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An investigation of the Nuclear Structure of Even-Even $^{156-170}$Er Isotopes

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ABSTRACT

With the use of the Interacting Boson Model (IBM-1), our objective is to investigate the possibility of describing the nuclear structure of erbium isotopes ranging from ^{156}Er to ^{170}Er. In order to study the nuclear structure of particular atomic nuclei, we considered three fundamental dynamic symmetries in the model. There is a significant alignment that exists between the calculated energy levels and the experimental data gathered for some erbium isotopes. In addition to this, we calculated and compared the theoretical calculations of the model with the probability of reduced electromagnetic transitions B(E2). The study's results show that the nucleus ^{156}Er may have features of the U(5) dynamic symmetry, while the nuclei $^{158, 160, 170}$Er may be considered as SU(3) dynamic symmetry. The erbium isotopes $^{156 - 160}$Er represent an example of the U(5)–SU(3) shape phase transition and exhibit a transition of nuclear shapes from spherical to axially symmetric deformed forms.

1. INTRODUCTION

Nuclear physicists used the Interacting Boson Model (IBM) as a framework to describes the collective excitations of atomic nuclei. The IBM is a powerful way to study nuclear structure and collective phenomena by looking at bosonic degrees of freedom. Specifically, it examines the interaction between pairs of nucleons, explaining it through collective bosonic excitations.

Early in the 1970s, scientists proposed the Interacting Boson Model as an alternative to the complex shell model of nuclear structure, which describes neutrons and protons as individual particles in quantized orbits. The IBM focuses instead on collective modes of excitation, which are often more relevant in understanding many nuclear properties [1-2].

IBM-1 is the first version of this model, which does not distinguish between the degrees of freedom of protons and neutrons. IBM has achieved significant progress in elucidating the characteristics of several atomic nuclei, especially those that exhibit collective phenomena like rotational and vibrational motion. There are certain limits in the IBM's capacity to describe the behavior of nuclei under situations of high excitation energy, which is when single-particle excitations become prominent [3-5].

Different symmetries are responsible for maintaining the Hamiltonian, which controls bosonic interactions [1]. The algebraic framework of the IBM is based on the Lie algebra associated with the group U(6). IBM-1 uses the U(6) algebra to describe the Hilbert space of available bosonic states. The bosons obey certain commutation relations that are similar to those in other systems of quantum mechanics [2].

For the purpose of constructing the nucleus, the IBM-1 version adds two distinct types of bosons: s-bosons, which are characterized by L=0 (monopole), and d-bosons, which are characterized by L=2 (quadrupole) [6, 7].

Combining the amplitudes of several direct product states into a single nuclear eigenstate is one method to produce collective phenomena. The three main models that IBM used to explain nuclear collectivity were the spherical vibrator U(5), the axially symmetric rotor SU(3), and the gamma-soft O(6) dynamic symmetries [1, 7, 8].

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We refer to the symmetries of the Hamiltonian as dynamical symmetries [2] because they originate from the symmetry of the Lie algebra. The study of dynamical symmetries has recently attracted a lot of attention from researchers. This is primarily because these symmetries have the potential to reveal systems that are quite complicated. To fully understand the quantum many-body system, it is important to look at deformed nuclei in the rare-earth mid-shell region, such as erbium isotopes [4].

Using dynamic symmetries to understand complicated systems can give us a lot of information and help us make analytical guesses about energies and transition rates [5]. The nuclear force builds a potential that allows the bosons to interact with each other. We use a mix of algebraic and group-theoretical approaches to characterize their interactions [2].

Through the use of this model, it is possible to examine medium and heavy nuclei that are defined by an even number of protons and neutrons. IBM has made major contributions to the description of the characteristics of atomic nuclei that exhibit collective phenomena, such as vibrational and rotational motion [10-12].

This study aims to examine the nuclear structure of the erbium isotopes ^{156}Er to ^{170}Er. We compute the energy of the low-lying states and the reduced electromagnetic transition ratio B(E2) for these nuclei utilizing IBM calculations. According to IBM-1, there is a possibility that the spectra of lighter isotopes, such as ^{156}Er, will exhibit vibrational characteristics with a dominating U(5) symmetry. On the other hand, as we study heavier isotopes, one can witness a nuclear shape phase transition from spherical to axially symmetric deformed forms.

