# A Quadratic Regression Approach Using Mathematical Goal Programming


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## Abstract

Goal programming is a method commonly used to solve multi-criteria decision problems in different areas, such as manufacturing, production planning, marketing, supply chains, healthcare, management science, environment and energy. Goal programming approach minimizes the absolute value of the deviations to reduce the effect of outliers. This study aims to use a suggested quadratic regression approach using goal programming by adopting the least absolute deviation method ($L_1$-norm) and the least square method ($L_2$-norm) to estimate the parameters. A simulation study was applied to evaluate the proposed approach's performance by generating data for dependent variables from the lognormal and Cauchy distributions with three different parameters and samples with different sizes. The results of the various simulation studies that are carried out to compare the efficiency of the least absolute deviation method and the least square method estimators, for various error distributions, indicate that the least absolute method is more efficient than the least square method estimator.

***Keywords:*** *Goal Programming, Quadratic Regression, Mean Square Error, Least Square Method, Least Absolute Method.*

## 1. Introduction:

Mathematical programming problems have been the focus of researchers in mathematics, economics, and operations research for more than a quarter of a century. It plays a vital role in engineering applications such as aircraft design, optimal trajectory of space vehicles, optimal design of electric networks, and planning the best strategy to obtain maximum profit. It also plays a vital role in statistics, many problems in regression analysis, sample surveys, cluster analysis, construction of designs, estimation, decision theory, and so on can be viewed as mathematical programming problems. The applications of mathematical programming are everywhere, many real-life problems can be converted into mathematical form and optimized by using the techniques of mathematical programming. The Simplex Method's development has led to unprecedented growth in both theory and methods of mathematical programming. Moreover, the emphasis has shifted to solving specific problems and finding efficient methods that are appropriate for computers. [Bhat and Ahmed (2012)] and [Janaki (2022)].

Goal Programming (GP) is one of the most important mathematical programming models and it is a widely used technique in operations research. Goal programming is a branch of multi-objective optimization, which is a branch of multi-criteria decision analysis (MCDA). The idea of goal programming was first suggested by Charnes and Cooper in 1961. The goal Programming (GP) models were popularized with applications by many authors Lee (1972-1973), Lee and Clayton (1972), Charnes and Cooper (1977), and Ignizio (1978). Goal Programming (GP) is being used as a statistical tool for estimation, which is another interesting development. Recent research suggests that goal programming could be a substitute for conventional statistical methods. [Lakshmi *et al.* (2021)].

Regression analysis aims to obtain a precise prediction of the level of output variables. Regression analysis investigates and models the relationships between dependent variables and a set of independent variables from several samples. It has many applications and is used in almost all fields, including engineering, physical and chemical sciences, economics, management, life and biological sciences, and social sciences, as one of the most widely used statistical techniques.

Quadratic regression models, a type of polynomial regression, are a type of statistical models that extend linear regression. These models aim to catch relationships where the response variable is not constant but change at a rate proportional to its value, leading to a nonlinear association. [Yang *et al.* (2017) and Montgomery *et al.* (2021)].

In recent years, many authors have shown keen interest in the study of quadratic regression and goal programming

Kassem *et al.* (2019) introduced a new method to estimate the parameter for a quadratic regression model by using Kuhn-Tucker conditions. According to the Durbin–Watson test it has shown that there is positively autocorrelation between the errors for the regression curve, this means that their estimators are a suitable estimator in the case of fitting data.

Hussain and Ali (2019) proposed Goal programing as an alternative estimation method to estimate the parameters of multiple linear regression, and nonparametric test is proposed in addition to the mean square error to assess the estimation methods. The results of the simulation study and the analysis of real data show that the goal programing method is not affected by the sample size because it gives approximately the same values of estimation for parameters of the model with smallest values of Mean square error (MSE) for all sample sizes. The results of the nonparametric test show that there is a difference between ordinary least square (OLS) and goal programing (GP) methods in small sample sizes meanwhile there is no difference when the sample size becomes large.

Zaliskyi *et al.* (2020) presented the problem of mathematical model building for COVID-19 diseases data. For data on new cases of diseases in Ukraine, Poland and Italy, a comparative analysis of the use of regression models based on polynomials of the 5th, 7th and 10th order, mathematical model building in a sliding window, as well as a segmented regression model was carried out. The segmented regression model is the most preferable from the point of view of both forecasting properties and considering the geometric structure of the initial data. The research results can be used in the process of solving the problems of predicting the spread of COVID-19 in different countries.

