Stochastic Modeling of Reserve Uncertainty: Univariate and Bivariate Approaches in the Egyptian General Insurance Market

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Scientific Journal for Financial and Commercial Studies and Research (SJFCSR)

Faculty of Commerce – Damietta University

Vol.6, No.2, Part 1., July 2025

APA Citation

Yousef, S.; **Elsayed**, M. and **Khalil**, D. (2025). Stochastic Modeling of Reserve Uncertainty: Univariate and Bivariate Approaches in the Egyptian General Insurance Market, ***Scientific Journal for Financial and Commercial Studies and Research***, Faculty of Commerce, Damietta University, 6(2)1, 871-894.

Website: https://cfdj.journals.ekb.eg/

Stochastic Modeling of Reserve Uncertainty: Univariate and Bivariate Approaches in the Egyptian General Insurance Market

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Abstract

This study focuses on estimating and comparing reserve risk uncertainty using two actuarial approaches: the univariate stochastic Mack model and the bivariate stochastic chain ladder model. The analysis is applied to three major lines of business; motor, medical, and fire in an Egyptian general insurance company by utilizing R-program during the period from 2015 to 2024. The stochastic Mack model is used independently on each line to assess the reserve estimate and its associated uncertainty. Then, a bivariate stochastic chain ladder model is employed incorporating two lines of business. This bivariate approach enables actuaries to capture the inherent dependencies across different lines, resulting in more accurate and robust reserve estimates. By joint modeling, a better understanding of reserve dynamics allowing for the quantification of diversification benefits is provided. The findings demonstrate that recognizing cross-line dependencies result in less uncertainty estimates of reserves, which could improve the predictive performance and provide a more realistic assessment of reserve risk.

Keywords: Reserve uncertainty, diversification effect, stochastic reserving, bivariate chain ladder, mean square error of prediction.

1. Introduction

Although the classical chain ladder (CL) was the most used loss-reserving technique, the stochastic models that underlie the chain ladder technique have been extensively implied recently. The distributional free CL model assumes that the cumulative claims of different accident years are independent. These cumulative claims are linked with certain development factors to estimate the conditionally expected future claims within an appropriate stochastic framework. This is to ensure the accuracy and the quality of these estimates.

Accordingly, the mean square error of prediction (MSEP) is calculated incorporating both: 1-The conditional process variance: which represents the stochastic error and explains the variation within the stochastic model, 2-The parameter estimation error: that explains the uncertainty in the estimation of the parameters and the conditional expectation.

The general insurance companies usually operate in multiple business lines where the risks can be dependent. Using silo approach reserving - where the reserve and its estimates are calculated for each line of business separately - will lead to aggregate the estimates ignoring the dependencies across business lines within the company's portfolio. Thus, modelling the claims jointly would allow for the cross borrowing of the information, and hence, considering the right dependence structure along business lines would lead to correctly assessed predictive estimates of reserves. This would help in mitigating of risk margins as well as ensuring that capital is used efficiently while meeting solvency requirements. Therefore, the motivation for studying bivariate and multivariate reserving models would enhance the accuracy and reliability of reserve estimates.

Moreover, by incorporating joint variables (two lines of business), bivariate models can provide a more comprehensive analysis to actuaries, leading to better predictions. Additionally, insurance data can be high-dimensional, and thus, these models could be more effective than univariate models allowing actuaries to model relationships among joint variables simultaneously, and to assess the risks associated with different segments of a portfolio. This should be useful to optimize capital allocation and risk management strategies. Bivariate models can also help the insurer meeting the regulatory standards and solvency assessments by providing a more robust framework for estimating reserves.

Furthermore, these models facilitate scenario analysis and stress testing by allowing actuaries to examine how changes in one variable may impact other related variables and the overall reserves.

In this research, we focus on estimating the uncertainty of reserve risk using two actuarial methods: the univariate stochastic Mack model and the bivariate stochastic chain ladder model. The study is conducted on three lines of business of one of the largest general insurance companies in Egypt, they are: Motor, Medical, and Fire. The triangular data of the three lines is collected from year 2015 to year 2024. These lines of business are selected as they constitute a significant portion of the company's overall portfolio .Also, their contribution to both premium volume and claim activity has a direct impact on the company's profitability, making them strategically important for reserve risk analysis.

Therefore, the stochastic Mack model is first applied independently to each line to evaluate the reserve estimate and the associated risk uncertainty. This is to test the accuracy of the reserve estimates and their underlying uncertainties in the real practice. Then, a bivariate stochastic chain ladder model is utilized to estimate the dependency between pairs of lines of business. Using the bivariate approach would allow actuaries to account for the inherent dependencies and correlations across various lines, resulting in more precise and robust reserve estimations. By employing R programming for the analysis, using the chain ladder bivariate model affords a detailed perspective on reserve dynamics and facilitates the measurement of diversification benefits.

This research is organized to provide analysis of claim reserving techniques, Section 2 provides a review of the theoretical background and relevant literature. Section 3 outlines the Mack model and its underlying framework with presenting the results derived from its application. Section 4 extends the analysis by incorporating a bivariate modeling approach and presenting the results obtained. The findings are then summarized in Section 5. The study concludes in Section 6 along with practical recommendations and potential directions for future research.

2. Background

Deterministic loss reserving approaches such as chain ladder algorithm, Bornhuetter-Ferguson algorithm and Berquist-Sharman technique were expertly reviewed by Taylor (2000). These traditional simple techniques of claims modelling have been used as the foundation for the evolution of many stochastic models. Conventional deterministic algorithms are utilized to get a single central estimation of the outstanding claims liability. However, the claim liability stochastic nature is not considered.

