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Modelling Maritime Traffic at Damietta Port as M/M/c/N Queuing Model with Encouraged Arrivals

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Abstract

Damietta Port is one of the most important ports in Egypt, with increased rates of ship arrival, loading and unloading during seasons and before holidays. We proposed an M/M/c/N queuing model which describes maritime traffic and ships handling. Numerical data were also discussed and plotted to investigate the attitude of the maritime traffic. The objective of the study is to improve the efficiency of logistical procedures and reduces truck waiting time and reduces ground congestion resulting from the transport, packing and unloading process.

Keywords: Queuing Model, M/M/c/N, Damietta port, Encouraged arrivals.

Introduction

Queuing systems are widely applied and clearly observable in daily life. Our model in this paper is based on the nature of the author's surrounding environment and the place of life that is Damietta port is a vital authority in Damietta Governorate. The movement of loading and unloading ships in the docks of Damietta port represents a major and clear challenge, that is encouraged arrivals in this model represented by the increase in the number and rates of ships arriving at the port during seasonal or holiday periods, when the demand for food commodities and seasonal needs is at its highest levels. Rates also reach their highest levels during the harvest period of agricultural crops that are exported to countries that lack these crops.

This model contributes to improving the efficiency of logistical procedures and determining appropriate times for trucks to enter the transport and unloading yards, which reduces truck waiting time and reduces ground congestion resulting from the transport, packing and unloading process.

Model Description

Our model is a Markovian queuing model with multi-server, finite capacity and encouraged arrivals it subject to the following assumptions:

- Let P_i denotes the probability that the system has i customers (ships) in the system, thus P_0 means the probability that the system is empty and P_N express the probability that the system is full.

- The arrivals of the customers (ships) occur according to Poisson process with parameter $\lambda(1+\alpha)$, where α is the percentage expresses the arrival rate of ships which calculated from observed and remarked data.
- In the case of decreases arrivals rate $\alpha = 0.75$, and in the case of increase arrivals rate we will take $\alpha = 1.5$.
- Service times are subjected to exponential distribution with parameter μ.
- Ships are serviced in the order of their arrival (first come first served).
- The quay has maximum capacity c of ships, which can be served at the same time.
- The total capacity of the operation of loading and unloading ships is N .

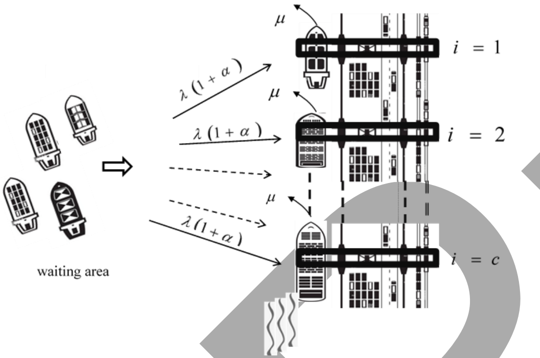


Fig. 1 arrivals and serving the ships as M/M/c/N queuing model

This model is discussed in Gross and Harris (1998) and Sharma and Tarabia (2000) where the model is described in its simplest form. In Som and Seth (2017, 2018) evolve Markovian queuing model with a multi-server Feedback and encouraged arrivals also with a good addition which is customer impatience, and retention of impatient customers. They obtained the stationary system size probabilities. Also, they calculate some measures of efficiency and gave numerical discussions to illustrate attitude of the system. Some special cases are obtained and discussed. Recently in (2024) Cruz and Julia are analysis and discussed the queuing model of M/M/c with encouraged arrival, they assumed that the arrival rate is a function of the customer's number in the system. Their model was discussed by the technique of continuous time Markov chain, and the steady state probabilities are obtained too.

In this paper, we developed a M/M/c/N queuing model to describe the maritime traffic and ship handling, and applied this model to the ship traffic in Damietta Port. The steady state probabilities of this model and the expected values of ship waiting times in the system and in the queue were calculated, and the numerical data of the model were discussed and plotted to investigate the maritime traffic situation.