2. Testing criteria for $^{156-170}$Er isotopes

In order to identify candidate nuclei to predict one of the basic dynamic symmetry of the IBM-1, they should meet all the following criteria:

1) It has at least nine levels with known spin and parity.
2) All of the levels predicted by IBM-1 are within the experimental sensitivity.
3) Appropriate justifications must be provided for levels that lie outside the model space.
4) Meeting the electromagnetic transition predicts made by IBM-1.
5) One of the IBM-1 formulas might be a good representation of the energy levels, namely equation (1) for vibrators, equation (2) for a deformed rotor, and equation (3) for γ -unstable nuclei [13-15].

$$E(U(5)) = \varepsilon n_d + \alpha n_d(n_d + 4) + 2\beta\tau(\tau + 3) + 2\gamma L(L + 1), \quad \textbf{(1)}$$

where n_d , τ and L are the quantum numbers for the number of d -bosons, the d -boson seniority, and the level spin, respectively. The factors ε , α , β and γ are adjustable parameters.

$$E(SU(3)) = E_0 - k[\lambda(\lambda + 3) + \mu(\mu + 3) + \lambda\mu - 2N(2N + 3)] + k'L(L + 1), \quad \textbf{(2)}$$

In this context, λ and μ serve as the quantum numbers that classify the rotational states, N signifies the total number of bosons, and L represents the level spin. The parameters k and k' function as adjustable variables. The equation formulated by IBM-1 for γ-unstable nuclei is,

$$E(O(6)) = E_0 + \frac{A}{4}(N - \sigma)(N + \sigma + 4) + B\tau(\tau + 3) + CL(L + 1), \quad \textbf{(3)}$$

The quantum numbers σ, τ, N, and L stand for the number of d-bosons, the seniority of the d-bosons, the total number of bosons, and the level momentum, respectively. Parameters A, B, and C are all controllable parameters.

Table (1): The model parameters for the even-even isotopes of $^{156-170}$Er.

		$U(5)$				$O(6)$			$SU(3)$	
	$R_{4/2}$	ε	α	β	γ	A	B	C	k	k'
$^{156}_{68}Er$	2.31	435.759	−2.138	−1.3926	2.4716	118.591	53.132	8.9539	7.58087	28.869
$^{158}_{68}Er$	2.74	351.6696	0.0926	1.5897	−0.649	108.124	44.729	5.092	6.0213	21.545
$^{160}_{68}Er$	3.10	204.99	26.727	−3.441	−2.041	----	47.137	0.113	5.962	17.164
$^{162}_{68}Er$	3.23	521.066	−14.588	2.792	−6.308	103.927	52.365	−4.463	6.008	14.688
$^{164}_{68}Er$	3.28	290.771	30.616	−8.9007	−5.252	88.518	57.679	−8.516	5.373	14.008
$^{166}_{68}Er$	3.29	98.859	80.759	−18.673	−8.774	107.087	63.95	−13.849	5.645	10.4
$^{168}_{68}Er$	3.31	240.732	38.412	−9.901	−5.996	80.227	56.07	−9.413	4.638	12.309
$^{170}_{68}Er$	3.31	356.538	0.71447	1.437	−4.885	63.481	47.566	−4.355	3.812	13.723

Table 1 presents the adjustable parameters for the $^{156\text{-}170}$Er isotopes. We employ two relevant values to evaluate the level of concordance between the observed energy levels and the fitted energy levels for each nucleus. The first one is the mean absolute deviation Δ,

$$\Delta = \frac{1}{N_L}\sum_i^{N_L}\left|E_i^{ex} - E_i^{fit}\right|, \qquad (4)$$

where E_i^{exp} and E_i^{fit} are the experimental and best-fit energies in KeV of the i^{th} level while N_L is the number of levels. The second is the quality factor defined by

$$Q = \frac{W_i}{N_L - b}\sum_i\left(E_i^{exp} - E_i^{fit}\right)^2, \qquad (5)$$

In this context, we select $W_i = 0.01$ as the weighting factor, corresponding to a uniform uncertainty of 10 KeV for the energy levels [6, 7, 8, 12, 13]. Table 2 Provide three sets of fitted energy levels for the even-even erbium isotopes $^{156\text{-}170}$Er, taking into account the quality factor Q and the absolute average deviation Δ.