Bornhuetter and Ferguson (1972) have developed the Bornhuetter-Ferguson algorithm which is one of the most commonly used techniques in practice. Unlike the deterministic chain ladder algorithm that employs the development factors to predict ultimate claims, the Bornhuetter-Ferguson algorithm employs the expected ultimate claims provided by specialists to predict the total outstanding claims.This is why the BF's estimates comprise both observations and experience. It could be considered as a more robust approach than the deterministic chain ladder approach, particularly when it comes to the fluctuations and instability in the proportions of ultimate claims that are paid in early development years.

Moreover, Berquist and Sherman (1977) created a technique to manage the consequences of changes in claim activities on the valuation of outstanding claims liabilities and the estimation of technical reserves. Despite the fact that Berquist-Sherman technique permits for changes in claim activities over time and as many adjustments and assumptions are implemented in this approach, it should be accomplished with a considerable degree of prudence and caution.

As mentioned by Mack (1993), deterministic chain ladder algorithm is accompanied by a number of weaknesses. The algorithm is contingent on the assumption that claims development patterns are the same over accident periods. Additionally, any variation in immature accident periods can lead to an ambiguous outstanding claims estimates. Mack developed a stochastic model to estimate the reserve and its accompanied uncertainty.

Friedland (2010) developed an excellent comprehensive illustration of deterministic techniques. He explained that there are two techniques that can be performed on the data; data rearrangement and data adjustment. This aims at obtaining data that is consistent and reliable.

In spite of the simplicity of deterministic models, it is very important for the insurer to consider and evaluate the uncertainty related to the point estimate of the outstanding claim liability. The negligence of the stochastic nature of the claim liability can result in consequential impacts on profits, accordingly can lead to a solvency issue.

Accordingly, the evolution of stochastic modeling techniques and the need to estimate uncertainty in reserves has increased. The first emergence of stochastic models in the loss reserving literature goes back to the 1980s with the outcome acquired by De Jong and Zehnwirth (1983). Based on the assumption that the claims are normally distributed, typically on the log scale, many researches cover stochastic models, (see: Verrall (1989, 1994); Ntzoufras and Dellaportas publication (2002); Atherino et al. (2010); De Jong (2006))

In all the previous silo approaches mentioned above, reserves and their anticipating variability are estimated for each business line individually. The bivariate models used for modeling claims from separate business lines have their privileges, they permit a cross-borrowing of data and communication that can enhance the accuracy and the exactness of outstanding claims valuation. The optimal comprehensive central estimate acquired from the joint modeling of claims from different business lines is not equivalent to adding up the optimal central estimates from the silo modeling. The reasons mentioned above have encouraged the evolution and the development of bivariate and multivariate stochastic loss reserving techniques for multiple dependent lines of business. This should be significantly important for risk management and regulatory requirements in insurance market.

In the following sections 3 and 4, both the univariate stochastic Mack model and bivariate chain ladder model are examined and results are presented and analyzed.

3. Stochastic Mack Model

In this section the stochastic Mack model is examined for the selected three lines of business: Motor comprehensive, Fire, and Medical using the triangular data collected from year 2015 to year 2024. The model framework is illustrated in subsection 3.1 followed by explanation of the results obtained in subsection 3.2.

3.1 Model Framework

Mack model is the first univariate stochastic model that underlies the CL technique. The model assumes that cumulative claims $C_{i,j}$ of different accident years are independent, and $C_{i,j}$ is a Markov chain where:

$$E[C_{i,j} \backslash C_{i,j-1}] = f_{j-1}\, C_{i,j-1} \tag{1}$$

$$V[C_{i,j} \backslash C_{i,j-1}] = \sigma^2{}_{j-1}\, C_{i,j-1} \tag{2}$$

where the parameters f_j and $\sigma^2{}_j$ can be estimated by:

$$\hat{f}_j = \frac{\sum_{i=0}^{I-j-1} C_{i,j+1}}{\sum_{i=0}^{I-j-1} C_{i,j}} \tag{3}$$

$$\hat{\sigma}_j^2 = \frac{1}{I-j-1} \sum_{i=0}^{I-j-1} C_{i,j} \left(\frac{C_{i,j+1}}{C_{i,j}} - \hat{f}_j\right)^2 \tag{4}$$

Here, it should be noted that the last variance parameter is not estimated by the same formula since the data may not have $I > J$, instead the following extrapolation that used by Mack could be used:

$$\hat{\sigma}_{J-1}^2 = \min\left\{ \hat{\sigma}_{J-2}^4 \big/ \hat{\sigma}_{J-3}^2 ;\ \hat{\sigma}_{J-3}^2 ; \hat{\sigma}_{J-2}^2 \right\} \tag{5}$$

This estimate is based on the fact that $\sigma_0, \sigma_1, \ldots\ldots\ldots. \sigma_{J-2}$ is a decreasing series.

Then it is important to derive an estimate for the conditional mean square error of prediction (MSEP) of $\widehat{C_{i,J}}^{CL}$, $1 \leq i \leq I$

For single accident year:

$$msep_{C_{i,J}\backslash\mathcal{D}_I}\left(\widehat{C_{i,J}}^{CL}\right) = E\left[\left(\widehat{C_{i,J}}^{CL} - C_{i,J}\right)^2 \backslash\mathcal{D}_I\right] = V\left(C_{i,J}\backslash\mathcal{D}_I\right) + (\widehat{C_{i,J}}^{CL} - E[C_{i,J}\backslash\mathcal{D}_I])^2 \qquad (6)$$

For aggregated accident years:

$$msep_{\sum C_{i,J}\backslash\mathcal{D}_I}\left(\sum \widehat{C_{i,J}}^{CL}\right) = E\left[\left(\sum \widehat{C_{i,J}}^{CL} - \sum C_{i,J}\right)^2 \backslash\mathcal{D}_I\right] \qquad (7)$$

