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Comparative Analysis of Picard and Adomian Decomposition Methods for Solving Fractional Differential Equations in a Neural Network Model

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ABSTRACT
Various fields of science and engineering use neural network technology to solve their problems. In this paper, the Adomian decomposition method (ADM) is applied to solve fractional differential equations (FDEs) of a deferred correction network (DC Net) model using Caputo-Fabrizo (CF). To improve the accuracy of the calculated solution, we compare it with the Picard method (PM). It was found that the two schemes are very close to each other based on the analytical results. Comparing these two approaches, numerical tests confirm the accuracy of the proposed (DC Net) model.

Keywords: DC Net; Fractional model; Adomian decomposition method and Picard method; Convergence analysis; Existence and uniqueness; Caputo-Fabrizio derivative.

1. Introduction

The solution of fractional differential equations (FDEs) has been investigated by many researchers in recent years, since equations of this type can be found in many fields, including physics, engineering, biology, and fluid dynamics [1]. There have been many suggested methods for solving FDEs in recent years, including variational iteration techniques [2, 3], homotopy perturbation techniques [4, 5], Adomian decomposition techniques [6], homotopy analysis techniques [7], and collocation techniques [8, 9]. There are many definitions, such as Caputo [10,11], Rieman-Liouville, Atangana, and Caputo-Fabrizo (CF) [12]. In this research, we use Caputo-Fabrizo, which adds a new dimension to the study of fractional differential equations (FDEs). An important feature of the new derivative is that it has a nonsingular kernel, which is formed by combining an ordinary derivative with an exponential function, but it also has the same supplementary motivating properties with various scales as Riemann-Liouville fractional derivatives and Caputo derivatives.

It is noted that the Adomian decomposition method (ADM) is effective for solving both ordinary and partial differential equations across a wide range of physical models. The ADM's ability to yield solutions that are nearly identical in accuracy to those obtained through the Picard method (PM), while requiring less computational time (as indicated by the comparison of execution times between ADM and PM), demonstrates its efficiency.

ADM improves the efficiency of solving fractional differential equations (FDEs) in the context of the deferred correction network (DC Net) model by providing a systematic approach that avoids the need for linearization and discretization [13]. This method involves decomposing the solution into a series of functions and applying an inverse operator to the differential equation. It allows for the direct handling of nonlinear terms, which is particularly beneficial in complex models.

This research aims to study a significant fractional-order model of neural network solve ordinary differential equations (ODEs) with the definition of CFD. Based on several numerical tests, the proposed DC Net model is approximately 100 to 10 times more accurate than the learning polynomial neural network (LPNet). We use ADM and PM that have several advantages, as they are used for solving different kinds of equations in deterministic or stochastic fields, whether they are linear or nonlinear, and they are free from linearization and discretization [14]. For this model, existence, series convergence, and error estimation are discussed. These fractional brain models are discussed in four different cases.

This research is continued as follows: Section 2 is a presentation of the main definitions and properties required through the paper. Section 3 introduces the two methods of solution: ADM and PM. Section 4 shows convergence analysis, and contains the existence of a unique solution, series solution convergence, and error estimation. In Section 5, we give the numerical solution of the FBM in four different cases, and a comparison between the ADM and PM solutions is given.

2. Material and methods

2.1. Definition of Caputo-Fabrizo

Definition 1 The definition of the CF derivative of order η is

$$ {}^{CF}D_a^{\eta}\mathrm{R}\left(t\right)=\frac{B\left(\eta\right)}{1-\eta}\int_a^t \exp\left(\frac{-\eta\left(t-s\right)}{1-\eta}\right)\mathrm{R}'(s)ds, \tag{1} $$

and the normalization function $B(\eta)>0$ satisfies $B(0)=B(1)=1$ (see [15], [16]). Its corresponding fractional integral (FI) is

$$ {}^{CF}I_a^{\eta}\mathrm{R}\left(t\right)=\frac{1-\eta}{B\left(\eta\right)}\mathrm{R}\left(t\right)+\frac{\eta}{B\left(\eta\right)}\int_a^t \mathrm{R}(s)ds,\ \ \eta\in(0,1), \tag{2} $$

where

$$ \left({}^{CF}I_a^{\eta}\right)\left({}^{CF}D_a^{\eta}\right)\mathrm{R}\left(t\right)=\mathrm{R}\left(t\right)-\mathrm{R}\left(a\right). $$

The main advantage of using this definition is that there is no singularity in its definition, as shown in (1) and (2).

