

Electrogravitational Oscillating Stability of Double streaming Fluids Cylinder

Samia S. Elazab^{1,a}, Alfaisal A. Hasan^{2,b}, Zeinab M. Ismail^{1,c}, and Marina L. Matta^{1,d,*}

¹ professor, women's College for Arts, Science and Education, Ain Shams University, Cairo, Egypt

² professor, Applied Sciences department, College of Engineering and Technology Arab Academy for science and Technology and Maritime Transport (AASTMT), Aswan, Egypt

³ Lecturer, women's College for Arts, Science and Education, Ain Shams University, Cairo, Egypt

⁴ Mathematics Department, women's College for Arts, Science and Education, Ain Shams University, Cairo, Egypt

E-mail: ^aSamia.El-azab@women.asu.edu.eg, ^balfaisal772001@aast.edu,

^cZeinab.ismail81@women.asu.edu.eg,

^{d,*}marina.lotfy@women.asu.edu.eg (Corresponding author)

Abstract

The problem of stability for two flowing fluids moving in an electrogravitational oscillating cylinder is presented. This type of research may be found to examine the interaction of a distinct self-gravitating, dielectric fluid, with densities ρ^e , with density ρ^i is displayed. A relation to detect the dispersion is constructed analytically explained, and confirmed these results with the numerical computations. The governing equations (equation of motion and equation of continuous) are obtained, providing that the boundary conditions are appropriate. The di-electric streaming system is caused due to the electrodynamic force. The resultant streaming system may give rise to reduce stability for both short and long wave durations. There are unstable and stable zones; however, the system is entirely unstable with an endlessly high growth rate when $\rho > 0.5$. The difference between these two states, stable and unstable, relies on the value of $\rho = \frac{\rho^e}{\rho^i}$. In this scenario, the gravitational instability of the current model, which forms the basis of this work, will be decreased.

Keywords: Electrohydrodynamic, Stability, Self-gravitating, two fluids, and Capillary.

1. Introduction

The concept of enclosing a full fluid jet in a gravitational medium with negligible inertia experiencing a pure self-gravitating instability has been firstly performed by Chandrasekhar and Fermi [1]. This has led Chandrasekhar [2] to investigate this topic under the influence of surface tension as well as in combination with other forces in the presence of the normal mode analysis. Several experiments using gravity-assisted are carried out using vertical vibrations with a considerable amplitude of a magnetic fluid section sustained by a magnetic field [5]. Yet Chandrasekhar searched for the existence of stability for an entire liquid jet under the influence of capillary forces, self-gravitation, and various axisymmetric and (non)axisymmetric disturbances. In addition he also examined the effects of an axisymmetric capillary instability brought on by a magnetic field perturbation as measured by toroidal and poloidal values, and solenoidal vectors, on a fixed complete fluid jet [11]. It is worth mentioning the stability of the electrohydrodynamic interface with two viscous fluids is superimposed in a tube subjected to a continuous electric field [9]. Meanwhile, Hasan [15] has investigated the effect of the axial electric field on the linear stability of self-gravitating compound dielectric immiscible jets. Moreover, El-Azab and Ismail investigated the gravitational oscillation of a fluid cylinder under varying electric fields, fluid cylinder stability. Implications of this approach may be found in the medical sector by studying the effect of propagating drugs into blood vessels, whether for treating heart attacks by means of electric shocks, and imposing magnetic resonance to alter blood flow.

1. The Governing Equation:

Let us propose a fluid-filled in which it has the following properties self-gravitating and incompressible, moving in cylinder has a radius of (radius R_0). In addition the fluid must be self-gravitating, permeated by a homogeneous electric field, with a di-electric constant on both the inside and the outside to be

$$\underline{E}_0^i = (0, 0, E_0). \quad (1)$$

A self-gravitating tenuous medium has been obtained by the transversely shifting electric field.

$$\underline{E}_0^e = \left(0, \frac{\beta E_0 R_0}{r}, 0\right) \quad (2)$$

Where β is any parameter, E_0 is the electric field's strength, and The components $\underline{E}_0^{(i)}$ and $\underline{E}_0^{(e)}$ along the cylindrical coordinate are considered (r, ϕ, z) system where the fluid cylinder's axis and the Z-axis coincide.