Under Mack's model, the following formula is used to estimate the conditional MSEP of the ultimate liability for aggregated accident years:

$$\widehat{msep}_{\sum C_{i,J}\backslash\mathcal{D}_I}(\sum \widehat{C_{i,J}}^{CL}) = \sum_{i=1}^{I} \widehat{msep}_{C_{i,J}\backslash\mathcal{D}_I}\left(\widehat{C_{i,J}}^{CL}\right) + 2\sum_{1\le i<k\le I} \widehat{C_{i,J}}^{CL}\widehat{C_{k,J}}^{CL} \sum_{j=I-i}^{J-1} \frac{\hat{\sigma}_j^2 / \hat{f}_j^{\,2}}{S_j^{[I-j-1]}} \qquad (8)$$

3.2 The Results

For this analysis, we focus on three specific lines of business: motor comprehensive insurance, fire insurance, and medical insurance using the collected data from an Egyptian general insurance company. By employing R-program, we obtain the results for the CL reserves and MSEP for each line respectively as shown below.

- **Motor Comprehensive insurance:**

The application of the stochastic Mack model in motor comprehensive insurance supports the validity of the model assumptions. The fitted values align well with actual data, and residuals exhibit random behavior without major trends or clustering, suggesting a good model fit as shown in **Figure 1**.These diagnostics, combined with a low coefficient of variation for the IBNR confirm that the Mack model provides stable and reliable reserve estimates for this portfolio.

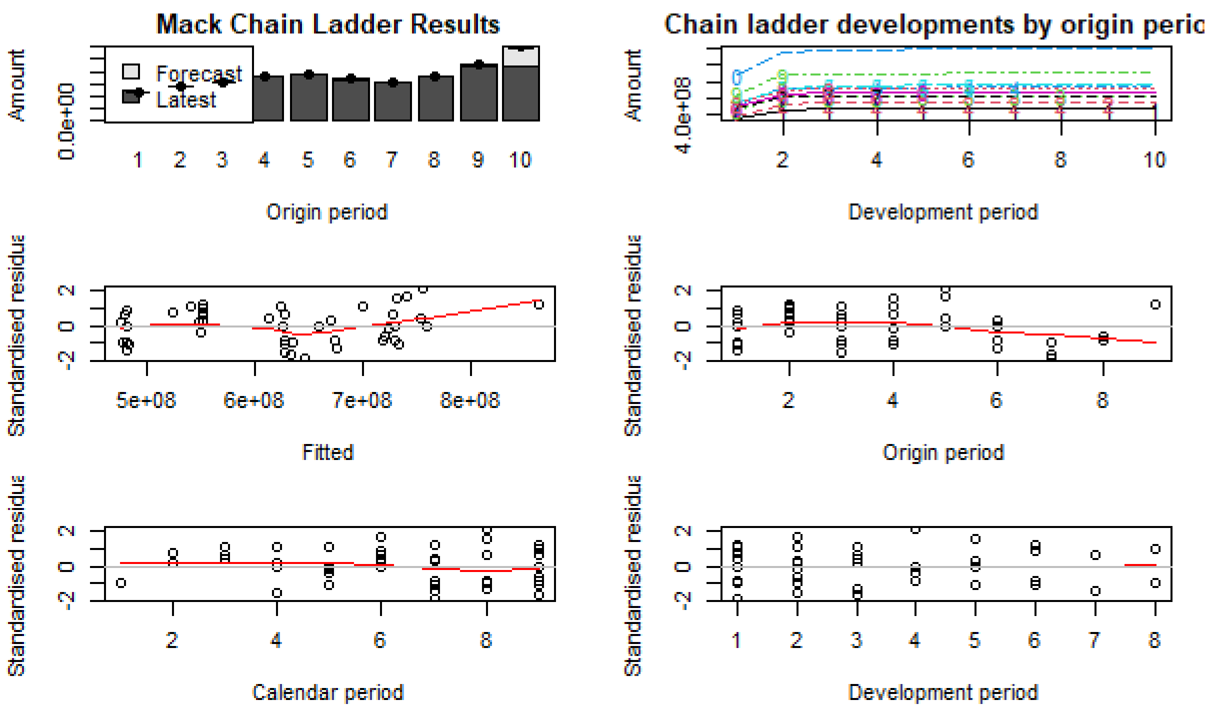

Figure 1: Results of Mack model on motor comprehensive insurance

Tables 1 and 2 show that the latest cumulative paid claims amount to approximately EGP 6.93 billion, whereas the estimated ultimate claims is EGP 7.28 billion. This implies an Incurred But Not Reported (IBNR) reserve of roughly EGP 346.6 million. The development factor of 0.95 indicates that the majority of claim development has already occurred, with only a small proportion of claims yet to emerge. The estimated standard error of the IBNR is approximately EGP 22.7 million reflecting the statistical uncertainty around the reserve estimate. The coefficient of variation (CV) for the IBNR (which is equivalent to MSEP estimate) is found to be 7% indicating a relatively low level of volatility in the reserve estimate. This low CV implies that the IBNR estimate is reasonably stable in this line and that the historical claims development pattern exhibits limited variability.