3. Solution Methods: ADM and PM

In this section, we review the solution algorithms for ADM and PM, which were used to solve the model.

3.1. The solution algorithm for ADM

In this section we shall review the procedure of the standard ADM for the initial value differential equations [17, 18].

Consider the differential equation

$$ L\varkappa+\mathrm{R}\left(t,\varkappa\left(t\right)\right)=g\left(t\right), \tag{3} $$

where L is the highest order derivative differential operator, which is assumed to be invertible, R is the linear differential operator whose order is less than L. We can apply the inverse operator L^{-1} to both sides of (3). After simple calculations, we obtain

$$ \varkappa=\sum_{i=0}^{n}\frac{c_i}{i!}t^i+L^{-1}g\left(t\right)-L^{-1}\mathrm{R}\left(t,\varkappa\left(t\right)\right), \tag{4} $$

where $\sum_{i=0}^{n}\frac{c_i}{i!}t^i$ arises from the given initial conditions (I.C).

ADM method assumes that the solution $\varkappa$ can be described by the series

$$ \varkappa\left(t\right)=\sum_{i=0}^{\infty}\varkappa_i\left(t\right). \tag{5} $$

By substituting the above equation in the equation (4), we attain the following recursive relationship

$$ \begin{aligned} \varkappa_0\left(t\right)&=\sum_{i=0}^{n}\frac{c_i}{i!}t^i+L^{-1}\left[g\left(t\right)\right], \\ \varkappa_\kappa\left(t\right)&=-L^{-1}\mathrm{R}\left(t,\varkappa\left(t\right)\right)_{\kappa-1}. \end{aligned} \tag{6} $$

Using Definition of CF where $L^{-1}(.)={}^{CF}I^{\eta}(.)$, it is reduced to the following fractional equation (FE), so we get

$$ \begin{aligned} \varkappa_0\left(t\right)&=\sum_{i=0}^{n}\frac{c_i}{i!}t^i+{}^{CF}I^{\eta}\left[g\left(t\right)\right], \\ \varkappa_\kappa\left(t\right)&=-{}^{CF}I^{\eta}\mathrm{R}\left(t,\varkappa\left(t\right)\right)_{\kappa-1}. \end{aligned} \tag{7} $$

where $t\in J=(0,T]$, $t\in\mathscr{R}^{+}$, $\mathbb{R}(t,\varkappa)$ is continuous function satisfies Lipschitz condition

$$\left|R\left(t,\grave{u}\right)-R\left(t,z\right)\right|\leq\Phi\left|\grave{u}-z\right|, \tag{8}$$

where Φ is the Lipschitz constant.

3.2. The solution algorithm for PM

Applying PM to FE (4), the solution is a sequence constructed by

$$\grave{u}_0(t)=\sum_{i=0}^{n}\frac{c_i}{i!}t^i+{}^{CF}I^{\eta}\left[g(t)\right],$$
$$\grave{u}_\kappa(t)=\grave{u}_0(t)-{}^{CF}I^{\eta}R\left(t,\grave{u}(t)\right)_{\kappa-1}. \tag{9}$$

The functions $\varkappa_\kappa(t)$ are continuous, and they are the sum of successive differences

$$\grave{u}_\kappa(t)=\grave{u}_0(t)+\sum_{\kappa=1}^{n}\left(\grave{u}_\kappa-\grave{u}_{\kappa-1}\right).$$

This means that the sequence $\varkappa_\kappa$ is equivalent to the infinite series $\sum_{\kappa=1}^{n}(\varkappa_\kappa-\varkappa_{\kappa-1})$ which is convergent. The final PM solution takes the form

$$\grave{u}(t)=\lim_{\kappa\to\infty}\grave{u}_\kappa(t).$$

From the above relations, we can deduce that if the series $\sum_{\kappa=1}^{n}(\varkappa_\kappa-\varkappa_{\kappa-1})$ is convergent, then the sequence $\varkappa_\kappa(t)$ would be convergent to $\varkappa(t)$. And to prove that the sequence $\{\varkappa_\kappa(t)\}$ is convergent, consider the series

$$\sum_{\kappa=1}^{\infty}\left[\grave{u}_\kappa(t)-\grave{u}_{\kappa-1}(t)\right].$$