Thus, the effects of velocity, electric forces, self-gravitating forces, and capillary forces on the fluid are cumulative. The system where the axis of the fluid cylinder and Z-axis coincide as shown in Figure 1. Furthermore capillary, self-gravitating, electric, and velocity forces are connected, the effect on the fluid is cumulative. depicted in Fig. 1.

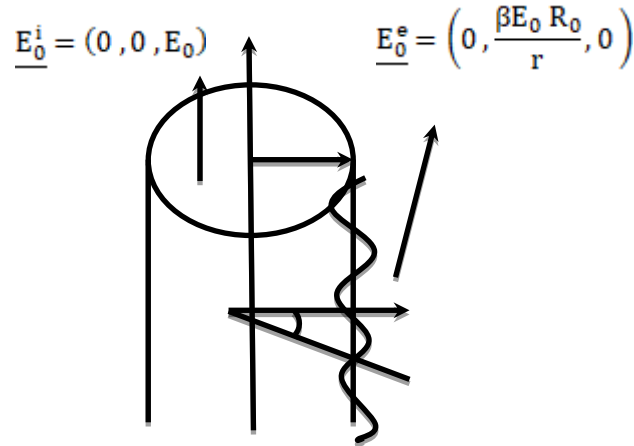


Fig.1. self-gravitation Electrogravitational cylindrical Fluid sketch

In the fluid.

Accordingly, the governing equation of motion becomes as follows

$$\rho \left[\frac{\partial \underline{u}}{\partial t} + (\underline{u} \cdot \nabla) \underline{u} \right] = -\nabla P + \rho \nabla V + \frac{1}{2} \nabla (\epsilon^i (\underline{E}^i \cdot \underline{E}^e)) \quad (3)$$

$$\nabla \cdot \underline{u} = 0 \quad (4)$$

$$\nabla \cdot (\epsilon \underline{E})^i = 0 \quad (5)$$

$$\nabla \Lambda (\epsilon^i \underline{E}^i) \quad (6)$$

$$\nabla^2 V^i = -4 \pi \rho^i G \quad (7)$$

Also, The surface of the fluid cylinder be

$$\mathbf{P}_s = T (\nabla \cdot \mathbf{N}) \quad (8)$$

which can be existed In the tenuous space such that

$$\nabla \cdot \underline{\underline{E}}^{(e)} = 0 \quad (9)$$

$$\nabla \Delta (\varepsilon^e \underline{\underline{E}}^e) = 0 \quad (10)$$

$$\nabla^2 V^e = -4 \pi \rho^e G \quad (11)$$

where $\rho^{i,e}$ is mass density, $\underline{\underline{u}}$ is vector of velocity, \mathbf{p} is kinetic force, V is self-gravitating potential, G stands for gravity constant, T surface tension coefficient, \mathbf{N} is the surface's outer unit vector is perpendicular to it. and $\underline{\underline{E}}^e$, V^e are the fluid cylinder surrounding tenuous mediums.

2. State of Equilibrium

In this section, we study the equilibrium state to determine the variables in a scenario. where $\underline{\underline{u}}_0 = 0$, $\frac{\partial}{\partial \varphi} = 0$ and $\frac{\partial}{\partial z} = 0$.

Thus, using Eq.(3) we get

$$V_i = -\pi G \rho^i r^2 \quad (12)$$

$$V_i^{(e)} = -\pi \rho^e G R_0^2 - 2\pi G a^2 (\rho^i - \rho^e) \left[\ln \frac{r}{R_0} - \frac{1}{2} \right] \quad (\text{SEQ Equation * ARABIC 13})$$

$$p_0^i = -\pi G \rho^i [\rho^i (r^2 - R_0^2) + \rho^e R_0^2] + \frac{\varepsilon^i}{2} E_0^2 \quad (14)$$

$$p_0^e = -\pi G \rho^e \left[\rho^e r^2 - 2R_0^2 (\rho^i - \rho^e) \left[\ln \frac{r}{R_0} - \frac{1}{2} \right] \right] + \frac{\beta^2 \varepsilon^e}{2} E_0^2 \quad (15)$$