Table 1: Error of prediction (MSEP) on Motor comprehensive insurance

Latest dev. to-date	Ultimate	IBNR	Mack.S.E	CV(IBNR)
1 4.82e+08	1.000 4.82e+08	0	0	NaN
2 5.53e+08	1.001 5.53e+08	-503,556	144,248	-0.2865
3 6.27e+08	1.001 6.27e+08	-338,156	227,351	-0.6723
4 7.33e+08	1.001 7.33e+08	-548,057	326,806	-0.5963
5 7.59e+08	1.000 7.59e+08	-364,707	484,403	-1.3282
6 6.74e+08	0.998 6.76e+08	1,602,668	1,508,426	0.9412
7 6.30e+08	0.995 6.33e+08	2,868,256	1,546,890	0.5393
8 7.17e+08	0.990 7.24e+08	6,974,415	2,870,852	0.4116
9 8.79e+08	0.974 9.02e+08	23,008,632	7,476,100	0.3249
10 8.77e+08	0.736 1.19e+09	313,907,865	20,144,898	0.0642

Table 2: R-results on motor comprehensive insurance

	Totals
Latest:	6,931,414,382.07
Dev:	0.95
Ultimate:	7,278,021,740.30
IBNR:	346,607,358.23
Mack.S.E	22,712,439.68
CV(IBNR):	0.07

- **Fire insurance:**

In this line, it is found that there are indications of increased uncertainty, especially in recent origin years and higher claim sizes. The residuals show mild patterns and deviations in recent development years as shown in **Figure 2**.

The application of the Mack model to the fire insurance line reveals a significantly higher level of uncertainty compared to other lines of business. The latest cumulative paid claims amount to approximately EGP 4.36 billion, while the estimated ultimate claims are projected to be around EGP 5.62 billion. This results in an Incurred But Not Reported (IBNR) reserve of approximately EGP 1.25 billion which constitutes nearly 22% of the total ultimate claims, indicating that a substantial portion of claims development is still outstanding as presented in **Table 3.**

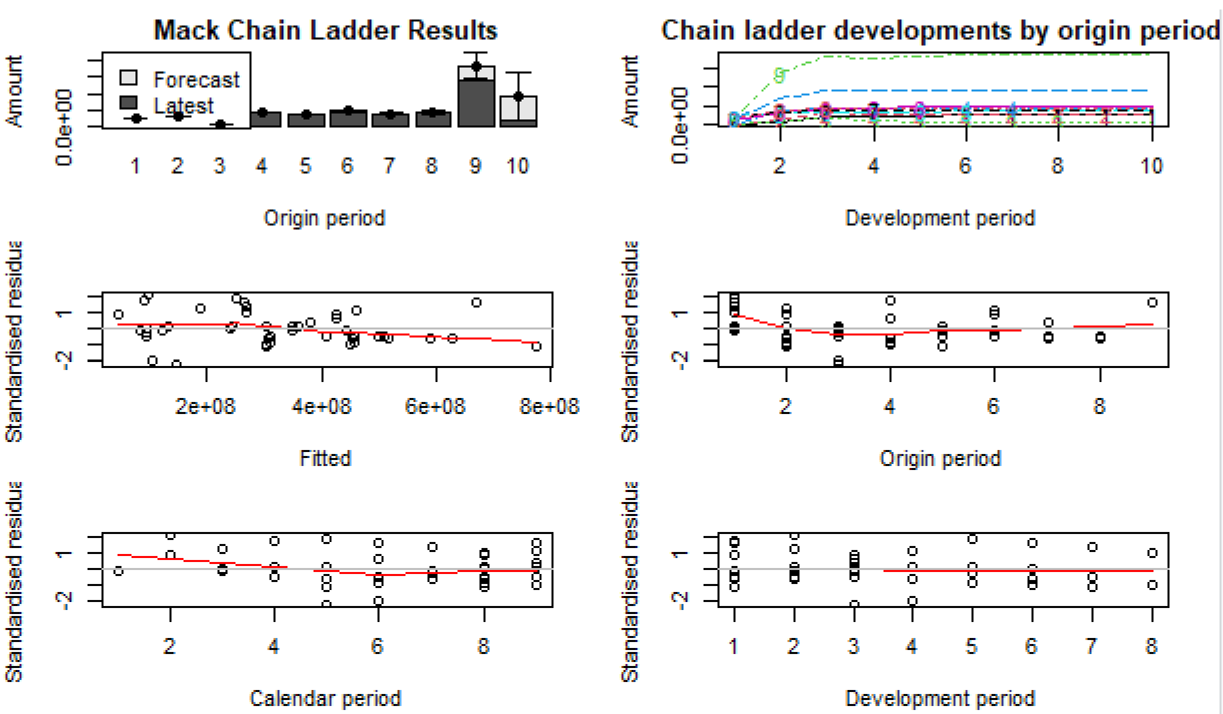

Figure 2: Results of Mack model on fire insurance

The calculated development factor indicates that only 78% of the total expected claims have been observed to date. This implies that fire insurance claims are developed more slowly and have longer tails compared to other lines.

The Mack standard error for the IBNR is approximately EGP 886 million, which is notably large in absolute terms. This is further emphasized by the coefficient of variation (MSEP) of 71%, indicating high relative volatility. The magnitude of this value suggests that the IBNR estimate is highly sensitive to changes in historical development patterns as shown in **Table 4**.

In practice, these results imply that the fire insurance portfolio carries considerable reserve risk. This highlights that a great attention in reserve estimation and risk management should be considered. Actuarial judgment, potential model validation, and scenario testing are essential for better assessment of the robustness of the results.