From (9) for $\kappa=1$, we get

$$\begin{aligned}
&\grave{u}_1(t)-\grave{u}_0(t)=-{}^{CF}I^{\eta}R\left(t,\grave{u}_0(t)\right)\\
&\left|\grave{u}_1(t)-\grave{u}_0(t)\right|=\left|{}^{CF}I^{\eta}R\left(t,\grave{u}_0(t)\right)\right|\\
&\leq\frac{1-\eta}{B(\eta)}\left[\left|h(t)\right|+n\left|\grave{u}_0(t)\right|\right]+\frac{\eta}{B(\eta)}\int_0^t\left[\left|h(\hat{s})\right|+n\left|\grave{u}_0(\hat{s})\right|\right]d\hat{s}\\
&\leq\left[\frac{1-\eta}{B(\eta)}+\frac{\eta T}{B(\eta)}\right]\left[H+\breve{r}n\right]\\
&\leq\frac{1+\eta(T-1)}{B(\eta)}\left[H+\breve{r}n\right]\\
&\leq\frac{(1+\eta T)\left[H+\breve{r}n\right]}{B(\eta)}\\
&\leq\phi,
\end{aligned} \tag{10}$$

where $\phi=\frac{(1+\eta T)[H+\breve{r}n]}{B(\eta)}$. Now, we get an estimate for $[\varkappa_\kappa(t)-\varkappa_{\kappa-1}(t)],\ \kappa\geqslant 2$,

$$\begin{aligned}
&\grave{u}_\kappa(t)-\grave{u}_{\kappa-1}(t)={}^{CF}I^{\eta}R\left(t,\grave{u}_{\kappa-1}(t)\right)-{}^{CF}I^{\eta}R\left(t,\grave{u}_{\kappa-2}(t)\right)\\
&\left|\grave{u}_\kappa(t)-\grave{u}_{\kappa-1}(t)\right|\leq{}^{CF}I^{\eta}\left|R\left(t,\grave{u}_{\kappa-1}(t)\right)-R\left(t,\grave{u}_{\kappa-2}(t)\right)\right|\\
&\leq\Phi\left[\frac{1-\eta}{B(\eta)}+\frac{\eta T}{B(\eta)}\right]\left|\grave{u}_{\kappa-1}(t)-\grave{u}_{\kappa-2}(t)\right|\\
&\leq\Phi\left[\frac{1+\eta T}{B(\eta)}\right]\left|\grave{u}_{\kappa-1}(t)-\grave{u}_{\kappa-2}(t)\right|\\
&\left\|\grave{u}_\kappa-\grave{u}_{\kappa-1}\right\|\leq\beta\left\|\grave{u}_{\kappa-1}-\grave{u}_{\kappa-2}\right\|.
\end{aligned} \tag{11}$$

In the above relation, if we put $\kappa=2$, and use (10) we get

$$\|\grave{u}_2-\grave{u}_1\|\leq \beta\|\grave{u}_1(t)-\grave{u}_0(t)\|$$
$$\|\grave{u}_2-\grave{u}_1\|\leq \beta\,\phi.$$

Make the same for $\kappa=3,4,\cdots,$

$$\|\grave{u}_3-\grave{u}_2\|\leq \beta\|\grave{u}_2-\grave{u}_1\|\leq \beta^2\phi$$
$$|\grave{u}_4-\grave{u}_3|\leq \beta\|\grave{u}_3-\grave{u}_2\|\leq \beta^3\phi$$
$$\vdots$$