3. Perturbed State

Following the previous section, we focus on examining the perturbed state as an approach to examine the reliability of stability as expressed in the following relation

$$Q(r, \varphi, z, t) = Q_s(r) + \eta_s(t) Q_1(r, \varphi, z) + \dots \quad (16)$$

where Q stands for $\rho, \underline{\underline{u}}, \underline{\underline{E}}^{(e)}, \underline{\underline{E}}^{(i)}, V^e, V^i, \underline{\underline{N}}_S$ and \mathbf{P}_S . $\eta(t)$ is a parameter with dimensions that measures the magnitude of the disturbance. $\eta(t)$ may be expressed as

$$\eta(t) = \eta_s e^{(\sigma t)} \quad (17)$$

Consequently, the starting amplitude is defined as ε_s , taking at $t=0$, and the growth is defined as $\sigma = i\omega$ while, a normal mode can be stated as

$$r = R_s + R_1, \quad R_1 \ll R_s \quad (18)$$

and

$$R_1 = \eta(t) e^{i(kz + m\varphi)} \quad (19)$$

where R_1 is the longitudinal wave's number where It is a real quantity, k , and the transverse wave number, an integer, that may be calculated from the state of equilibrium elevation surface wave.

The basic equations (3) to (11), which are important perturbation equations, are provided by the expansion (16).

$$\frac{\partial \underline{u}_1}{\partial t} = -\nabla \pi_1^i \quad (20)$$

$$\pi_1^i = \frac{p_1^i}{\rho^i} + V_1^i + \frac{1}{\rho^i} (\varepsilon^i (\underline{E}_1^i \cdot \underline{E}_1^i)) \quad (21)$$

$$\nabla \cdot \underline{u}_1 = 0 \quad (22)$$

$$\nabla \cdot (\varepsilon^i \underline{E}_1^i) = 0 \quad (23)$$

$$\nabla \Delta (\varepsilon \underline{E}_1)^i = 0 \quad (24)$$

$$\nabla^2 V_1^i = 0 \quad (25)$$

$$\nabla \cdot (\varepsilon^e \underline{E}_1^e) = 0 \quad (\text{SEQ Equation * ARABIC 26})$$

$$\nabla \Delta \underline{E}_1^e = 0 \quad (\text{SEQ Equation * ARABIC 27})$$

$$\nabla^2 V_1^e = 0 \quad (28)$$

3.1. Fourier Analysis

A method for resolving the cylinder stability issue that makes use of the appropriate perturbed quantity's space-time dependence (18) and (19) as well as linear perturbation $Q_1(r, \varphi, z, t)$ Possibly exhibited by

$$Q_1(r, \varphi, z, t) = Q_1(r) e^{i(kz + m\varphi) + \sigma t} \quad (29)$$

Using equations (23) and (22), we can obtain the divergence of equation (20).

$$\nabla^2 \pi_1^i = 0 \quad (30)$$

The related electric field perturbation, $\underline{E}_1^{i,e}$, is stated in equations (23) and (28) to be derivable from the as color function, that is,

$$\underline{E}_1^{i,e} = -\nabla \psi_1^{i,e} \quad (31)$$

If the equations (31),(24),and (26), we obtain

$$\nabla^2 \psi_1^{i,e} = 0 \quad (32)$$

We are able to solve equations (29), (31), (32), and (33) using the expansion (30).

$$r^{-1} \frac{d}{dr} \left(r \frac{dQ_1(r)}{dr} \right) - \left(\frac{m^2}{r^2} + k^2 \right) Q_1(r) = 0 \quad (33)$$

Where, $Q_1(r)$ stands for $\pi_1^i(r), V_1^i(r), V_1^e(r)$, and $\psi_1^i(r)$. Using standard Bessel functions of order m, the solution of the common second-order differential equation (34) is shown. A single resolve is one thing, but we also

$$\pi_1^e = B^e k(kr) e^{i(kz + m\varphi) + \sigma t} \quad (34)$$

$$\pi_1^i = B^i I_m(kr) e^{i(kz + m\varphi) + \sigma t} \quad (35)$$

$$V_1^i = A^i I_m(kr) e^{i(kz + m\varphi) + \sigma t} \quad (36)$$

$$V_1^e = A^e K_m(kr) e^{i(kz + m\varphi) + \sigma t} \quad (37)$$

$$\psi_1^i = C^i I_m(kr) e^{i(kz+m\varphi)+\sigma t} \quad (38)$$

$$\psi_1^e = C^e K_m(kr) e^{i(kz+m\varphi)+\sigma t} \quad (39)$$

3.2. Stability Criterion

3.2.1. Self-gravitating condition

(i) For $r = R_0$, a constant self-gravitating potential is required to examine the equilibrium surface.

