z; t)$ may be described as

$$Q(r, \varphi, z; t) = q_1(r) \, exp \, exp\left(\sigma t + i(kz + m\varphi)\right) \tag{36}$$

From Laplace equation in cylinder coordinate equation

$$V_1^{(i)} = A\varepsilon_0 I_M(x) \, exp \, exp\left(\sigma t + i(kz + m\varphi)\right), \tag{37}$$

$$V_1^{(e)} = B\varepsilon_0 k_m(x) \, exp \, exp\left(\sigma t + i(kz + m\varphi)\right). \tag{38}$$

Thus, from equations (34), (35) we get

$$\underline{H}_1 = \frac{ikH_0}{(\sigma + ikU \cos \cos \Omega t)}\underline{u}_1 \tag{39}$$

By take the divergence to eq. (31) we get

$$\nabla^2 \Pi_1^{(i)} = 0, \tag{40}$$

In which

$$\underline{H}_1^{(e)} = \nabla \Psi_1^{(e)} \tag{41}$$

And Equation (38) becomes

$$\nabla^2 \Psi_1^{(e)} = 0 \tag{42}$$

Since the fluid is incompressible, in viscid and irrational

$$u_1 = \nabla \Phi \tag{43}$$

Combining equations (33), (44)

$$\nabla^2 \Phi_1 = 0 \tag{44}$$

From Equation 28), the variable $\Phi_1 , \pi_1 \, and \, \Psi_1$

Therefore, the non-singular solutions of equations (40), (41) and (44)   are obtained in the following way:

$$\phi_1^{(i)} = c_4 \varepsilon_0 I_m(kr) exp(\sigma t + i(kz + m\varphi)) \tag{45}$$

$$\Pi_1^{(i)} = c_5 \varepsilon_0 I_m(x) exp\left(\sigma t + i(kz + m\varphi)\right) \tag{46}$$

$$\Phi_1^{(e)} = c_6 \varepsilon_0 k_m(x) exp\left(\sigma t + i(kz + m\varphi)\right) \tag{47}$$

$$\Pi_1^{(e)} = c_7 \varepsilon_0 k_m(x) exp\left(\sigma t + i(kz + m\varphi)\right) . \tag{48}$$

Where c_4, c_5 , c_6 , and c_7 are integration constants and m is the first and second types of order, $I_m(kr)$ and $k_m(kr)$ are Bessel functions.
Where $(x = kR_0)$

5. Boundary conditions

Now, it is worth mentioning that the solution of the fundamental equation (4) and (14) must satisfy the boundary conditions. Simple equations in the un-perterbuted state by Equations (1-3), (17) and (23-26) while in perturbed state given by (47) and (48)

5.1.1. Magnetic condition

Due to considering the equation of motion is affected magnetically, this will add up a vital factor in considering the boundary condition, which regulates along the fluid the fluid contact. This issue is appearing as the normal magnetic field component to continuous at $r = R_0$.

Which is expressed as follows

$$\underline{N}_0 \cdot \underline{H}_1^{(i)} + \underline{N}_1 \cdot \underline{H}_0^{(i)} = \underline{N}_0 \cdot \underline{H}_1^{(e)} + \underline{N}_1 \cdot \underline{H}_0^{(e)} \tag{49}$$

Such that

$$N_0 = (1,0,0) \qquad , \qquad N_1 = \left(0, \frac{-im}{R_0}, -ik\right) \tag{50}$$

Then,

$$c_6 = \frac{i\alpha H_0}{k'_m(x)} \text{ Where } (x{=}kr) \tag{51}$$

5.1.2. Kinematic State

The typical element of the fluid's velocity and the velocity of the perturbed boundary fluid connection must be similar. (29) At $r = R_0$ i.e.

$$u_{1r} = (\sigma + ikU coscos \, \Omega t \,)\varepsilon_0 expexp \left(\sigma t + i(kz + m\varphi)\right) \tag{52}$$

Combining eq. (57)

$$u_{1r} = \frac{\partial \phi_1}{\partial r}$$

We get

$$c_4 = \frac{(\sigma + ikU coscos \, \Omega t \,)}{k \, I'_m(x)} \tag{53}$$

From eq. (31), (40) we get

$$\rho \left[\frac{\partial u_{1r}}{\partial t} + U coscos \, \Omega t \, \frac{\partial u_{1r}}{\partial z}\right] - \frac{ik\mu H_0^2}{(\sigma + ikU coscos \, \Omega t \,)} \frac{\partial u_{1r}}{\partial z} = -\frac{\partial \Pi}{\partial r} \tag{54}$$

From which we get

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$$c_5 = \frac{-\rho^i}{k I_m'(x)}\left[\sigma^2 + 2ik\sigma U\cos\cos\Omega t - ikU\Omega\sin\sin\Omega t - k^2 U^2\cos^2\Omega t\right] - \frac{\mu k H_0^2}{I_m'(x)} \tag{55}$$

5.1.3. Self-gravitating conditions

i. The equilibrium surface must have a continuous self-gravitating potential.