So, the general form of this relation is

$$|\grave{u}_{\kappa}-\grave{u}_{\kappa-1}|\leq \beta^{\kappa-1}\phi.$$

Since $\beta<1$, then the series

$$\sum_{\kappa=1}^{\infty}\left[\grave{u}_{\kappa}(t)-\grave{u}_{\kappa-1}(t)\right],$$

is convergent. Hence, the sequence $\{\varkappa_{\kappa}(t)\}$ uniformly converges. Since $\mathbb{R}(t,\varkappa(t))$ is continuous in $\varkappa$, then

$$\grave{u}(t)=\lim_{\kappa\to\infty}{}^{CF}I^{\eta}\mathrm{R}\left(t,\grave{u}_{(\kappa-1)}(t)\right)={}^{CF}I^{\eta}\mathrm{R}\left(t,\grave{u}(t)\right).$$

4. Convergence analysis

4.1. Existence and uniqueness

Theorem 1:

Let $\mathbb{R}(t,\varkappa)$ satisfy the Lipschitz condition (8), and if $T^{\eta}<\frac{\Gamma(\eta+1)}{\Phi}$, then the solution $\varkappa$ of the FE (4) is unique.

Proof. From (4), we can define a mapping Ψ as

$$\Psi\grave{u}=\sum_{i=0}^{n}\frac{c_i}{i!}t^i+{}^{CF}I^{\eta}\left[g(t)-\mathrm{R}\left(t,\grave{u}(t)\right)\right],$$

Let $\varkappa,z\in\breve{E}$, then

$$\Psi\grave{u}-\Psi z=-{}^{CF}I^{\eta}\mathrm{R}\left(t,\grave{u}(t)\right)+{}^{CF}I^{\eta}\mathrm{R}\left(t,z(t)\right)$$
$$\|\Psi\grave{u}-\Psi z\|\leq \max_{t\in J}{}^{CF}I^{\eta}\left|\mathrm{R}\left(t,\grave{u}(t)\right)-\mathrm{R}\left(t,z(t)\right)\right|$$
$$\leq \Phi\|\grave{u}-z\|\left[\frac{1-\eta}{B(\eta)}+\frac{\eta}{B(\eta)}\int_0^t d\hat{s}\right]$$
$$\leq \Phi\left[\frac{1-\eta}{B(\eta)}+\frac{\eta T}{B(\eta)}\right]\|\grave{u}-z\|$$
$$\leq \Phi\left[\frac{1+\eta(T-1)}{B(\eta)}\right]\|\grave{u}-z\|$$
$$\leq \Phi\left[\frac{1+\eta T}{B(\eta)}\right]\|\grave{u}-z\|$$
$$\leq \Phi\|\grave{u}-z\|.$$

If $0<\delta<1$, then the mapping Ψ would be contraction, moreover if $T<\frac{B(\eta)-\Phi}{\Phi\eta}$, then there exists a unique solution to (4).

4.2. Solution convergence

Theorem 2:

If the solution of FE (4) exists, and $|\varkappa_1(t)|<L$, where L is a positive constant, then the ADM series solution (5) of FE (4) converges.

Proof. Define a sequence $Q_{\breve{n}} = \sum_{\kappa=0}^{\breve{n}} \varkappa_\kappa(t)$ is the sequence of partial sums from the ADM series solution, and we have

$$\mathrm{R}\left(t, \grave{u}(t)\right) = \sum_{\kappa=0}^{\infty} \Lambda_\kappa .$$

Taking two partial sums; $Q_{\breve{n}}$ and Q_ν, such as $\breve{n} > \nu$. Now, our goal is to prove that $Q_{\breve{n}}$ is a Cauchy sequence in the Banach space $\Im$.