We quantify the nucleus to belong to certain dynamical symmetry if $Q \leq 150$ and $\Delta \leq 100$ [7].

3. The calculation of the number of bosons N

The IBM-1 views the nucleus as a core, encircled by a number of N bosons. To calculate the number of bosons, we consider the number of protons or neutrons required to reach the nearest closed shell. We consider the number of protons N_p closest to the shell—either complementary or excess—and also calculate the number of neutrons N_n that are superfluous or complementary to the shell [1-4]. Hence the number of bosons:

$$N = \frac{N_n + N_p}{2}, \qquad (6)$$

4. P- value

It measures the strength of the interaction between valence protons and neutrons. Valence nucleons denote the number of protons or neutrons required to reach the nearest closed shell. Their interaction is the primary cause of deformation. High P-value indicates that valence protons and neutrons exhibit strong interactions, resulting in more nuclear deformation. This occurs because protons and neutrons are more inclined to deviate from their equilibrium locations and assume a more irregular configuration. Equation (7) clearly illustrates the p-n interaction through the utilization of the P-factor, as noted in reference [16-18].

$$P = \frac{N_n N_p}{N_n + N_p}, \qquad (7)$$

The collectivity and the beginning of deformation are predicted to be significantly affected by the numbers of valence protons and neutrons, N_p and N_n, respectively, as the P-factor increases [19-22].

Table (2): The number of bosons N, the p- value, the absolute average deviation (Δ) and quality factor (Q) for three sets of calculated energy levels for even-even Erbium $^{156\text{-}170}$Er isotopes.

Nucleus			$U(5)$		$O(6)$		$SU(3)$	
	N	P	Δ	Q	Δ	Q	Δ	Q
$^{156}_{68}Er$	10	4.2	**83.27**	**114.87**	139.89	443.68	132.39	235.958
$^{158}_{68}Er$	11	5.09	119.33	260.36	124.01	338.19	**57.51**	**47.72**
$^{160}_{68}Er$	12	5.83	131.78	320.37	147.01	455.16	**46.04**	**31.11**
$^{162}_{68}Er$	13	6.46	210.54	1010.45	205.33	906.80	136.25	381.80
$^{164}_{68}Er$	14	7.00	148.21	367.93	116.20	273.10	139.68	392.51
$^{166}_{68}Er$	15	7.47	179.02	661.73	132.97	352.36	236.49	934.82
$^{168}_{68}Er$	16	7.88	142.38	347.18	109.28	248.48	147.69	386.17
$^{170}_{68}Er$	17	8.235	150.83	414.48	147.91	578.34	**80.85**	**99.19**

Table (3): A comparison between the experimental data with calculated B(E2) values for U(5), SU(3), and O(6) for the even-even $^{156-170}$Er isotopes.

Nucleus	N	$R_{4/2}$	R_E	R $U(5)$	R $O(6)$	R $SU(3)$
$^{156}_{68}Er$	10	2.31	**1.78±0.12**	**1.800**	1.377	1.398
$^{158}_{68}Er$	11	2.74	**1.44±0.11**	1.818	1.385	**1.403**
$^{160}_{68}Er$	12	3.10	**1.43±0.07**	1.833	1.391	**1.407**
$^{162}_{68}Er$	13	3.23		1.846	1.396	1.410
$^{164}_{68}Er$	14	3.28	1.26±0.15	1.857	1.400	1.412
$^{166}_{68}Er$	15	3.29	1.44±0.06	1.867	1.404	1.414
$^{168}_{68}Er$	16	3.31	1.50±0.05	1.875	1.406	1.416
$^{170}_{68}Er$	17	3.31		1.882	1.409	1.417

5. Transition ratio

Collective motion within the nucleus can be better understood with the help of the electric quadrupole transition strengths, which are frequently represented by the symbol B(E2). To a large extent, the values of B(E2) will be predictive of changes that occur as the nuclei undergo gradual deformation [6,12,13]. It is employed to ascertain the transition between low-lying energy levels. Through the analysis and comparison of the ratio, we can identify the collective characteristics of the selected nuclei.