   At $r = R_0$

$$V_1 + R_1 \frac{\partial V_0}{\partial r} = V_1^{(e)} + R_1 \frac{\partial V_0^{(e)}}{\partial r} \tag{56}$$

ii. The self-gravitating potential's derivative needs to be continuous over the surface of the initial equilibrium at $r = R_0$

$$\frac{\partial V_1}{\partial r} + R_1 \frac{\partial^2 V_0}{\partial r^2} = \frac{\partial V_1^{(e)}}{\partial r} + R_1 \frac{\partial V_0^{(e)}}{\partial r} \tag{57}$$

Sub. From eqs. (22), (23), (28), (37) and (38) we get

$$A = 4\pi G(\rho^e - \rho^i) R_0 k_m(x) \tag{58}$$

$$B = 4\pi G(\rho^e - \rho^i) R_0 I_m(x) \tag{59}$$

Lastly, we must apply a condition requiring compatibility between the jump in total fluid stress and the framing of P_{1s} across the fluid cylindrical interface (29) at $r = R_0$

$$p_1 + R_1 \frac{\partial p_0}{\partial r} + \mu(H_0.H_1) - \mu(H_0.H_1)^{(e)} = p_{1s} \tag{60}$$

The condition can be written

$$\rho^e\left[\Pi_1^{(e)} - V_1^{(e)}\right] - \rho^i\left[\Pi_1^{(i)} - V_1^{(i)}\right] = R_1 \frac{\partial p_0^i}{\partial r} - R_1 \frac{\partial p_0^{(e)}}{\partial r} - \mu(H_0.H_1)^{(i)} + \mu(H_0.H_1)^{(e)} \tag{61}$$

Then we get

$$\sigma^2 + 2ik\sigma U\cos\cos\Omega t - ikU\Omega\sin\sin\Omega t - k^2 U^2\cos\cos\Omega t = \frac{x I_m'(x) k_m'(x)\rho^i}{[I_m(x)k_m'(x) - \rho I_m'(x)k_m(x)]}\Bigg[4\pi G(1 - \rho)\left((1-\rho)I_m(x)K_m(x) - \frac{1}{2}(2\rho + 1)\right) - \frac{H_0^2 x^2(1-\alpha^2)I_m(x)k_m(x)}{(\rho^i)^2 R_0^2[I_m(x)k_m(x) - I_m(x)k_m(x)]}\Bigg] \tag{62}$$

Since the density relation of a self-gravitating oscillating fluid is equal to $\rho = \left(\frac{\rho_e}{\rho_i}\right)$, Eq. (62) is the dispersion relation of a self-gravitating fluid cylinder; each is acting upon magnetic forces. The first and second forms of

modified Bessel functions, as well as the longitudinal and transverse wave numbers x and m, all have a relationship with the growth of rate σ. $I_m(x)$, $k_m(x)$ of order m, and their derivatives $I_m'(x)$, $k_m'(x)$. the fluid density ρ^i the fluid cylinder radius R_0, the uniform streaming U, and the self-gravitating constant g. we put U=0, $\rho = 0, \alpha = 0 \underline{H_0} = 0$ and m=0 we get

$$\sigma^2 = 4\pi G\, \rho^i \left[\frac{xI_0'(x)}{I_0}\right] \left(I_0(x)k_0(x) - \frac{1}{2}\right) \tag{63}$$

The dispersion relation was obtained by Chandrasekhar and Fermi, and it is the same. In an actuality different an approach then we have here. They applied the technique of expressing solenoidal. Poloidal and toroidal values of vectors.