$$\begin{aligned}
Q_{\breve{n}} - Q_\nu &= \sum_{\kappa=0}^{\breve{n}} \grave{u}_\kappa - \sum_{\kappa=0}^{\nu} \grave{u}_\kappa \\
&= {}^{CF}I^\eta \sum_{\kappa=0}^{\breve{n}} \mathrm{R}\left(t, \grave{u}_{(\kappa-1)}(t)\right) - {}^{CF}I^\eta \sum_{\kappa=0}^{\nu} \mathrm{R}\left(t, \grave{u}_{(\kappa-1)}(t)\right) \\
&= {}^{CF}I^\eta \left[\sum_{\kappa=\nu}^{\breve{n}-1} \mathrm{R}\left(t, \grave{u}_{(\kappa-1)}(t)\right)\right] \\
&= {}^{CF}I^\eta \left[\mathrm{R}\left(t, Q_{\breve{n}-1}\right) - \mathrm{R}\left(t, Q_{\nu-1}\right)\right] \\
\left\| Q_{\breve{n}} - Q_\nu \right\| &\le \max_{t\in J} {}^{CF}I^\eta \left| \mathrm{R}\left(t, Q_{\breve{n}-1}\right) - \mathrm{R}\left(t, Q_{\nu-1}\right) \right| \\
&\le \Phi \left[\frac{1-\eta}{B(\eta)} + \frac{\eta T}{B(\eta)}\right] \left\| Q_{\breve{n}-1} - Q_{\nu-1} \right\| \\
&\le \Phi \left[\frac{1+\eta T}{B(\eta)}\right] \left\| Q_{\breve{n}-1} - Q_{\nu-1} \right\| \\
&\le \delta \left\| Q_{\breve{n}-1} - Q_{\nu-1} \right\| .
\end{aligned}$$

Let $\breve{n} = \nu + 1$, then

$$\left\| Q_{\nu+1} - Q_\nu \right\| \le \delta \left\| Q_\nu - Q_{\nu-1} \right\| \le \delta^2 \left\| Q_{\nu-1} - Q_{\nu-2} \right\| \le \cdots \le \delta^\nu \left\| Q_1 - Q_0 \right\| .$$

Using the triangle inequality, we get

$$\begin{aligned}
\left\| Q_{\breve{n}} - Q_\nu \right\| &\le \left\| Q_{\nu+1} - Q_\nu \right\| + \left\| Q_{\nu+2} - Q_{\nu+1} \right\| + \cdots + \left\| Q_{\breve{n}} - Q_{\breve{n}-1} \right\| \\
&\le \left[\delta^\nu + \delta^{\nu+1} + \cdots + \delta^{\breve{n}-1}\right] \left\| Q_1 - Q_0 \right\| \\
&\le \delta^\nu \left[1 + \delta + \cdots + \delta^{\breve{n}-\nu-1}\right] \left\| Q_1 - Q_0 \right\| \\
&\le \delta \left[\frac{1-\delta^{\breve{n}-\nu}}{1-\delta}\right] \left\| \grave{u}_1 \right\| .
\end{aligned}$$

Now $0 < \delta < 1$, and $\breve{n} > \nu$ implies that $(1 - \delta^{\breve{n}-\nu}) \le 1$. Hence

$$\left\| Q_{\breve{n}} - Q_\nu \right\| \le \frac{\delta^\nu}{1-\delta} \left\| \grave{u}_1 \right\| \le \frac{\delta^\nu}{1-\delta} \max_{t\in J} \left| \grave{u}_1(t) \right| .$$

If $|\varkappa_1(t)| < L$ and as $\nu \to \infty$ then, $\|Q_{\breve{n}} - Q_\nu\| \to 0$, and therefor $Q_{\breve{n}}$ is a Cauchy sequence in the Banach space $\Im$, then the ADM series solution converges.

4.3. Error estimation

Theorem 3:

The maximum absolute error of the ADM series solution is

$$\max_{t\in J} \left| \grave{u}(t) - \sum_{\kappa=0}^{\nu} \grave{u}_\kappa(t) \right| \le \frac{\delta^\nu}{1-\delta} \max_{t\in J} \left| \grave{u}_1(t) \right| .$$

Proof. From theorem 2, we have

$$\|Q_{\breve{n}} - Q_{\nu}\| \leq \frac{\delta^{\nu}}{1-\delta} \max_{t\in J} |\grave{u}_1(t)|.$$

But $Q_{\breve{n}} = \sum_{\kappa=0}^{\breve{n}} \varkappa_{\kappa}(t)$ as $\breve{n} \to \infty$ then, $Q_{\breve{n}} \to \varkappa(t)$, hence

$$\|\grave{u}(t) - Q_{\nu}\| \leq \frac{\delta^{\nu}}{1-\delta} \max_{t\in J} |\grave{u}_1(t)|.$$

So, the MAE in the interval J is

$$\max_{t\in J} \left| \grave{u}(t) - \sum_{\kappa=0}^{\nu} \grave{u}_{\kappa}(t) \right| \leq \frac{\delta^{\nu}}{1-\delta} \max_{t\in J} |\grave{u}_1(t)|.$$

Examples and applications

Example 5.1 [12] Consider the fractional equation:

$$^{CF}D^{\eta}\grave{u} = -\grave{u}, \quad t \in (0,1]$$
$$\grave{u}(0) = 1, \tag{12}$$

with exact solution e^{-t}.