Mathematical Formulation of the Model

This applicable model can be described by the following balance equations:

$$\mu P_1 = \lambda(1+\alpha) P_0, \qquad i = 0$$

$$(i+1)\mu P_{i+1} = \left(\lambda(1+\alpha) + i\mu\right) P_i - \lambda(1+\alpha) P_{i-1}, \qquad 1 \le i \le c-1$$

$$c\mu P_{i+1} = \left(\lambda(1+\alpha) + c\mu\right) P_i - \lambda(1+\alpha) P_{i-1}, \qquad c \le i \le N-1$$

$$c\mu P_N = \lambda(1+\alpha) P_{N-1}, \qquad i = N$$

Solving the system recurrently, then

$$P_i = \begin{cases} \dfrac{1}{i!}\left(\dfrac{\lambda(1+\alpha)}{\mu}\right)^i P_0, & 1 \le i \le c \\[2ex] \dfrac{1}{c^{i-c} c!}\left(\dfrac{\lambda(1+\alpha)}{\mu}\right)^i P_o & c < i \le N \end{cases}$$

where

$$P_0 = \Pr\{\text{system is empty}\}$$

$$P_N = \Pr\{\text{system is full}\},$$

Now, to calculate P_0 , we will use the normality condition $\sum_{i=0}^{N} P_i = 1$, then

$$P_0 = \left(\sum_{i=0}^{c} \frac{1}{i!}\left(\frac{\lambda(1+\alpha)}{\mu}\right)^i + \sum_{i=c+1}^{N} \frac{1}{c^{N-c} c!}\left(\frac{\lambda(1+\alpha)}{\mu}\right)^i \right)^{-1}$$

Some Performance Measures

Here, we will introduce some useful performance measures, which can be calculated easily to Controlling the facilitating of Maritime Logistics:

- Expected system size $L_s = \sum_{i=1}^{N} iP_i$

  Expected queue length $L_q = \sum_{i=c}^{N} (i-c)P_i$
- Expected waiting time in the system $W_s = \frac{L_s}{\lambda(1+\alpha)}$
- Expected waiting time in the queue $W_q = \frac{L_q}{\lambda(1+\alpha)}$

Numerical Illustrations

In this section we will introduce some numerical examples which illustrate the behavior of traffic flow in the port. In Table 1 and 2 we illustrate the case of un encouraged arrivals by assume $\alpha = 0.75$, in Table 1, we assume $N = 15, c = 4, \lambda = 5, 3 \le \mu \le 6$, and in Table 2 we assume $N = 15, c = 4$, $\mu = 3, 3 \le \lambda \le 6$. Table 3 and 4 are explaining the behavior in the encouraged arrivals case ($\alpha = 1.5$). In Table 1 we can see that the predicted waiting time in the system and in the queue are decreasing while the service rate (μ) is increases with stable arrivals rate (λ). Data in Table 2 provided that the expected waiting time in the system and in the queue are increasing while the arrivals rate (λ) is increases with stable service rate (μ). In Table 3 once $\mu \ge c$ the length of the queue will excessively get low as will as waiting time in the system and in the queue which give an approximated behavior for decreasing the expected waiting time in the system and in the queue when service rate (μ) is increases with stable arrivals rate (λ) in the case of encouraged arrivals. Also Table 4 illustrate the case of encouraged arrivals that the predicted waiting time in the system and in the queue are increasing while the arrivals rate (λ) is increases with stable service rate (μ).

Table 1. Relation between variant values of μ at $\alpha = 0.75, N = 15, c = 4, \lambda = 5$ and performance measures.

μ	P_0	L_s	L_q	W_s	W_q
3	0.0646707	2.45420	0.09633140	0.280480	0.0110093
3.5	0.0919778	2.14252	0.01465560	0.244860	0.00167492
4	0.1207580	1.93880	0.00282091	0.221577	0.00032239
4.5	0.1502350	1.77124	0.00065603	0.202427	0.00007497
5	0.1796830	1.62734	0.00017807	0.185982	0.00002035
5.5	0.2085750	1.50241	0.000055019	0.171704	0.00000629
6	0.2365700	1.39334	0.000018997	0.159239	0.00000217

Table 2. Relation between variant values of λ at $\alpha = 0.75, N = 15, c = 4, \mu = 3$ and performance measures.

μ	P_0	L_s	L_q	W_s	W_q
3	0.179683	1.62734	0.00017807	0.309969	0.000039177
3.5	0.137568	1.83976	0.00120067	0.300369	0.000196028
4	0.106228	2.03448	0.00626306	0.290640	0.000894722
4.5	0.0826829	2.22582	0.02667790	0.282644	0.003387670
5	0.0.064671	2.45420	0.09633140	0.280480	0.011009300
5.5	0.0504011	2.81845	0.30137800	0.292826	0.031312000
6	0.0384047	3.51853	0.82029200	0.335098	0.078123000
6.5	0.0276457	4.837701	1.91069000	0.425232	0.167973000

Table 3. Relation between variant values of μ at $\alpha = 1.5, N = 25, c = 4, \lambda = 5$ and performance measures.