$$R = \frac{B(E2;4_g^+ \rightarrow 2_g^+)}{B(E2;2_g^+ \rightarrow 0_g^+)}, \qquad \textbf{(8)}$$

the theoretical values for the three dynamical symmetry limits are:

1- For the $U(5)$ limit :

$$R = \frac{2(N-1)}{N}, \qquad \textbf{(9)}$$

2-For the $SU(3)$ limit :

$$R = \frac{10}{7}\frac{(2N^2+3N-5)}{(2N^2+3N}, \qquad \textbf{(10)}$$

3-For the $O(6)$ limit :

$$R = \frac{10}{7}\frac{(N^2+4N-5)}{(N^2+4N}, \qquad \textbf{(11)}$$

The B(E2) transition ratio between low-lying levels is computed for the isotopes $^{156-170}$Er. The experimental data is compared with the predictions produced by the three dynamical symmetry thresholds. The results of the comparisons conducted are presented in Table 3. In certain instances, the alignment between the calculated results and the experimental data does not meet expectations for the model. Nonetheless, three nuclei exhibit a close alignment with the predictions derived from the model's data. Specifically, ^{156}Er approaches the U(5) limit, with ^{158}Er and ^{160}Er corresponding to the SU(3) limit, exhibiting an error margin of less than 0.04 (see Table 3).

6. Discussion

This section examines the isotopes of Erbium, specifically $^{156-170}$Er, characterized by a proton number Z=68 and a neutron number varying from 88 to 102. The $R_{4/2}$ value ranges from 2.31 to 3.31 (see Table 1), while the P value extends from 4.2 to 8.235 (see Table 2). All nuclei, except for the ^{156}Er and ^{158}Er nuclei, shows significant deformation.

Figure 1 illustrates the relation between the neutron number, the $R_{4/2}$ ratio, and the P-values for the isotopes of $^{156-170}$Er. It highlights the transition of vibrational nuclear structure to rotational one, predicting nuclear phase transition.

The nucleus ^{156}Er exhibits U(5) dynamical symmetry, as indicated in Table 4, with Q = 114.87 and Δ = 83.27. The calculated quadruple transition ratio, $R(U(5)) = 1.80$, closely aligns with the experimental value, $R_E = 1.78 \pm 0.12$.

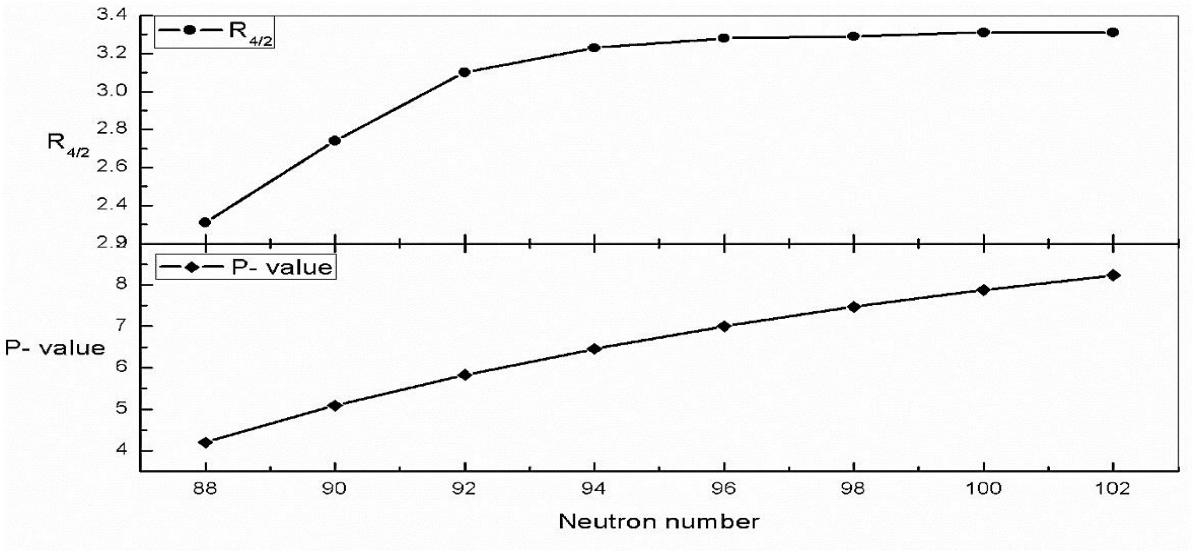

Fig. (1): The relationship between the neutron number, the ratio $R_{4/2}$, and the P-values for the Erbium $^{156-170}$Er isotopes.