Elrefaey *et al.* (2023) presented the goal programming approach to estimate the parameters of the two regression models and use the results to impute the missing values and complete data in matched files. The goal programming approach has the advantage of minimizing the effect of outliers on estimates because it uses minimization of the sum of absolute deviations that is less affected by outliers. Results show the efficacy of the approach in accurately estimating missing values while maintaining data quality and minimizing errors.

This paper is organized as follows: the next Section is devoted to illustrate Goal Programming and Quadratic Regression Models. Section 3 introduces Least absolute deviation and Least Squares Methods. Section 4 suggests a Quadratic Regression Approach using Goal Programming. A simulation study in Section 5 to evaluate the performance of the suggested approach, Section 6 closes with the final conclusions.

## 2. The Goal Programming and Quadratic Regression Models:

This section explains the Goal Programming and Quadratic Regression Models.

### 2.1 Goal Programming Model:

The technique of goal programming is extensively used to solve the problems of decision making. The use of goal programming model approach is widely employed in problems of multi-criteria decision-making. The refined method of Linear programming for decision-making and solving multi-objective problems is developed in goal programming. [Dave (2015)].

Linear programming methods are used to solve and find an optimal solution of a single dimensional or multi-dimensional objective function. with a given set of linear constraints.

On the other hand, goal programming technique allows the decision maker to find feasible and optimal solutions to multiple and conflicting objectives, The objective function in goal programming technique contains all the management goals, and the constraints are all those environmental conditions that are outside the management's control. Each goal set by the management is given priority according to the higher and lower level. [Al-Sabbah *et al* (2021)].

The technique of goal programming can be utilized to find the solution for all linear goal constraints, depending on the importance or contribution of each goal. It aims to minimize both the sum of all deviations and all possible priority deviations. The decision maker or manager has the ability to make concessions between goals to influence multi-objective problem solutions. goal programming problems deal with the extent and direction of these deviations. Goal Programming and decision-making share great similarities although goal programming has its characteristics.

The general goal programming model which can be expressed mathematically as:

$$mininize \quad Z = \sum_{i=1}^{m} d_i{}^{+} + d_i{}^{-}$$

Subject to:

$$\sum_{j=1}^{n} a_{ij}x_j - d_i{}^{+} + d_i{}^{-} = b_i \, , for\ i = 1, \dots, m$$

$$\sum_{j=1}^{n} a_{ij}x_j \begin{pmatrix} \leq \\ \geq \end{pmatrix} b_i \, , for\ i = m+1, \dots, m+p$$

$$d_i^+, d_i^-, x_j \geq 0 \ , \ for\ i = 1, \dots, m \ , \ for\ j = 1, \dots, n \qquad (\mathbf{1})$$

where:

$Z$ = objective function = Summation of all deviations.

$a_{ij}$ = the coefficient associated with variable $j$ in the $i^{th}$ goal.

$x_j$ = the $j^{th}$ decision variable.

$b_i$ = the associated right hand side value.

$d_i^-$ = negative deviational variable from the $i^{th}$ goal (underachievement).

$d_i^+$ = positive deviational variable from the $i^{th}$ goal (overachievement).

[Dave (2015)].

## 2.2 Quadratic Regression:

Quadratic Regression is a statistical method used to model the relationship between a dependent variable and an independent variable using a quadratic function. It is used to fit a quadratic equation to a set of data points, providing a model that can describe non-linear relationships. Quadratic regression models play an important role in many fields such as economics, biology, and engineering, where data often exhibit quadratic trends. In math education, quadratic regression helps understand how to apply mathematical models to analyze real-world data, enhancing their analytical and critical thinking skills. By using technology to perform quadratic regression, they interpret the resulting equation, assess the fit of the model, and make predictions based on the data. The quadratic model is also sometimes called a second-order model in one variable.

The general form of a quadratic regression model is:

$$y_i = \beta_0 + \beta_1 x_i + \beta_2 {x_i}^2 + \varepsilon_i \ , \qquad i = 1,2, \dots, n \qquad (\mathbf{2})$$

where:

$y_i$ = represents the dependent variable for the $i\ th$ observation.

$x_i$ = represents the independent variable for the $i\ th$ observation.

$\varepsilon_i$ = represents the independent random error term for the $i\ th$ observation.