$$V_1^i + R_1 \frac{\partial V_0^i}{\partial r} = V_1^e + R_1 \frac{\partial V_0^e}{\partial r} \quad (40)$$

(ii) Using common Bessel functions of order m , the conventional second-order differential equation (34) is solved. We also provide a single resolution and $r = R_0$.

$$\frac{\partial V_1^i}{\partial r} + R_1 \frac{\partial^2 V_0^i}{\partial r^2} = \frac{\partial V_1^e}{\partial r} + R_1 \frac{\partial^2 V_0^e}{\partial r^2} \quad (41)$$

From which, we get

$$A^i = 4\pi(\rho^i - \rho^e) R_0^2 G K_m(x) \quad (42)$$

$$A^e = 4\pi(\rho^i - \rho^e) R_0^2 G I_m(x) \quad (43)$$

3.2.2. Kinematic State

The normal component of the fluid velocity vector and the fluid tenuous medium contact velocity must be consistent. at $r = R_0$

i.e.

$$\underline{u}_{1r} = (\sigma + ikU \cos \cos \omega t) \varepsilon_0 e^{i(kz+m\varphi)+\sigma t} \quad (44)$$

such that

$$\underline{u}_{1r} = \frac{\partial \Phi_1}{\partial r}. \quad (45)$$

Thus , we obtain

$$B^i = \frac{(\sigma + ikU \cos \cos \omega t)}{k I_m'(x)} \quad (46)$$

$$B^e = -\frac{\rho^i}{k I_m(x)} [\sigma^2 + 2ik\sigma U \cos \cos \omega t - ik\omega U \sin \sin \omega t - k^2 U^2 \omega t] - \frac{ikE_0^2}{I_m(x)} \quad (47)$$

3.2.3. Electric State

Additionally, the electric field potential's normal component needs to remain constant over the equilibrium surface. at $r = R_0$

$$\underline{N} \cdot (\varepsilon^i \underline{E}^i - \varepsilon^e \underline{E}^e) = 0 \quad (48)$$

$$\underline{E} = \underline{E}_s + R_1 \frac{\partial \underline{E}}{\partial r} + \underline{E}_1 \quad (49)$$

If

$$\psi_0^i = -E_0 r \quad (50)$$

$$\psi_0^e = -aE_0 \left[1 + \log \log \frac{r}{R_0} \right] \quad (51)$$

$$\psi_1^i + R_1 \frac{\partial \psi_0^i}{\partial r} = \psi_1^e + R_1 \frac{\partial \psi_0^e}{\partial r} \quad (52)$$

$$c^i = \frac{i E_i [\varepsilon^i - \beta \varepsilon^e] K_m(x)}{\varepsilon^i I_m'(x) K_m(x) - \varepsilon^e K_m'(x) I_m(x)} \quad (53)$$

$$c^e = \frac{i E_e [\varepsilon^i - \beta \varepsilon^e] I_m(x)}{\varepsilon^i I_m'(x) K_m(x) - \varepsilon^e K_m'(x) I_m(x)} \quad (54)$$

Where $x = ka$ is dimensionless, the number of longitudinal waves. by replacing with equations (15), (21), and (26) we get

$$\begin{aligned} \sigma^2 + 2ik\sigma U \cos \cos \omega t - ik\omega U \sin \sin \omega t - k^2 U^2 \omega t = \\ \frac{x I_m'(x) K_m'(x) \rho^i}{K_m'(x) I_m(x) - \rho I_m'(x) K_m(x)} \left[4\pi G(1 - \rho) \left((1 - \rho) I_m(x) K_m(x) - \frac{1}{2} (2\rho + 1) \right) - \right. \\ \left. \frac{E_i^2 x^2 [\varepsilon^i - \beta \varepsilon^e]^2 I_m(x) K_m(x)}{\varepsilon^i (\rho^i)^2 R_0^2 [I_m'(x) K_m(x) - \varepsilon K_m'(x) I_m(x)]} \right] \end{aligned} \quad (55)$$

Where $\rho = \frac{\rho^e}{\rho^i}$ is the proportion of the densities of the self-gravitating dielectric fluids, and $\varepsilon = \frac{\varepsilon^e}{\varepsilon^i}$ is the proportion between the dielectric constants of fluids.