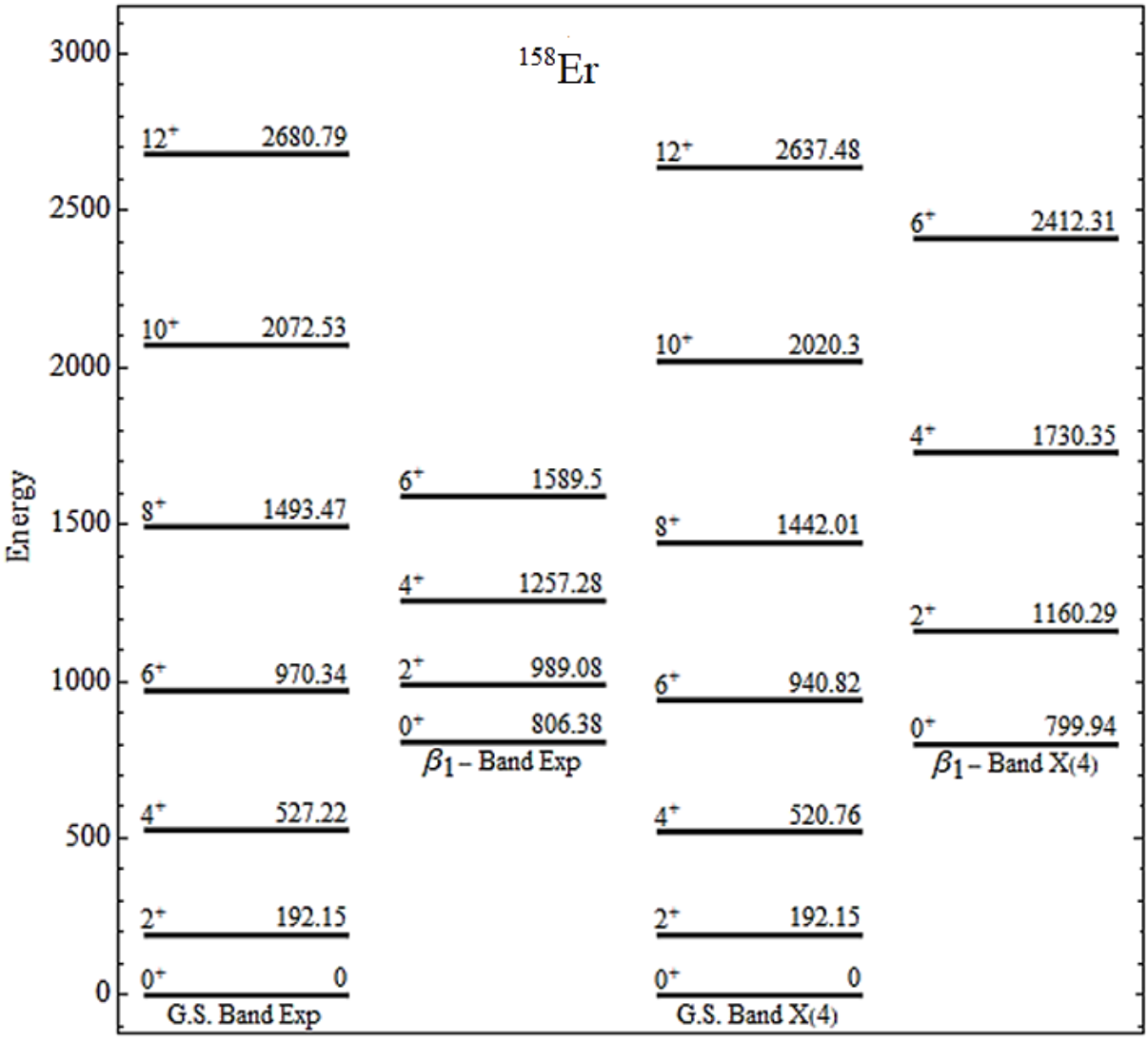

Fig. (2): A comparison between the experimental energy levels and the X(4) model energy levels