μ	P_0	L_s	L_q	W_s	W_q
3	0.0129190	13.8132	10.4064000	1.16506	0.832512000
3.5	0.0381802	3.34712	0.68578000	0.267769	0.054862400
4	0.0552472	2.47769	0.03714990	0.198216	0.002971990
4.5	0.0730428	2.27734	0.00273396	0.182188	0.000218717
5	0.0921072	2.12550	0.00026265	0.17004	0.00002101
5.5	0.1120620	1.98963	0.00003142	0.159171	0.00000250
6	0.1325310	1.86662	0.00000451	0.149329	0.00000036

Table 4. Relation between variant values of λ at $\alpha = 1.5, N = 25, c = 4, \mu = 3$ and performance measures.

μ	P_0	L_s	L_q	W_s	W_q
3	0.0921072	2.1255	0.0002627	0.283400	0.00003502
3.5	0.0652450	2.35135	0.0080701	0.268726	0.00092229
4	0.0468044	2.68957	0.1537980	0.268957	0.01537980
4.5	0.0315383	4.68408	1.8857400	0.416362	0.16762100
5	0.0129190	13.8132	10.406400	1.105060	0.83251200
5.5	0.0021707	22.3296	18.448600	1.623970	1.34171000
6	0.00027818	24.3546	20.372600	1.623640	1.35818000

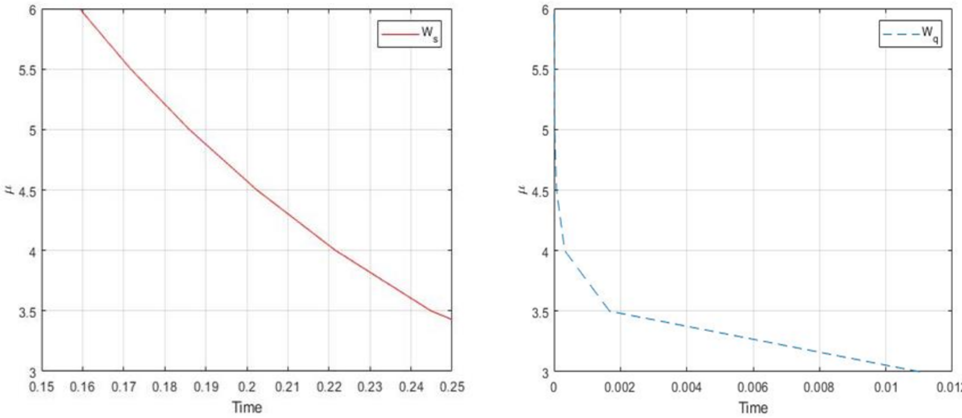


Fig 2, 3 The performance of waiting time at the system and waiting time at the queue in case $\alpha = 0.75, N = 15, c = 4, \lambda = 5$.

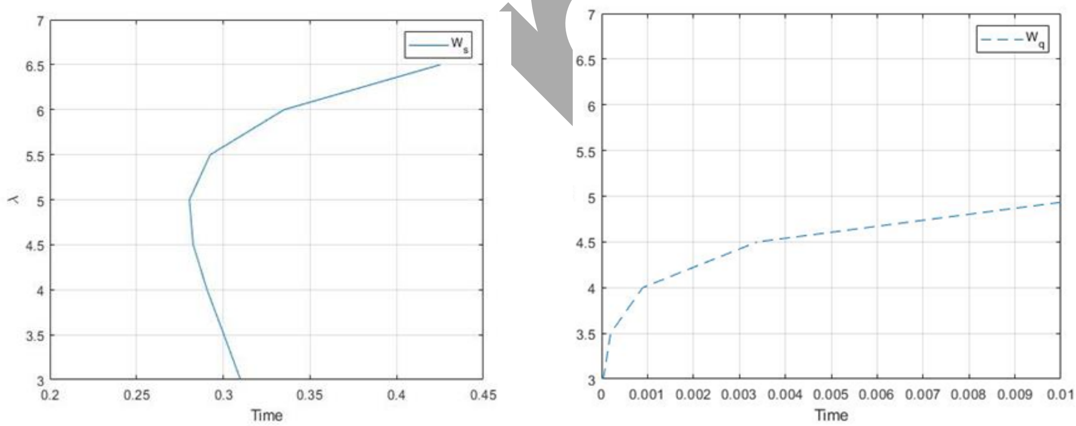


Fig. 4, 5 The performance of waiting time at the system and waiting time at the queue in case $\alpha = 0.75, N = 15, c = 4, \mu = 3$.