4. General Discussions.

Following significant simplifications as taking into account the following items:

i) $\rho = 0, \underline{E}_0 = 0, \lambda = 0, U = 0$ and $m = 0$ the following dispersion relation is obtained from Eq.(55)

$$\sigma^2 = 4\pi G \rho^i \left[\frac{x I_0'(x)}{I_0(x)} \right] \left(I_0(x) K_0(x) - \frac{1}{2} \right) \quad (56)$$

ii) $\rho = 0, \underline{E}_0 = 0, \lambda = 0$ and $m \geq 0$ the following dispersion relation is obtained from Eq. (55)

$$(\sigma + ikU)^2 = 4\pi G \rho^i \left[\frac{x I_m'(x)}{I_m(x)} \right] \left(I_m(x) K_m(x) - \frac{1}{2} \right) \quad (57)$$

iii) $\rho = 0, G = 0, U = 0$ and $m \geq 0$

This will reduce the dispersion relation as shown in (55) to take the following form

$$\sigma^2 = \frac{E_i^2 x^2 [\varepsilon^i - \beta \varepsilon^e]^2 I_m(x) K_m(x)}{(\rho^i)^2 R_0^2 [I_m'(x) K_m(x) - \varepsilon K_m'(x) I_m(x)]} \quad (58)$$

5. Numerical Discussions

In order to examine the consistency of (55) has been written in dimensionless form to calculate the combined impact relating to the effects of gravity and electrodynamics:

$$\begin{aligned} \sigma^* = -U^* + \frac{x I_0'(x) K_0'(x)}{K_0'(x) I_0(x) - \rho I_0'(x) K_0(x)} \left[(1 - \rho) \left((1 - \rho) I_0(x) K_0(x) - \frac{1}{2} (2\rho + 1) \right) - \right. \\ \left. M \frac{[\beta - \alpha \varepsilon]^2 I_0(x) K_0(x)}{[I_0'(x) K_0(x) - \varepsilon K_0'(x) I_0(x)]} \right] \end{aligned} \quad (59)$$

Provided that:

$$\sigma^* = \frac{\sigma^2}{4\pi G \rho^i}, M = \frac{E_i^2}{E_0^2}, E_3^2 = 2\sqrt{\frac{\pi G}{\varepsilon^i}} \rho^i a^2, U^* = \sqrt{\frac{2ik\sigma U \cos \cos \omega t - ik\omega U \sin \sin \omega t - k^2 U^2 \omega t}{4\pi G \rho^i}}$$

The entered relation (59) was computed by the computer. This has been done for several values of ρ and U . The numerical regions of those of collected, 6). From this qualities and significant: For different have been range of

(i) For

$\rho = 0, U = 0, M = 0.1, 0.5, 0.7, 0.9, 1.2, \text{ and } 2, \varepsilon = 0.2$ it is discovered that There exist stable domains: $0 < x < \infty$

Thus, as seen in Figure 2 , The equalities line up with the states of marginal stability.

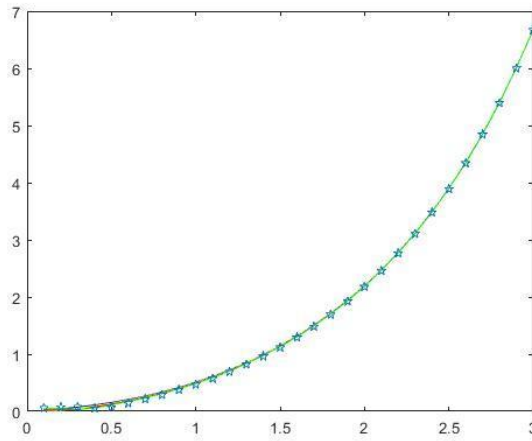
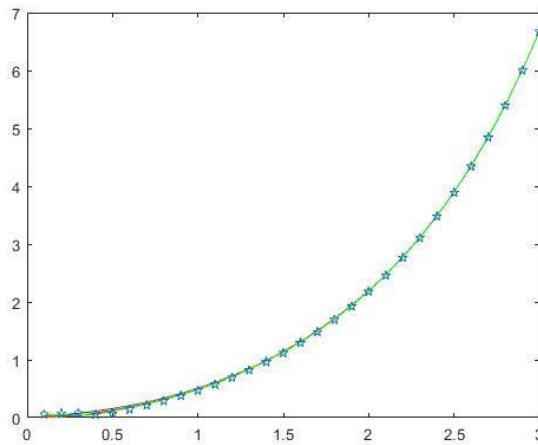