For the ^{158}Er nucleus, based on its Q and Δ values (Table 4), this nucleus fits the SU(3) dynamic symmetry very well. However, references [23, 24, 25] support Budaca's research, indicating that it is a transitional nucleus of type X(4) which can describe the shape phase transition from spherical to axially symmetric shapes. For transition nuclei of type X(4), its $R_{4/2}$ = 2.71, and for the nucleus ^{158}E, $R_{4/2}$ = 2.744, with most of its energy levels produce those of X(4) except some of the energy levels of the beta 1 band as shown in Figure 2. Also, the value for quadruple transition ratio for X(4) model ($R_{X(4)}$ = 1.7) which is far away from the experimental value for ^{158}Er. For this reason we consider this nucleus as SU(3) candidate, since its values of Q = 47.72 and Δ = 57.51 are small compared to the quantifying values and its quadruple transition ratio with experimental value of $R_E = 1.44 \pm 0.11$ very close to the SU(3) calculated value $R_{SU(3)}$=1.403. However, the correlation between its energy levels and that of X(4) supported the phase transition between U(5) symmetry ^{156}Er to SU(3) symmetry ^{160}Er, (see Tables 2, 4, 5).

Table 5 makes it clear that ^{160}Er fits well with the dynamic symmetry SU(3) because its energy levels are compatible with the calculated values of the SU(3) which can be tested by its (Q = **31.11** and Δ = 46.04) values which are very small. Additionally, the experimental quadruple transition ratio (R_E= **1.43 ± 0.07**) is close to the theoretical values $R_{SU(3)}$ = **1.407,** see Table (3). So, more nucleons in ^{160}Er create clear rotational bands, which supports the idea of SU(3) symmetry. Hence, the erbium isotopes $^{156 - 160}$Er exhibit the U(5)–SU(3) shape phase transition, demonstrating a transformation of nuclear shapes from spherical to axially symmetric deformed configurations.

In the same way, the last nucleus we study ^{170}Er, has energy levels that indicate the SU(3) dynamical symmetry Fairley, as shown in Table 7. However, we were not able to find the values of its experiential B(E2) transition ratio R_E.

For the isotopes $^{162-168}$Er, its Q and Δ values are very high (see Tables 2, 5, 6, 7), which is an indication of a more distorted geometric structure. Their experimental energy levels, especially the beta 1 band, are Pushed up, which breaks the degeneracy between the even energy levels in gamma 1 and beta 1 bands. A. Leviatan has developed a method known as partial dynamic symmetry to characterize such nuclei [26]. One of the recently known isotopes to have such characterization is ^{168}E [26].

Table (4): The experimental, the fitted energy levels, the quality factor Q, and the absolute average deviation for the fitted energy levels of the $^{156-158}$Er nuclei.