$\beta' s$ = are the called the regression coefficients of the regression line, they are the unknown parameters which are going to be estimated.

$n$ = represents the number of observations. [Kassem *et al.* (2019)].

## 3. Least absolute deviation and Least Squares Methods:

This section explains the least absolute deviation and least square methods.

### 3.1Least Absolute Deviations Method:

The least absolute deviations method is also called $L_1$-norm and originally proposed by Joseph Boscovich in 1757.

The least absolute error method is based on choosing the values of $\hat{\beta}$ which minimize the absolute difference between the observed $y$ , and estimated value of $\hat{y}$, for $n$ observations as follows:

$$\sum_{i=1}^{n} |\varepsilon_i| = \sum_{i=1}^{n} \left| y_i - \beta_0 - \sum_{j=1}^{k} \beta_j x_{ij} \right| \quad , \quad i = 1, \dots, n , \quad j = 1, \dots, k \qquad (3)$$

where:

$x_i$ =represents the independent variable for the $i\ th$ observation.

$y_i$ = represents the dependent variable for the $i\ th$ observation.

$\varepsilon_i$ = represents the independent random error term for the $i\ th$ observation.

$\beta_j$ = are the parameters of the regression model and $j = 1, \ldots, k$ ,

$k$ - dimensional space of the regressor variables $x_j$ .

$n$ = represents the number of observations.

The least absolute deviation ($L_1$-norm) estimators of the regression coefficients have some valuable properties. First, they are the estimates that give the smallest sum of absolute residual. Second, the least absolute deviation regression is efficient when the error distribution is heavy tailed. Third, the ($L_1$-norm) method is more resistant to outliers in data. Last, the least absolute value overcomes the problems of the least squares estimators concerning outliers because it is less sensitive to extreme errors. [Ismail (2003)].

**3.2 Least Squares Method:**

The least squares method is a form of mathematical regression analysis used to determine the line of best fit for a set of data, providing a visual demonstration of the relationship between the data points. Each point of data represents the relationship between a known independent variable and an unknown dependent variable. The method of minimal squares is the basis for minimizing the total squares of deviations or errors in the result of each equation.

MSSE is one of many names given to the least squares method which aims to minimize the sum of squared errors and is also referred to as the least squares deviation criterion or $L_2$-norm. The least squares method was proposed by Cari Friedrich Gauss in 1795, and Adrien Marie Legendre in 1805. [Ismail (2003)].

The least squares estimators of $\beta$ are defined to be the values of $\hat{\beta}$ which minimize the square difference between the observed $y$, and estimated value of ($\hat{y}$ ), for $n$ observations, and $x_{ij}$ denotes the $i\ th$ observation or level of regressor $x_j$ .The Least Squares function is as follows: [Dave (2015)].

$$S(\beta_0, \beta_1, \dots, \beta_k) = \sum_{i=1}^{n} \varepsilon_i{}^2 = \sum_{i=1}^{n} (y_i - \beta_0 - \sum_{j=1}^{k} \beta_j x_{ij})^2, \qquad i = 1, \dots, n, \qquad j = 1, \dots, k \quad (\mathbf{4})$$

## 4. The Suggested Approach of Quadratic Regression using Goal Programming:

This section suggests the approach of quadratic regression using goal programming. The suggested approach adopts the least absolute deviation method ($L_1$-norm) and the least square method ($L_2$-norm) to estimate the parameters. ($L_1$-norm) and ($L_2$-norm) regression coefficients are estimated using an estimated mathematical goal programming approach.

### 4.1 Suggested Approach of Quadratic Regression using Goal Programming in Least Absolute Deviation Method ($L_1$-norm):

The suggested approach adopts the least absolute deviation method ($L_1$-norm) to estimate the regression parameters. The ($L_1$-norm) regression coefficients are using a special case of quadratic regression with mathematical goal programming approach as follows:

$$mininize \quad \sum_{i=1}^{n} \left| d_i{}^+ + \ d_i{}^- \right|$$

Subject to:

$$y_i - \sum_{i=1}^{n} bx_i^{\,2} \; - \; d_i^{\,+} + d_i^{\,-} = 0 \,, \qquad i = 1,2, \dots, n$$

$$d_i^{\,+}, d_i^{\,-} \; \geq 0 \qquad , and \; b \; \in F \; (F \; is \; a \; feasible \; set \,), \qquad (\mathbf{5})$$

where:

$x_i$ are the independent variables.