Fig.2.



$$\rho=0, U=0, M = 0.1, 0.5, 0.7, 0.9, 1.2, \text{ and } 2, \varepsilon = 0.2$$

(ii) For $\rho = 0, U = 0, \varepsilon = 0.8 M = 0.1, 0.5, 0.97, 1.2, \text{ and } 2$ it is discovered that There exist stable domains: $0 < x < \infty$

Thus, as seen in Figure 3 , The equalities line up with the states of marginal stability.

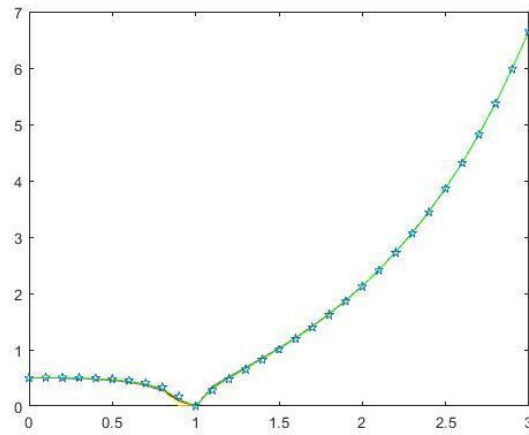


Fig.3. $\rho = 0, U = 0, M = 0.1, 0.5, 0.7, 0.9, 1.2, \text{ and } 2, \varepsilon = 0.2$

(iii) For $\rho = 0, U = 0.25, M = 0.1, 0.5, 0.7, 0.9, 1.2, \text{ and } 2, \varepsilon = 0.2$ it is discovered that
 There exist unstable domains $0 < x < 1$
 while domains that are stable are $1 < x < \infty$
 Thus, as seen in Figure 4, The equalities line up with the states of marginal stability.

Fig.4 $\rho = 0, U = 0.25, M = 0.1, 0.5, 0.7, 0.9, 1.2, \text{ and } 2, \varepsilon = 0.2$.

(iv) For $\rho = 0, U = 0.25, M = 0.1, 0.5, 0.7, 0.9, 1.2, \text{ and } 2, \varepsilon = 0.2$ it is discovered that
 There exist unstable domains $0 < x < 1$
 while domains that are stable are $1 < x < \infty$
 Thus, as seen in Figure 5, The equalities line up with the states of marginal stability.

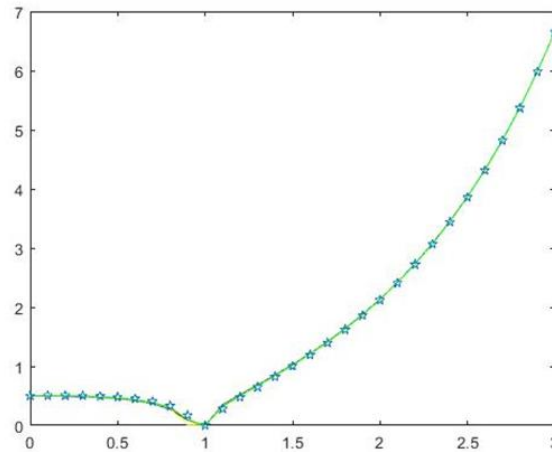


Fig.5. $\rho = 0, U = 0.25, M = 0.1, 0.5, 0.7, 0.9, 1.2, \text{ and } 2, \varepsilon = 0.2$

6. Conclusions

The numerical solution which has been obtained by using MATLAB Package 2017 may service as a tool to compared with its analytical counterpart to reach the following in which it may give rise to be in favor of the reliability of the above mentioned analytical method.

From this perspective, we have figured out that the model completely stabilises for both very long and short wavelengths with the same values of ($\rho = 0.2, U < 0.3$). It is discovered that the unstable domains are growing with rising U values for the same values of ($\rho = 0.2, U = 0.25$). For the same values of , it is discovered that the model becomes entirely stable not only for short wavelengths but also for very long wavelengths, indicating that streaming has a destabilising impact on the model for all short and long wavelengths.

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