Table 3: Error of prediction (MSEP) on fire insurance

Latest dev. to-date	**Ultimate**	**IBNR**	**Mack.S.E**	**CV(IBNR)**
1 2.68e+08	1.000 2.68e+08	0	0	NaN
2 3.05e+08	1.000 3.05e+08	1.22e+05	5.55e+02	0.00455
3 9.40e+07	1.000 9.40e+07	3.88e+04	3.20e+03	0.08260
4 4.48e+08	0.999 4.49e+08	2.78e+05	1.17e+05	0.42193
5 3.61e+08	0.996 3.62e+08	1.31e+06	1.68e+06	1.28371
6 4.76e+08	0.977 4.88e+08	1.15e+07	1.28e+07	1.11748
7 3.90e+08	0.957 4.08e+08	1.73e+07	1.84e+07	1.06116
8 4.46e+08	0.971 4.60e+08	1.34e+07	4.43e+07	3.31525
9 1.38e+09	0.748 1.85e+09	4.65e+08	4.08e+08	0.87686
10 1.92e+08	0.205 9.35e+08	7.43e+08	7.38e+08	0.99221

Table 4: R-results on fire insurance

	Totals
Latest:	4,363,343,945.55
Dev:	0.78
Ultimate:	5,615,509,416.78
IBNR:	1,252,165,471.23
Mack.S.E	886,280,987.92
CV(IBNR):	0.71

- **Medical Insurance:**

The Mack chain ladder model provides a reasonable and well-fitting representation of the development patterns in the medical insurance data. **Figure 3** shows the absence of strong patterns in the residual plots, which implies that assumptions of the model hold reasonably well. However, predictions should be interpreted carefully for the latest origin years.

The application of the Mack chain ladder model to the medical insurance line provides results that indicate a relatively mature claims development process with moderate reserve uncertainty. As provided in **Tables 5 and 6**, the total latest cumulative paid claims is approximately EGP 6.38 billion whereas the total estimated ultimate claims is EGP 6.86 billion. This results in an IBNR reserve is equal to EGP 480 million which implies that nearly 7% of the total expected claims are yet to be reported or developed indicating a high degree of claims maturity.

The development factor supports this conclusion indicating that 93% of the claims have already been observed. This relatively high development level implies a shorter tail compared to other lines, where claims may take longer to be fully developed.

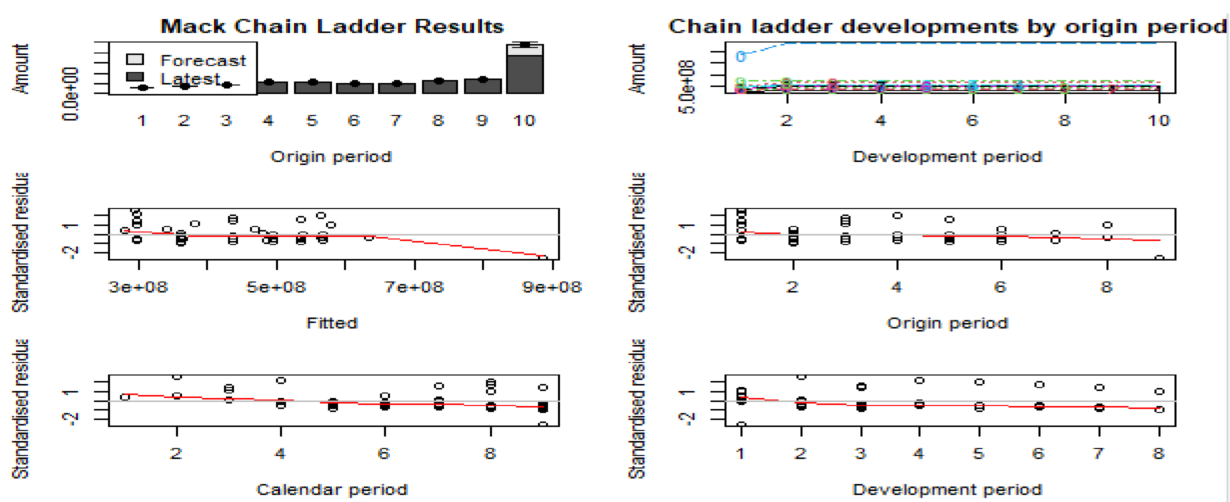

Figure 3: Results of Mack model on medical insurance

The Mack standard error of approximately EGP150 million is observed providing a relatively moderate level of uncertainty around the IBNR estimate in the context of the portfolio size. This is further confirmed by the coefficient of variation (CV) of 31%, which reflects moderate variability in the reserve estimate. Although some variability is still present due to model and process uncertainty, CV suggests a reasonable degree of confidence in the IBNR prediction.

Table 5: Error of prediction on medical line

Latest dev. to-date	**Ultimate**	**IBNR**	**Mack.S.E**	**CV(IBNR)**
1 2.98e+08	1.000 2.98e+08	0	0	NaN
2 3.63e+08	1.000 3.63e+08	0	0	NaN
3 4.31e+08	1.000 4.31e+08	1.19e-07	0	0
4 5.49e+08	1.000 5.49e+08	-1.09e+01	2.15e+01	-1.968
5 5.35e+08	1.000 5.35e+08	-6.99e+02	1.19e+03	-1.696
6 4.92e+08	1.000 4.92e+08	1.55e+04	3.61e+04	2.327
7 4.87e+08	1.000 4.87e+08	-4.03e+03	4.87e+04	-12.075
8 6.32e+08	1.001 6.32e+08	-3.34e+05	8.96e+05	-2.685
9 7.13e+08	1.006 7.09e+08	-4.38e+06	7.49e+06	-1.709
10 1.88e+09	0.795 2.36e+09	4.85e+08	1.50e+08	0.309

Table 6: R-results on medical insurance line

	Totals
Latest:	6,376,486,154.15
Dev:	0.93
Ultimate:	6,856,596,719.89
IBNR:	480,110,565.74
Mack.S.E	150,415,299.10
CV(IBNR):	0.31

4. The Bivariate chain ladder Model

4.1 Model Framework

By incorporating **joint variables** (two lines of business), the implementation of bivariate models could provide a more comprehensive view compared to the univariate model explained earlier. Calculating the reserve estimate and its underlying uncertainty using bivariate framework would expect to minimize the standard error leading to better predictions.

Therefore, we assume that the general insurer has $N>1$ lines of business, and the data collected is n triangles of the same size where:

n, $1 \leq n \leq N$, is the n^{th} line of business,

i, $0 \leq i \leq I$, is the accident year,

j, $0 \leq j \leq J$, is the development year.