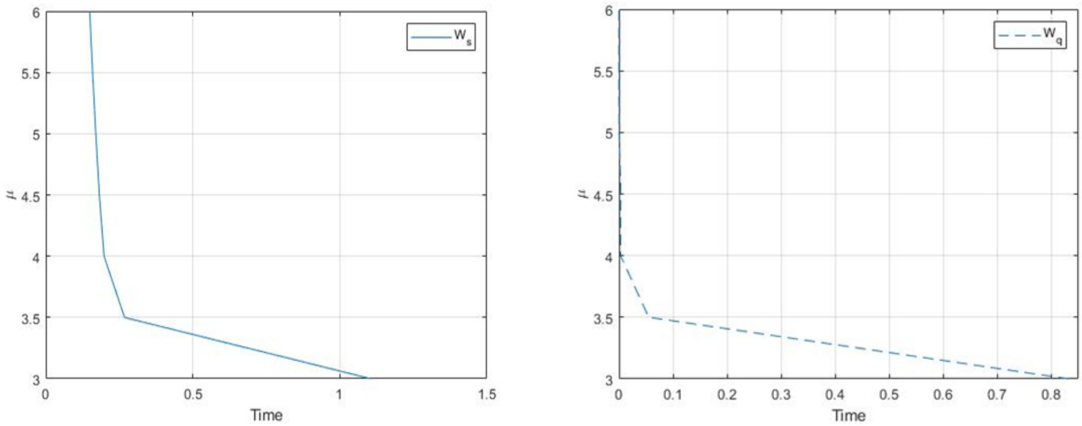
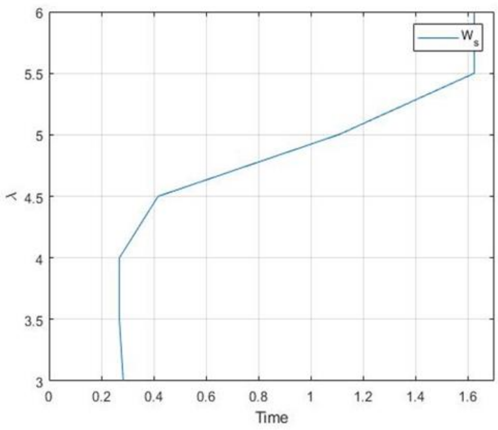


Fig. 6, 7 The performance of waiting time at the system and waiting time at the queue in case $\alpha = 1.5, N = 25, c = 4, \lambda = 5$.



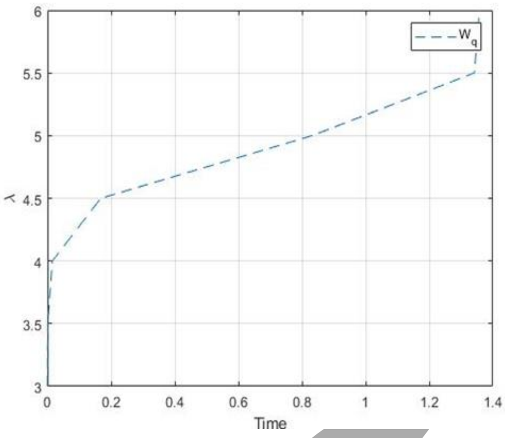


Fig. 8, 9 The performance of waiting time at the system and waiting time at the queue in case $\alpha = 1.5, N = 25, c = 4, \mu = 3$.

Conclusion

We describe the maritime traffic and ship handling in Damietta port as M/M/c/N Markovian queuing model. The steady state probabilities are calculated and some measures of fulfillment status like expected system size L_s, expected queue length L_q, expected waiting time in the system W_s, expected waiting time in the queue W_q.The numerical data of the model were discussed and plotted to investigate the maritime traffic situation which raising the efficiency of logistical procedures and optimizing the wasted waiting time in the future.

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الملخص العربي

عنوان البحث: نمذجه حركة المرور البحريه فى ميناء دمياط كنموذج لطابورM/M/c/N مع تشجيع الواصلين

غدير عبد الوهاب الشريف*١، أحمد محمد كامل طرابية١

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في هذا البحث تم تقديم نموذج طابور متعدد الخادم كتمثيل لحركه الملاحه البحريه للسفن فى ميناء دمياط حيث تم وصف حركة الملاحه البحريه و فتره انتظار السفن للشحن والتفريغ على ارصفة الميناء كنموذج رياضي يتمثل فى طابور متعدد الخادم وتم حساب المده المتوقعه للبقاء فى النظام والمده المتوقعه للبقاء على رصيف التفريغ كما تم القيام ببعض الحسابات الرقميه من واقع المعدلات الحقيقيه لميناء دمياط.