Applying ADM to equation (12), we get

$$\grave{u}_0(t) = 1,$$
$$\grave{u}_{\kappa}(t) = -^{CF}I^{\eta}\left[\grave{u}_0(t)\right], \ \kappa \geq 1.$$

Using PM to equation (12), we have

$$\grave{u}_0(t) = 1,$$
$$\grave{u}_{\kappa}(t) = \grave{u}_0(t) - ^{CF}I^{\eta}\left[\grave{u}_0(t)\right], \quad \kappa \geq 1.$$

Figures (1,2) show ADM and PM solutions at ($\eta = 1, 0.97$, 0.95, and 0.9).

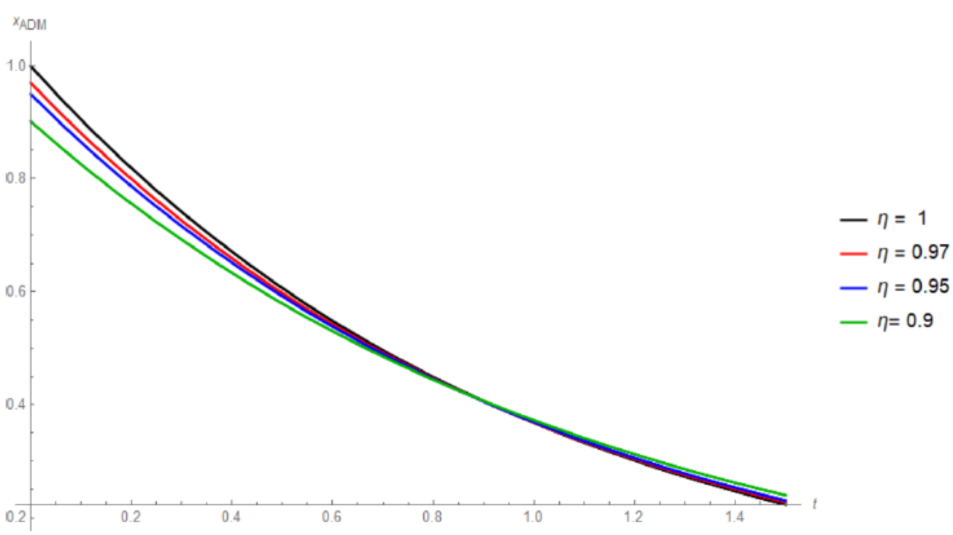

Figure 1: ADM solution at different η

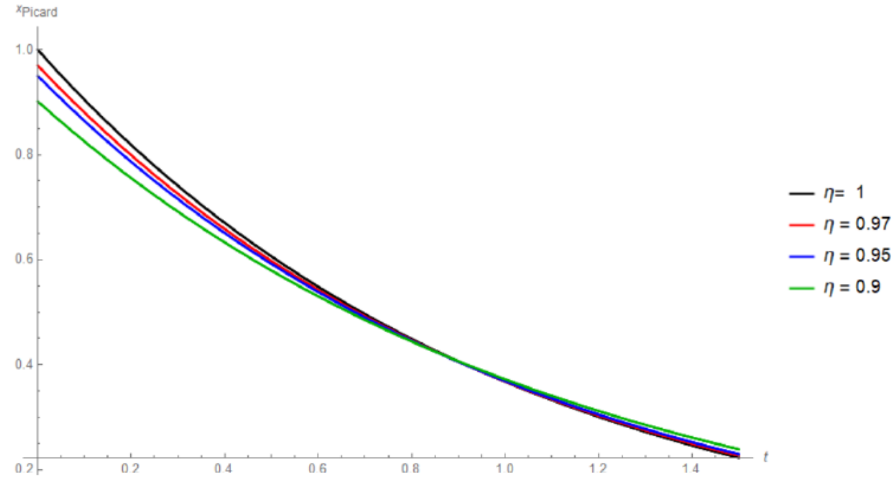

Figure 2: PM solution at different η

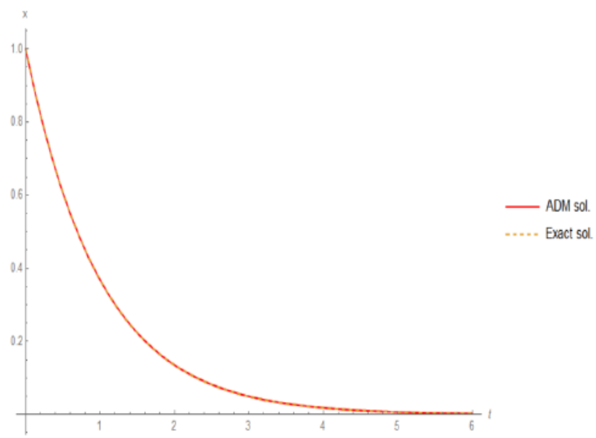

Figure 3: ADM solution ($\eta = 1$) with exact solution

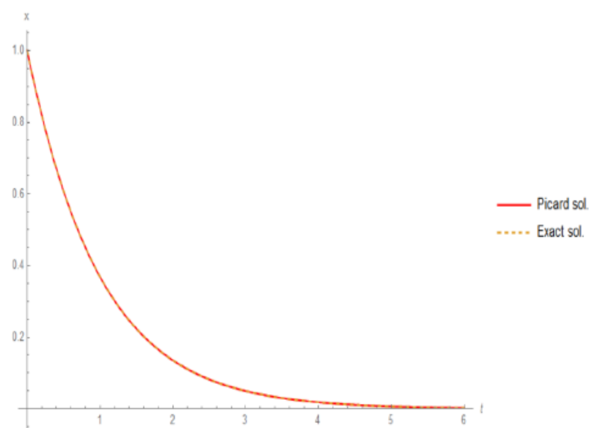

Figure 4: Picard solution ($\eta = 1$) with exact solution

Table 1: RE between (ADM and PM) at η = 0.9

t	$\left\lvert \frac{\varkappa_{ADM} - \varkappa_{Picard}}{Exact} \right\rvert$
2	3.53139×10^{-17}
4	9.91413×10^{-14}
6	7.13859×10^{-11}
8	9.18725×10^{-10}