	$^{156}_{68}Er$				$^{158}_{68}Er$			
Levels	**Exp**	**U(5)**	**O(6)**	**SU(3)**	**Exp**	**U(5)**	**O(6)**	**SU(3)**
$E(2^+_1)$	344.53	443.59	266.252	173.216	192.15	357.065	209.466	129.271
$E(0^+_2)$	930.07	845.86	956.38	864.22	806.38	704.451	805.115	758.686
$E(2^+_2)$	930.48	847.67	585.046	1037.44	820.12	728.459	477.838	887.957
$E(4^+_1)$	797.39	916.87	710.401	577.385	527.22	710.294	549.126	430.902
$E(0^+_3)$					1386.9	1114.18	1297.49	1372.86
$E(2^+_3)$	1220.7	1280.89	1541.43	1037.44	989.08	1061.89	1282.95	887.957
$E(3^+_1)$	1351.3	1271.56	1063.83	1210.65	1043.39	1098.61	866.219	1017.23
$E(4^+_2)$	1406.2	1311.11	1135.46	1441.61	1183.78	1088.23	906.955	1189.59
$E(6^+_1)$	1340.9	1419.86	1332.44	1212.51	970.34	1059.69	1018.98	904.895
$E(2^+_4)$	1570.8	1626.29	1570.75	1719.72	1417.55	1490.88	1506.96	1502.13
$E(2^+_5)$								
$E(4^+_3)$	1546.7	1695.49	1666.78	1441.61	1257.28	1472.71	1354.24	1189.59
$E(5^+_1)$	1835.2	1744.92	1756.32	1730.3	1438.22	1459.74	1405.16	1405.04
$E(6^+_2)$	1885.9	1804.24	1863.77	2076.73	1589.02	1444.17	1466.26	1663.58
$E(8^+_1)$	1959.2	1952.54	2132.38	2078.59	1493.47	1405.24	1619.03	1551.25
Δ		**83.27**	139.89	132.39		119.33	124.03	**57.51**
Q		**114.87**	443.68	235.96		260.36	338.19	**47.72**

Table (5): The same as table 4 for $^{160\text{-}162}$Er nuclei

	$^{160}_{68}Er$				$^{162}_{68}Er$			
Levels	**Exp**	**U(5)**	**O(6)**	**SU(3)**	**Exp**	**U(5)**	**O(6)**	**SU(3)**
$E(2^+_1)$	125.47	286.6	189.232	102.986	102.04	394.768	182.681	88.1285
$E(0^+_2)$	893.5	730.704	848.495	822.781	1087.16	867.076	942.574	901.25
$E(2^+_2)$	854.2	637.387	472.063	925.767	900.72	847.223	496.873	989.379
$E(4^+_1)$	389.37	580.228	473.644	343.288	329.62	670.603	434.387	293.762
$E(0^+_3)$					2114.11	1357.36	1454.98	1658.3
$E(2^+_3)$	1007.93	1124.21	1320.56	925.767	1171.02	1203.49	1439.45	989.379
$E(3^+_1)$	987.15	1003.37	849.85	1028.75	1002.06	1205.98	889.015	1077.51
$E(4^+_2)$	1128.54	970.705	850.753	1166.07	1128.11	1105.05	853.309	1195.01
$E(6^+_1)$	765.01	880.883	853.236	720.904	666.68	827.503	755.117	616.9
$E(2^+_4)$					1429.79	1698.11	1637.66	1746.43
$E(2^+_5)$					1500.58	1597.59	1951.85	1746.43
$E(4^+_3)$	1229.68	1400.87	1322.14	1166.07	1369	1521.49	1376.96	1195.01
$E(5^+_1)$	1316.36	1360.04	1323.27	1337.71	1286.22	1395.33	1332.33	1341.89
$E(6^+_2)$	1499.24	1311.05	1324.62	1543.68	1459.58	1243.94	1278.77	1518.15
$E(8^+_1)$	1229.06	1188.56	1328.01	1235.84	1096.7	865.47	1144.87	1057.54
Δ		131.78	147.01	**46.04**		210.54	205.33	136.25
Q		320.37	455.16	**31.11**		1010.45	906.80	381.80