$y_i$ are the dependent variable.

$b$ is the parameter of the regression model.

$d_i^{\,+}$ *and* $d_i^{\,-}$ are the positive and negative deviational variables.

### 4.2 Suggested Approach of Quadratic Regression using Goal Programming in Least Squares Method ($L_2$-norm):

The suggested approach adopts the least square method ($L_2$-norm) to estimate the regression parameters. The ($L_2$-norm) regression coefficients are using a special case of quadratic regression with mathematical goal programming approach as follows:

$$mininize \quad \sum_{i=1}^{n} \; \left(d_i^{\,+} + d_i^{\,-}\right)^2$$

Subject to:

$$y_i - \sum_{i=1}^{n} bx_i^{\,2} \; - \; d_i^{\,+} + d_i^{\,-} = 0 \,, \qquad i = 1,2, \dots, n$$

$$d_i^{\,+}, d_i^{\,-} \; \geq 0 \;\; , \quad and \; b \; \in F \; (F \; is \; a \; feasible \; set \,), \qquad (\mathbf{6})$$

## 5. A Simulation Study:

A simulation study introduces the quadratic regression model using the mathematical goal programming approach. The least absolute method ($L_1$-norm) and the least square method ($L_2$-norm) are used to estimate and improve the parameters of the suggested model. The study assumes $\varepsilon$ generates from two distributions: Lognormal and Cauchy, which are heavy-tailed or peaked-tailed. The following steps are used to compare the least absolute method and least square method in the case of a heavy-tailed distribution of different sizes $[n = 25, 50, 100, 300]$. The *Mean Square Error* (MSE) and the *Mean Absolute Error* (MAE) are used as criteria to compare the solutions of ($L_1$-norm) and ($L_2$-norm) approaches, where the method with the lowest values of these criteria is the preferred method.

### 5.1 Steps of Simulation Study:

The steps of the simulation study are as follows:

- Putting the initial values for coefficient regression $[b = 0.9, 2]$.
- Generating the explanatory variable $x$ assuming it follows the normal distribution (0,1).
- Generating ε from Lognormal distribution with parameters (1.3,0.9), (1.7,1) and (2.1,1.5).
- Generating ε from Cauchy distribution with parameters (4,1), (6,2), and (2.5,1.2).
- Considering different sample sizes: 25, 50, 100 and 300.

- Computing The *Mean Square Error* (MSE) and the *Mean Absolute Error* (MAE) for each estimate using the following formulae:

$$MSE = \frac{1}{R}\sum_{R=1}^{R}\left(\hat{b} - b\right)^2, \qquad MAE = \frac{1}{R}\sum_{R=1}^{R}\left|\hat{b} - b\right|,$$

where R is the number of replications and $\hat{b}$ is the estimates calculated from the sample.

- The sampling runs 500 replications for each distribution with three different parameters and different sample sizes.
- All Computations of the simulation study are performed using a GAMS 45 program *(General Algebraic Modeling System).*

The results of the simulation study are summarized in the following tables:

**Table (1):** The estimate MSE and MAE for **Lognormal distribution,** the initial value ($\mathbf{b = 0.9}$).

| **Distributions** | **Sample sizes** | **Least Absolute Method ($L_1$-norm)** | | **Least Squares Method ($L_2$-norm)** | |
|---|---|---|---|---|---|
| | | **MSE** | **MAE** | **MSE** | **MAE** |
| **Lognormal (1.3,0.9)** | n=25 | **5.0168** | **0.2171** | **8.0857** | **0.2411** |
| | n=50 | **4.5935** | **0.2123** | **5.9332** | **0.2165** |
| | n=100 | **4.4936** | **0.2009** | **4.7839** | **0.2155** |
| | n=300 | **4.2832** | **0.1901** | **4.4944** | **0.1872** |
| **Lognormal (1.7,1)** | n=25 | **9.6819** | **0.2772** | **22.2916** | **0.3637** |
| | n=50 | **9.4248** | **0.2195** | **13.2451** | **0.3223** |
| | n=100 | **8.2414** | **0.2759** | **10.6850** | **0.3144** |
| | n=300 | **7.8132** | **0.2589** | **9.8037** | **0.2841** |
| **Lognormal (2.1,1.5)** | n=25 | **12.4034** | **0.2209** | **258.3723** | **1.0259** |
| | n=50 | **8.0434** | **0.2219** | **138.2197** | **0.8773** |
| | n=100 | **6.7555** | **0.2139** | **89.4391** | **0.8698** |
| | n=300 | **5.9642** | **0.2078** | **77.9893** | **0.7824** |