A comparison between the RE (relative error) of ADM and PM solutions (where η = 0.9) is given in table 1. Although the two solutions are nearly identical in accuracy, when the time used in two cases is compared, ADM takes a shorter amount of time than PM (ADM time = 0.171 sec., PM time = 0.312 sec.).

<u>Example 5.2:</u> [12] Consider the fractional equation:

$$
\begin{gathered}
{}^{CF}D^{\eta}\grave{u}\;(t) = -10^{3}\left(\grave{u}\;(t) - \sin(t)\right) + \cos(t), \quad \mathrm{t} \in (0,1] \\
\grave{u}\;(0) = 0.
\end{gathered} \tag{13}
$$

Applying ADM to equation (13), the solution algorithm is

$$
\begin{aligned}
\grave{u}_{\,0}(t) &= {}^{CF}I^{\eta}\left[-10^{3}\sin(t)\right] + {}^{CF}I^{\eta}\left[\cos(t)\right], \\
\grave{u}_{\,\kappa}(t) &= {}^{CF}I^{\eta}\left[-10^{3}\grave{u}_{\,0}(t)\right], \;\; \kappa \geq 1.
\end{aligned}
$$

Using PM to equation (13), so we have

$$\grave{u}_0(t) = {}^{CF}I^{\eta}\left[-10^3 \sin(t)\right] + {}^{CF}I^{\eta}\left[\cos(t)\right],$$

$$\grave{u}_{\kappa}(t) = \grave{u}_0(t) + {}^{CF}I^{\eta}\left[-10^3 \grave{u}_0(t)\right], \ \kappa \geq 1.$$

Figures (5,6) show ADM and PM solutions at (η = 1,0.97, 0.95, and 0.9).