Table (6): The same as table 4 for $^{164\text{-}166}$Er nuclei

	$^{164}_{68}Er$				$^{166}_{68}Er$			
Levels	**Exp**	**U(5)**	**O(6)**	**SU(3)**	**Exp**	**U(5)**	**O(6)**	**SU(3)**
$E(2^+_1)$	91.38	309.62	179.62	84.0463	80.58	247.98	172.71	62.4009
$E(0^+_2)$	1246.06	948.93	1038.22	870.399	1460.03	1166.82	1151.11	982.256
$E(2^+_2)$	860.25	707.89	525.69	954.445	785.91	688.07	556.42	1044.66
$E(4^+_1)$	299.43	560.82	406.47	280.154	264.99	442.39	362.53	208.003
$E(0^+_3)$	1416.57	1194.82	1327.77	1611.85	1713.40	1320.28	1713.40	1829.03
$E(2^+_3)$	1314.56	1381.01	1563.91	954.445	1528.401	1737.83	1707.53	1044.66
$E(3^+_1)$	946.34	1068.76	936.03	1038.49	859.39	1109.69	984.93	1107.06
$E(4^+_2)$	1058.49	984.721	867.90	1150.55	956.23	969.30	874.14	1190.26
$E(6^+_1)$	614.39	753.62	680.56	588.324	545.45	583.23	569.46	436.806
$E(2^+_4)$	1483.69	1581.32	1507.39	1695.9				
$E(2^+_5)$	1788.35	1901.74	1853.46	1695.9				
$E(4^+_3)$	1469.72	1434.25	1444.69	1150.55	1678.77	1583.05	1513.65	1190.26
$E(5^+_1)$	1197.48	1329.2	1359.53	1290.63	1075.28	1407.56	1375.16	1294.26
$E(6^+_2)$	1358.73	1203.14	1257.34	1458.72	1215.97	1196.97	1208.97	1419.06
$E(8^+_1)$	1024.62	888	1001.87	1008.56	911.21	670.51	793.51	748.81
Δ		148.21	116.20	139.68		179.02	132.97	236.49
Q		367.93	273.10	392.51		661.73	352.36	934.82

Table (7): The same as table 4 for $^{168-170}$Er nuclei.

	$^{168}_{68}Er$				$^{170}_{68}Er$			
Levels	**Exp**	**U(5)**	**O(6)**	**SU(3)**	**Exp**	**U(5)**	**O(6)**	**SU(3)**
$E(2^+_1)$	79.80	281.63	167.80	73.85	78.59	312.99	164.13	82.34
$E(0^+_2)$	1217.17	942.40	1009.27	862.63	890.88	721.65	856.19	754.77
$E(2^+_2)$	821.169	672.43	504.23	936.48	934.02	691.77	449.53	837.11
$E(4^+_1)$	264.09	504.55	372.44	246.18	260.14	554.99	388.56	274.47
$E(0^+_3)$	1422.12	1172.39	1363.86	1613.94	1324.3	1136.4	1142.70	1418.10
$E(2^+_3)$	1276.27	1377.68	1513.49	936.48	959.99	1037.5	1305.70	837.11
$E(3^+_1)$	895.80	1028.50	896.31	1010.33	1010.5	1019.1	803.93	919.45
$E(4^+_2)$	994.75	932.57	821.01	1108.80	1103.4	940.95	769.09	1029.20
$E(6^+_1)$	548.75	668.76	613.92	516.98	540.68	726.02	673.27	576.38
$E(2^+_4)$	1493.13	1565.68	1531.66	1687.80	1332	1470.9	1306.80	1500.40
$E(2^+_5)$	1848.35	1922.13	1868.09	1687.80	1385.4	1419.1	1592.20	1500.40
$E(4^+_3)$	1411.10	1397.8	1381.71	1108.80	1127.3	1334.1	1244.80	1029.20
$E(5^+_1)$	1117.57	1277.89	1287.58	1231.89	1236.7	1236.4	1201.20	1166.50
$E(6^+_2)$	1263.91	1134.00	1174.62	1379.60	1350.5	1119.2	1148.90	1331.20
$E(8^+_1)$	928.30	774.26	892.23	886.25	914.97	826.06	1018.30	988.08
Δ		142.38	109.28	147.69		150.83	147.91	**80.85**
Q		347.18	248.48	386.17		414.48	578.34	**99.19**

7. Conclusion

This study investigates the structural changes in the Erbium isotope chain $^{156-170}$Er using the IBM-1 framework, which is commonly employed to analyze the structure of low-lying excited states in even-even nuclei. The analysis has concluded that the nucleus ^{156}Er may have characteristics of $U(5)$ dynamic symmetry while $^{158,160,\ 170}$Er nuclei having characteristics of $SU(3)$ dynamic symmetry. These erbium isotopes represent an example of the U(5)–SU(3) shape phase transition and exhibit a transition of nuclear shapes from spherical to axially symmetric deformed forms. For the remaining nuclei, we cannot explain their behavior using the IBM-1, instead they may be explained in the framework of the partial dynamic symmetry [25, 26].

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