**Table (1):** shows the estimate of *Mean Square Error* (MSE) and the estimate of *Mean Absolute Error* (MAE) for the suggested quadratic regression approach using goal programming for **Lognormal distribution** with three different parameters (1.3,0.9), (1.7,1) and (2.1,1.5), four samples with different sizes n = 25 , 50 , 100 , 300 and Putting the initial values for coefficient regression (b = 0.9) depend on least absolute deviations ($L_1$-norm) and the square ($L_2$-norm) methods estimators.

**Table (2):** The estimate MSE and MAE for **Cauchy distribution,** the initial value ($\mathbf{b = 0.9}$).

| **Distributions** | **Sample sizes** | **Least Absolute Method ($L_1$-norm)** | | **Least Squares Method ($L_2$-norm)** | |
|---|---|---|---|---|---|
| | | **MSE** | **MAE** | **MSE** | **MAE** |
| **Cauchy (4,1)** | n=25 | **4.6934** | **0.2233** | **29.1275** | **0.3904** |
| | n=50 | **4.9650** | **0.1903** | **13.0725** | **0.2380** |
| | n=100 | **4.6340** | **0.1873** | **10.3500** | **0.4620** |
| | n=300 | **4.3594** | **0.1746** | **9.8149** | **0.1885** |
| **Cauchy (6,2)** | n=25 | **10.2514** | **0.2900** | **74.0454** | **0.5873** |
| | n=50 | **9.6895** | **0.2899** | **47.7821** | **0.4317** |
| | n=100 | **8.9973** | **0.2645** | **30.0623** | **0.2951** |
| | n=300 | **8.8912** | **0.2532** | **26.7026** | **0.2616** |
| **Cauchy (2.5,1.2)** | n=25 | **2.0915** | **0.1336** | **20.7721** | **0.1466** |
| | n=50 | **2.0215** | **0.1156** | **16.3761** | **0.2363** |
| | n=100 | **1.5039** | **0.1190** | **10.0027** | **0.1516** |
| | n=300 | **1.4824** | **0.1139** | **9.5249** | **0.1392** |

**Table (2):** shows the estimate of *Mean Square Error* (MSE) and the estimate of *Mean Absolute Error* (MAE) for the suggested quadratic regression approach using goal programming for **Cauchy distribution** with three different parameters (4,1), (6,2) and (2.5,1.2), four samples with different sizes n = 25 , 50 , 100 , 300 and Putting the initial values for coefficient regression (b = 0.9) depend on least absolute deviations ($L_1$-norm) and the square ($L_2$-norm) methods estimators.

**Table (3):** The estimate MSE and MAE for **Lognormal distribution,** the initial value (**b = 2**)**.**

| Distributions | Sample sizes | Least Absolute Method ($L_1$-norm) | | Least Squares Method ($L_2$-norm) | |
|---|---|---|---|---|---|
| | | MSE | MAE | MSE | MAE |
| **Lognormal (1.3,0.9)** | n=25 | **7.5830** | **0.2532** | **8.4308** | **0.2491** |
| | n=50 | **6.7739** | **0.2374** | **6.7694** | **0.2425** |
| | n=100 | **5.8699** | **0.2320** | **5.1051** | **0.2315** |
| | n=300 | **5.0857** | **0.2255** | **4.9681** | **0.2152** |
| **Lognormal (1.7,1)** | n=25 | **12.8772** | **0.3185** | **22.6376** | **0.3677** |
| | n=50 | **11.7340** | **0.3170** | **13.7150** | **0.3303** |
| | n=100 | **10.6618** | **0.3048** | **11.0629** | **0.3225** |
| | n=300 | **9.3598** | **0.3029** | **9.9105** | **0.2941** |
| **Lognormal (2.1,1.5)** | n=25 | **15.0087** | **0.3153** | **211.8054** | **1.0059** |
| | n=50 | **11.7333** | **0.3119** | **137.7702** | **0.8574** |
| | n=100 | **9.3083** | **0.2996** | **89.4391** | **0.8698** |
| | n=300 | **9.2084** | **0.2173** | **77.8765** | **0.7765** |

**Table (3):** shows the estimate of *Mean Square Error* (MSE) and the estimate of *Mean Absolute Error* (MAE) for the suggested quadratic regression approach using goal programming for **Lognormal distribution** with three different parameters (1.3,0.9), (1.7,1) and (2.1,1.5), four samples with different sizes n = 25 , 50 , 100 , 300 and Putting the initial values for coefficient regression (b = 2) depend on least absolute deviations ($L_1$-norm) and the square ($L_2$-norm) methods estimators.