The cumulative claims for accident year i, development year j, and n^{th} run-off triangle are given by: $C_{i,j}^{(n)} = \sum_{k=o}^{j} X_{i,k}^{(n)}$ where $X_{i,j}^{(n)}$ is zero for all j $> J$.

For accident year i and development year j the development factors are given by:

$$F_{i,j}^{(n)} = \frac{C_{i,j}^{(n)}}{C_{i,j-1}^{(n)}} \text{ where } F_{i,j} = (F_{i,j}^{(1)}, F_{i,j}^{(2)}, \dots . F_{i,j}^{(N)})'$$

Since the cumulative claims of different accident years are independent and we have N- dimensional vector of the development factors $f_j = (f_j^{(1)}, f_j^{(2)}, ,,,,, f_j^{(N)})'$ then:

- $E\left[C_{i,j} \setminus C_{i,j-1}\right] = D\left(f_{j-1}\right) C_{i,j-1},$
- $and\ Cov\ (C_{i,j}, C_{i,j} \setminus C_{i,j-1}) = D(C_{i,j-1})^{1/2} \sum_{j-1} D(C_{i,j-1})^{1/2}$

Where the conditional dependence between different lines of business is described by the correlation matrix $\sum_{j-1}$, where it directly links the cells that have the same coordinates across different run off triangles.

Accordingly and under the model assumptions we have:

$$E[C_{i,j} \setminus \mathcal{D}_I{}^N] = E\left[C_{i,J} \setminus C_{i,I-i}\right] = \prod_{j=I-i}^{J-1} D(f_j) C_{i,I-i} \quad (9)$$

The Conditional mean square error of prediction:

$$\widehat{msep}_{\sum_i \sum_n C_{i,J}^{(n)} \setminus \mathcal{D}_I{}^N}\left(\sum_{i=1}^{I} \sum_{n=1}^{N} \hat{C}_{i,J}^{(n)^{CL}}\right) = \sum_{i=1}^{I} \widehat{msep}_{\sum_n C_{i,J}^{(n)} \setminus \mathcal{D}_I{}^N}\left(\sum_{n=1}^{N} \hat{C}_{i,J}^{(n)^{CL}}\right) + 2 \sum_{1 \le i \le k \le I} 1' D(C_{i,I-i}) \left(\widehat{\Delta}_{i,J}^{(n,m)}\right)_{1 \le n,m \le N} D(C_{k,I-k}) \prod_{j=I-k}^{I-i-1} D(\hat{f}_j) 1 \quad (10)$$

Where: $\widehat{\Delta}_{i,J}^{(n,m)} = \prod_{l=I-i}^{J-1} \left(\hat{f}_l^{(n)} \hat{f}_l^{(m)} + \frac{\hat{\rho}_l^{(n,m)} \hat{\sigma}_l^{(n)} \hat{\sigma}_l^{(m)}}{S_l^{[I-l-1](n)} S_l^{[I-l-1](m)}}\right) \times$

$$\left(\sum_{k=0}^{I-l-1} \sqrt{\hat{C}_{k,l}^{(n)^{CL}} \hat{C}_{k,l}^{(m)^{CL}}}\right) - \prod_{l=I-i}^{J-1} \hat{f}_l^{(n)} \hat{f}_l^{(m)}, \quad (11)$$

$$\widehat{\Sigma_J} = D(\hat{\sigma}_j) \widehat{Cov}(\varepsilon_{i,j+1}, \varepsilon_{i,j+1}) D(\hat{\sigma}_j) \quad (12)$$

The unbiased estimator of $\sigma_j^2 = \left(\left(\sigma_j^{(1)}\right)^2, \dots \left(\sigma_j^{(N)}\right)^2\right)'$ is given by:

$$\hat{\sigma}_j^2 = \frac{1}{I-j-1} \sum_{i=0}^{I-j-1} (D(F_{i,j+1}) - D(\hat{f}_j^{(0)}))^2 C_{i,j} \quad (13)$$

The estimate $\widehat{Cov}(\varepsilon_{i,J}, \varepsilon_{i,J})^{(k)}$ for the covariance matrix $\widehat{Cov}(\varepsilon_{i,J}, \varepsilon_{i,J})$ is given by:

$$\widehat{Cov}(\varepsilon_{i,J}, \varepsilon_{i,J})^{(k)} = (\hat{\rho}_{J-1}^{(n,m)(k)})_{1 \le n,m \le N} \quad (14)$$

$$\hat{\rho}_{J-1}^{(n,m)(k)} = \frac{\hat{\varphi}_{J-1}^{(n,m)(k)}}{\hat{\sigma}_{J-1}^{(n)(k)} \hat{\sigma}_{J-1}^{(m)(k)}} \quad (15)$$

And for $1 \le n < m \le N$ the definition of $\hat{\varphi}_{J-1}^{(n,m)(k)}$ is:

$$\hat{\varphi}_{J-1}^{(n,m)(k)} = \min\left\{\left|\hat{\rho}_{J-2}^{(n,m)(k)} \hat{\sigma}_{J-2}^{(n)(k)} \hat{\sigma}_{J-2}^{(m)(k)}\right|, \left|\hat{\rho}_{J-3}^{(n,m)(k)} \hat{\sigma}_{J-3}^{(n)(k)} \hat{\sigma}_{J-3}^{(m)(k)}\right|, \frac{\left|\hat{\rho}_{J-2}^{(n,m)(k)} \hat{\sigma}_{J-2}^{(n)(k)} \hat{\sigma}_{J-2}^{(m)(k)}\right|^2}{\left|\hat{\rho}_{J-3}^{(n,m)(k)} \hat{\sigma}_{J-3}^{(n)(k)} \hat{\sigma}_{J-3}^{(m)(k)}\right|}\right. \quad (16)$$

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4.2 The Results

Employing the bivariate chain ladder model using the triangular data during collected from an Egyptian general insurance company during the period from 2015 to 2024; we select the following joint lines:

- Motor comprehensive insurance and fire insurance,
- Motor comprehensive insurance and Medical insurance,
- Fire insurance and Medical insurance.