If we assume that $\rho = 0, H_0 = 0, \alpha = 0, and\ m \geq 0$, the relation (63) produces where the ratio of the densities of the self-gravitating dielectric fluids is equal to

$\rho = \frac{\rho^e}{\rho^i}$ and $\varepsilon = \frac{\varepsilon^e}{\varepsilon^i}$ is the proportion between the dielectric constants of fluids.

$$(\sigma + ikU)^2 = 4\pi G\rho\left[\frac{xI_0'(x)}{I_0(x)}\right]\left(I_m(x)k_m(x) - \frac{1}{2}\right) \tag{64}$$

This is consistent with the conclusions made by Chandrasekhar [2] and Hassan [5].If we assume U=0, =0, G=0, and m=0, the relation (63) produces.

$$\sigma^2 = -\frac{H_0^2}{\rho R_0^2}\frac{x^2(1-\alpha^2)I_m(x)k_m(x)}{[I_m'(x)k_m(x) - I_m(x)k_m'(x)]} \tag{65}$$

This is the fluid cylinder's magneto hydrodynamic dispersion relation

6. Numerical Solutions

From solving the equants of motion (14) numerically using Matlab package 2-17 as a tool to be compared with the analytical results, it has been found out that

$$\sigma^* = V + sqrt\left[U^* + \left[\frac{x\acute{I_0}(x)\acute{k_0}(x)}{\left(I_0(x)\acute{k_0}(x) - \rho\acute{I_0}(x)k_0(x)\right)}\right]\left[(1-\rho)\left[(1-\rho)I_0(x)k_0(x) - \frac{1}{2}(2\rho-1)\right] - \right.\right.$$

$$\left.\left. M\frac{(1-\alpha^2)I_0(x)k_0(x)}{\left(\acute{I_0}(x)k_0(x) - I_0(x)\acute{k_0}(x)\right)}\right]\right]$$

(66)

Where $= \dfrac{-ikU cos\Omega t}{(4\pi G\rho^i)^{\frac{1}{2}}}$ $U^* = \dfrac{ikU\Omega sin\Omega t}{4\pi G\rho^i}$ $M = \left[\dfrac{H_0}{H_s}\right]^2$ $H_s = 2\rho^i R_0 \sqrt{\dfrac{\pi G}{\mu}}$

$\rho = \dfrac{\rho^i}{\rho^e}$
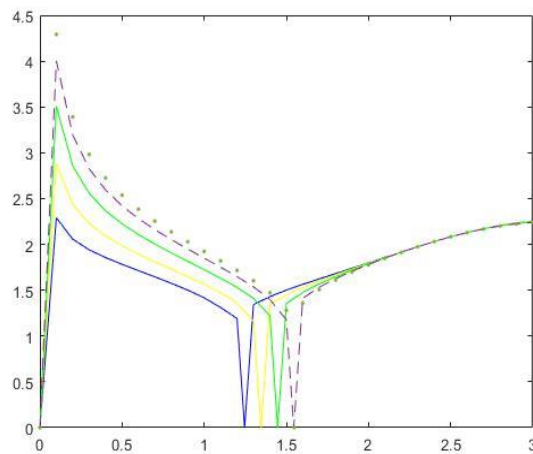
Fig. 2.U=0, ρ=0.2 conformable with M=0.1, 0.4, 0.7, 0.9 and 1.2

(i) For U=0, ρ=0.2 conformable with M=0.1,0.4,0.7,0.9 and 1.2 it is found unstable domain is $0 < x < 1.24$, $0 < x < 1.346$, $0 < x < 1.447$, $0 < x < 1.545$, $0 < x < 1.548$ The contiguous stable domain are $1.246 < x < \infty$, $1.346 < x < \infty$, $1.447 < x < \infty$, $1.545 < x < \infty$, $1.548 < x < \infty$.
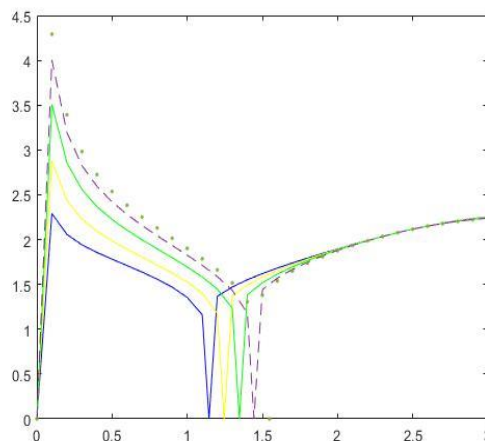
Fig. 3. For U=0, ρ=0.4 conformable with M=0.1, 0.4, 0.7, 0.9 and 1