**Table (4):** The estimate MSE and MAE for **Cauchy distribution,** the initial value ($\mathbf{b = 2}$).

| Distributions | Sample sizes | Least Absolute Method ($L_1$-norm) | | Least Squares Method ($L_2$-norm) | |
|---|---|---|---|---|---|
| | | MSE | MAE | MSE | MAE |
| Cauchy (4,1) | n=25 | **7.4195** | **0.2419** | **28.6747** | **0.3864** |
| | n=50 | **6.9544** | **0.2255** | **13.0029** | **0.2360** |
| | n=100 | **6.0450** | **0.2212** | **7.8778** | **0.2298** |
| | n=300 | **5.3395** | **0.2076** | **7.6011** | **0.1518** |
| Cauchy (6,2) | n=25 | **10.9760** | **0.2879** | **110.1976** | **0.7074** |
| | n=50 | **10.5707** | **0.2668** | **47.6696** | **0.4237** |
| | n=100 | **10.3349** | **0.2591** | **34.2531** | **0.2951** |
| | n=300 | **9.1665** | **0.2562** | **25.6412** | **0.2809** |
| Cauchy (2.5,1.2) | n=25 | **2.8041** | **0.1449** | **38.0966** | **0.3866** |
| | n=50 | **2.1596** | **0.1386** | **16.3761** | **0.2363** |
| | n=100 | **1.9374** | **0.1324** | **11.6014** | **0.1476** |
| | n=300 | **1.8993** | **0.1320** | **8.4922** | **0.1375** |

**Table (4):** shows the estimate of *Mean Square Error* (MSE) and the estimate of *Mean Absolute Error* (MAE) for the suggested quadratic regression approach using goal programming for **Cauchy distribution** with three different parameters (4,1), (6,2) and (2.5,1.2), four samples with different sizes n = 25 , 50 , 100 , 300 and Putting the initial values for coefficient regression (b = 2) depend on least absolute deviations ($L_1$-norm) and the square ($L_2$-norm) methods estimators.

According to the results in tables (1) to (4),

- The least absolute method is more efficient than the least square method estimators.
- The least absolute estimators of the regression coefficients give the smallest sum of absolute residual, which is efficient when the error distribution is heavy-tailed.

- One can observe that when the sample size increases the MSE decreases.
- It was noticed that the estimates of MAE are better than the MSE.

## 6. General Conclusion:

This section discusses the General Conclusion of the simulation study using of the two models under consideration; The least absolute method ($L_1$-norm) and least square method ($L_2$-norm) estimated or improved the parameters of the quadratic regression using goal programming when lognormal and Cauchy distributions, four samples with different sizes n = 25, 50, 100, 300, with three different parameters for each distribution, and putting the two initial values for coefficient regression [$b = 0.9 , 2$].

The comparison between the least absolute method ($L_1$-norm) and the least square method ($L_2$-norm) estimators were performed. In addition, the Mean Square Error (MSE) and Mean Absolute Error (MAE) are calculated.

The Conclusion of the various simulation studies carried out to compare the efficiency of the least absolute deviation method estimators and that the least square method estimators, indicate that the least absolute method is more efficient than least square method estimators. The least absolute estimators of the regression coefficients give the smallest sum of absolute residual, which is efficient when the error distribution is heavy-tailed, and it is more resistant to outliers in data. The least absolute estimator overcomes the problems of the least squares estimator concerning outliers because it is less sensitive to extreme errors. one can observe that, One can observe that when the sample size increases the MSE decreases. It was noticed that the estimates of MAE are better than the MSE.

Another point of view, with respect to the initial values *b*, it is noticed that; no mather how the *b* values change, the least absolute deviation method yielded better outcomes than the least squares method.

The general Conclusion demonstrated that the suggested Quadratic Regression Approach using Goal Programming which is the least absolute method is more efficient than the least square method in estimating the quadratic regression equation using lognormal and Cauchy distributions.

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