For the development and execution of this model, the statistical programming language R is utilized to capture the dependency and estimate the error of prediction around the reserve estimate jointly for each two lines of business.

- **Motor comprehensive insurance and fire insurance**

The bivariate model is applied to the combined motor and fire insurance data. As shown in **Table 7**, the total latest cumulative claims is approximately EGP 11.4 billion, with an ultimate estimate of EGP 13.1 billion, resulting in an IBNR of EGP 1.69 billion. The CV of the IBNR is 33%, which indicates a moderate level of uncertainty in the reserve estimates across the portfolio. These results underscore the usefulness of the bivariate model in capturing the interdependencies between motor and fire lines, especially in the more recent accident years where data availability is limited and uncertainty is higher.

Table 7: Summary statistics for triangle motor + fire

Latest dev. to-date	**Ultimate**	**IBNR**	**S.E**	**CV**
1 7.51e+08	1.000 7.51e+08	0	0	0
2 8.69e+08	1.000 8.69e+08	1.30e+05	1.29e+04	0.0987
3 7.78e+08	1.000 7.79e+08	2.52e+05	2.46e+05	0.9779
4 1.18e+09	1.000 1.18e+09	5.16e+05	3.00e+05	0.5810
5 1.12e+09	0.999 1.12e+09	1.68e+06	1.74e+06	1.0384
6 1.15e+09	0.989 1.16e+09	1.33e+07	1.29e+07	0.9716
7 1.02e+09	0.983 1.04e+09	1.82e+07	1.47e+07	0.8072
8 1.16e+09	0.972 1.20e+09	3.37e+07	1.84e+07	0.5457
9 2.26e+09	0.807 2.80e+09	5.39e+08	4.06e+08	0.7531
10 1.07e+09	0.497 2.15e+09	1.08e+09	7.55e+08	0.٤966
Total 1.14e+10	0.871 1.31e+10	1.69e+09	8.97e+08	0.٣303

- **Motor comprehensive insurance and Medical insurance**

As shown in **Table 8**, the application of the bivariate model to the joint motor and medical insurance data provides the following results: the total latest cumulative claims is approximately EGP13.3 billion, with an estimated ultimate claims value of EGP14.2 billion, resulting in an IBNR of EGP 860 million. The CV is relatively low at 17.7% indicating a high degree of reliability in the reserve estimates for this combined portfolio.

Table 8: Summary statistics for triangle motor + medical

Latest dev. to-date	Ultimate	IBNR	S.E	CV
1 7.81e+08	1.000 7.81e+08	0	0	0
2 9.17e+08	1.000 9.17e+08	3.68e+03	1.28e+04	3.489
3 1.06e+09	1.000 1.06e+09	1.89e+05	2.46e+05	1.300
4 1.30e+09	1.000 1.30e+09	2.45e+05	2.75e+05	1.124
5 1.30e+09	1.000 1.30e+09	5.00e+05	4.12e+05	0.824
6 1.17e+09	0.998 1.17e+09	2.39e+06	1.49e+06	0.624
7 1.12e+09	0.997 1.12e+09	3.62e+06	1.53e+06	0.424
8 1.35e+09	0.994 1.36e+09	8.05e+06	2.88e+06	0.357
9 1.59e+09	0.984 1.62e+09	2.55e+07	7.71e+06	0.302
10 2.75e+09	0.771 3.57e+09	8.20e+08	1.52e+08	0.185
Total 1.33e+10	0.939 1.42e+10	8.60e+08	1.52e+08	0.177

- **Fire insurance and Medical insurance**

The joint modelling of fire and medical insurance reveals total latest cumulative claims of EGP10.8 billion, with an ultimate claims estimate of EGP12.7 billion as shown in **Table 9**. This implies a total IBNR of EGP1.84 billion, and a CV of 49.32% that indicates a moderate to high level of uncertainty in the aggregate reserve estimate for this portfolio.

Table 9: Summary statistics for triangle fire + medical

Latest dev. to-date	Ultimate	IBNR	S.E	CV
1 5.67e+08	1.000 5.67e+08	0	0	0
2 6.79e+08	1.000 6.80e+08	1.27e+05	4.32e+02	0.0034
3 5.87e+08	1.000 5.87e+08	6.25e+04	3.72e+03	0.0596
4 1.01e+09	1.000 1.01e+09	2.71e+05	1.19e+05	0.4380
5 8.98e+08	0.999 8.99e+08	1.18e+06	1.69e+06	1.4346
6 9.71e+08	0.989 9.82e+08	1.09e+07	1.28e+07	1.1729
7 8.77e+08	0.984 8.92e+08	1.46e+07	1.46e+07	0.9980
8 1.08e+09	0.976 1.11e+09	2.61e+07	1.82e+07	0.6947
9 2.10e+09	0.802 2.61e+09	5.17e+08	4.06e+08	0.7858
10 2.07e+09	0.619 3.34e+09	1.27e+09	7.70e+08	0.6045
Total 1.08e+10	0.855 1.27e+10	1.84e+09	9.09e+08	0.4932

Summary statistics for the combined triangles of the pairs of insurance lines mentioned above show that the prediction errors have decreased to 33%, 17.7%, and 49.3%, respectively. It is noted that these values are lower than the weighted averages of the individual prediction errors for the corresponding lines when modeled separately. This reduction in predictive uncertainty underscores the diversification benefit derived from the joint modeling of reserves. By accounting for the correlations between lines of business, the bivariate approach yields to a more stable and robust reserve estimates, aligning with the risk aggregation principles promoted under IFRS 17.