(ii)   For ρ=0.4,U=0 conformable with M=0.1, 0.4, 0.7, 0.9 and 1.2 it is found unstable domain is $0 < x <$ 1.450, $0 < x < 1.246$, $0 < x < 1.347$, $0 < x < 1.444$, $0 < x < 1.547$ The contiguous stable domain are $1.450 < x < \infty$, $1246 < x < \infty$, $1.347 < x < \infty$, $1.444 < x < \infty$, $1.547 < x < \infty$.
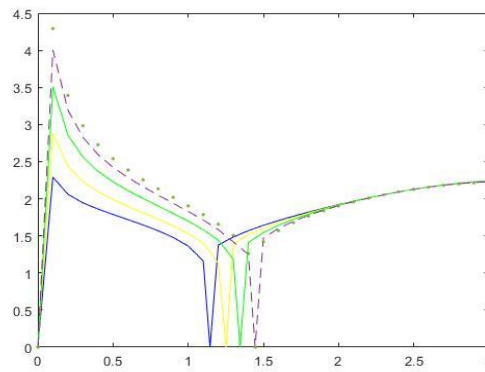

Fig.4.For ρ=0.5, U=0 conformable with M=0.1, 0.4, 0.7, 0.9 and 1

(iii)   For ρ=0.5,U=0 conformable with M=0.1, 0.4, 0.7, 0.9 and 1.2 it is found unstable domain is $0 < x <$ 1.145, $0 < x < 1.252$, $0 < x < 1345$, $0 < x < 1445$, $0 < x < 1447$ The contiguous stable domain are $1.145 < x < \infty$, $1252 < x < \infty$, $1345 < x < \infty$, $1445 < x < \infty$, $1447 < x < \infty$.
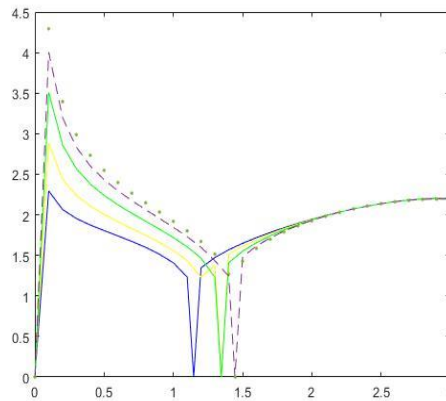

Fig.5. for U=0, ρ=0.7 conformable with M=0.1, 0.4, 0.7, 0.9 and 1

(iv)   For ρ=0.7,U=0, conformable with M=0.1, 0.4, 0.7, 0.9 and 1.2 it is found unstable domain is $0 < x <$ 1.147, $0 < x < 1.347$, $0 < x < 1.346$, $0 < x < 1.445$, $0 < x < 1.447$ The contiguous stable domain are $1.147 < x < \infty$, $1.347 < x < \infty$, $1.346 < x < \infty$, $1.445 < x < \infty$, $1447 < x < \infty$
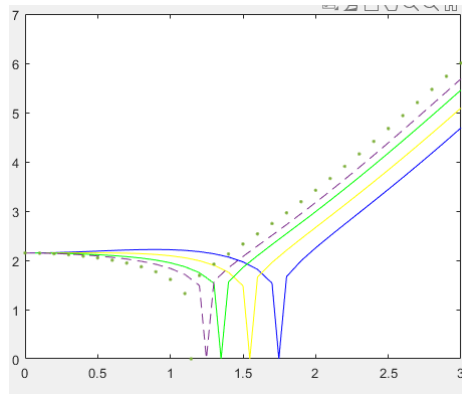
Fig.6.For ρ=0.7, U=0, conformable with M=0.1, 0.4, 0.7, 0.9 and 1.2

Accordingly, the numerical results go in agreement with the analytical ones as shown in the previous sections.

7. Conclusions

In this section we have found out that the unstable domains are reduced as N value grows for a given value of U*, which means that that the magnetic field's impact stabilises the system. Such reducing the N, the capillary force (M) which demonstrates the stability of the magnetic force, the model by increasing the regions of stable domains while reducing the regions of unstable ones.

Meanwhile. The capillary force has a large stabilising effect on the model. While it has been discovered that unstable domains expand for the same N values that U* values expand. Owing to this result, it reveals the puzzle of the streaming effect which appear as in terms short and long waves to become unstable.

Finally, we have figured out that the capillary force is indicated by the growth of the unstable domain with increasing M values for a given value of N.

Acknowledgement

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