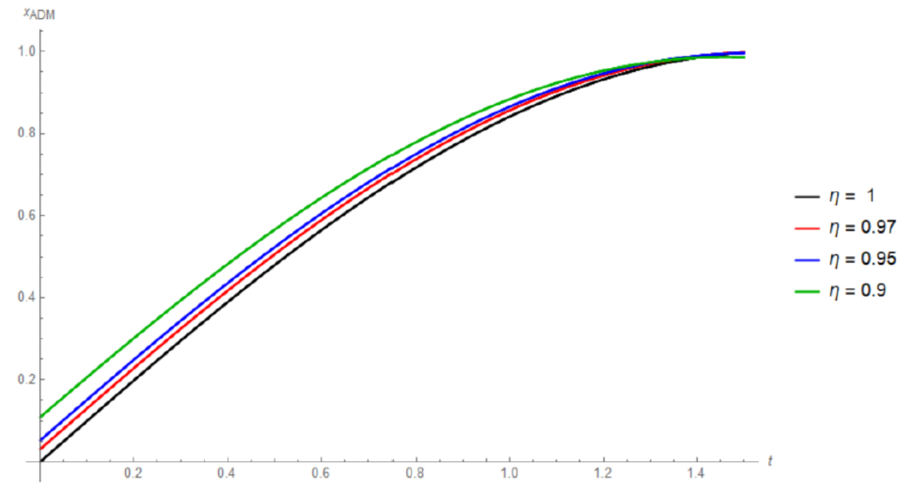

Figure 5: ADM solution at different η

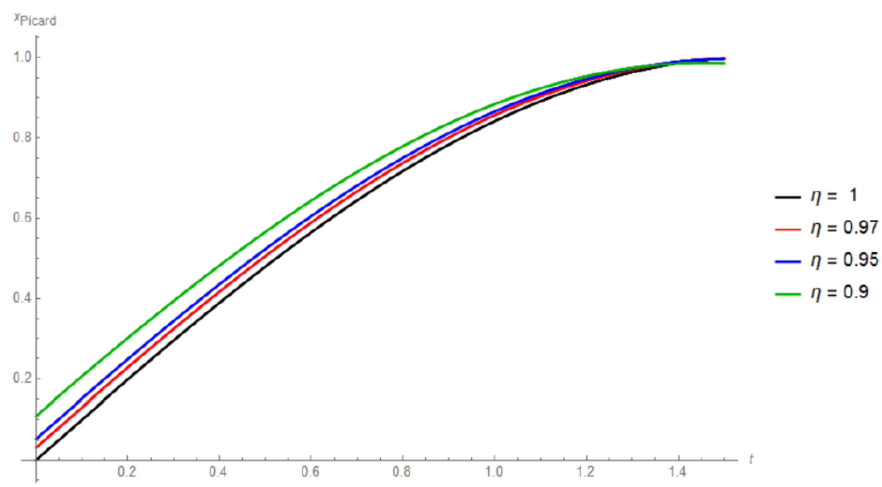

Figure 6: PM solution at different η

Table 2: AD between (ADM and PM) at η = 0.9

t	$\lvert\varkappa_{ADM} - \varkappa_{Picard}\rvert$
2	1.00754×10^{-16}
4	8.42711×10^{-17}
6	3.09201×10^{-17}
8	1.09703×10^{-16}

A comparison between the AD (Absolute difference) of ADM and PM solutions (where η = 0.9) is given in table 2. Although the two solutions are nearly identical in accuracy, when the time used in two cases is compared, ADM takes a shorter amount of time than PM (ADM time = 0.187 sec., PM time = 0.282 sec.).

Conclusion

This research is presented on the use of ADM and PM to solve FDEs, particularly around a deferred correction network (DC Net). It highlights the advantages of using the Caputo-Fabrizo derivative, which offers a nonsingular kernel and retains essential properties like other fractional derivatives.

The findings indicate that both ADM and PM are effective in providing accurate solutions to FDEs, but ADM demonstrates increased flexibility and efficiency. The comparative analysis shows that ADM converges faster and requires less computational time than PM. The research shows the importance of these methods in various scientific and engineering applications. It emphasizes their role in enhancing the reliability of models predicting brain metabolite variations.

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