5. Analysis

The Mack Chain Ladder model is applied to three key lines of business, motor, fire, and medical insurance, to estimate incurred but not reported (IBNR) claims and assess reserve uncertainty. Implementing the Mack model – which is considered the most common model used in practice – provides a significant method to estimate the reserve and its sufficiency under uncertainties to ensure the company's solvency in different lines of business.

For motor insurance, the estimated IBNR amounts to EGP 347 million with a low CV of 7% indicating high model precision and limited uncertainty. The figures demonstrate a stable development pattern with residuals randomly scattered. In contrast, fire insurance exhibits a significantly higher IBNR of EGP 1.25 billion and a CV of 71% reflecting greater volatility and higher uncertainty in reserve estimates. The figures show variability in the development and residual patterns, particularly in the later origin periods. This suggests that caution is needed when interpreting these results. For medical insurance, the IBNR reaches nearly EGP 480 million with a moderate CV of 31%. The development patterns are relatively smooth and the residual plots show no major violations of model assumptions. This indicates a reliable but moderately uncertain reserve estimate.

Overall, the Mack model provides a reasonable fit across all three lines under study. It is found that there are various degrees of uncertainty among them, with the motor being the most stable and the fire is the most volatile. This is aligned with the fact that the claims of motor insurance are usually frequent with minor and predictable losses. On the other hand, claims of fire insurance are less frequent and vary in their severity. This leads to a longer tail of claims settlement especially with large losses.

Alternatively, the application of bivariate model implies a reduction in prediction error which reflects a clear diversification benefit achieved through jointly estimating reserves. Considering the interdependencies between different lines of business, a particularly low CV of 17.7% is recorded in the motor and medical lines, 33% for the motor and fire lines, and 49.3% for the fire and medical lines. This indicates a high degree of stability of the development patterns between motor and medical lines. Furthermore, in combinations involving more volatile lines, such as fire insurance, the joint model delivers an appropriate level of stability compared to the results obtained from the univariate model.

These findings confirm the statistical advantages of bivariate modelling and directly support the regulatory perspectives by producing more robust and economically realistic reserve estimates, which can ultimately lead to more capital efficiency and accurate solvency projections.

6. Conclusion and recommendation

This study aims at examining reserve uncertainty estimate by applying both the stochastic Mack model and bivariate chain ladder approach to three lines of business; Motor, Medical, and Fire, within an Egyptian general insurance company. The stochastic Mack model provides individual IBNR estimates and associated prediction errors for each line. However, by adopting a bivariate approach, that jointly models these lines; the analysis demonstrates a notable reduction in prediction error due to the interdependencies between lines and their diversification effect. The results prove that using bivariate modeling better captures the underlying dependency structures between lines of business. Ultimately, this produces more stable and reliable reserve estimates. Moreover, this approach aligns with the risk aggregation and diversification principles within the new regulations issued by Financial Regulatory Authority regarding the application of the standards of IFR17.

The research recommends that insurers should consider incorporating bivariate models in their reserving framework when managing multiple lines of business with potential dependencies. This should improve reserve accuracy and reflect the diversification benefit. Additionally, insurers should utilize methods that explicitly capture interdependencies across portfolios to enhance the credibility of the reserve evaluation process, especially with the increasing regulatory focus on risk aggregation under IFRS 17.

Thus, future research may extend the univariate and bivariate framework of estimating the reserve and its underlying uncertainty to multivariate modelling. Incorporating different lines of business would allow for more comprehensive assessment of diversification effects across the entire portfolio. Additionally, further enhancements may involve imbedding dynamic factors such as: time into the model to better capture temporal variations in estimating reserves.

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استخدام النماذج العشوائية لتقديرعدم اليقين في الاحتياطيات: مناهج أحادية وثنائية المتغير في سوق التأمين العام المصري

المستخلص

تركّز هذه الدراسة على تقدير ومقارنة عدم اليقين المرتبط بمخاطر الاحتياطيات باستخدام منهجين اكتواريين: نموذج Mack العشوائي الأحادي المتغير، ونموذج chain ladder العشوائي الثنائي المتغيرBivariate chain ladder. تم تطبيق التحليل على ثلاث فروع رئيسية من التأمين: السيارات، والتأمين الطبي، والتأمين ضد الحريق في إحدى شركات التأمينات العامة المصرية، وذلك باستخدام برنامج R للفترة من ٢٠١٥ إلى ٢٠٢٤. تم استخدام نموذج Mack العشوائي على كل فرع تأمين بشكل مستقل لتقدير قيمة الاحتياطي وعدم اليقين المصاحب له، ثم تم تطبيق نموذج chain ladder العشوائي الثنائي المتغيرBivariate chain ladder بدمج كل فرعين تأمين معًا. يتيح هذا المنهج الثنائي للاكتواريين التقاط الترابطات الكامنة بين فروع التأمين المختلفة، مما يؤدي إلى تقدير الاحتياطى بشكل ادق. النمذجة المشتركة تؤدى الى فهم أعمق لديناميكيات الاحتياطي، مع إمكانية قياس تأثيرات التنويع. تُظهر النتائج أن أخذ الترابطات بين الفروع المختلفة في الاعتبار يؤدي إلى تقليل عدم اليقين في تقديرات الاحتياطي، مما يُحسّن من الأداء التنبؤي ويوفّر تقييماً أكثر واقعية لمخاطر الاحتياطي.

الكلمات المفتاحية:
عدم اليقين في الاحتياطيات، أثر التنويع، النماذج العشوائية لتقدير الاحتياطى، التسلسل السلمى، متوسط مربع خطأ التنبؤ.