A SEQUENTIAL TEST FOR KENDALL'S CORRELATION COEFFICIENT

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INTRODUCTION

Cox (1952) gave a sequential probability ration test (SPRT) for the correlation coefficient ρ which computes sample correlation coefficients successively. Choi (1971) developed an SPRT for ρ which is essentially a test for the cauchy parameter. Kowalski (1971) constructed a test which reduces to the SPRT for the binomial parameter. Meena and Sathe (1975) constructed a test that differs from Kowalski's in using two pairs of observation each time. All these tests assumes that we are sampling a bivariate normal distribution. In the non-normal case all these tests fail and we still want to test for the presence of association in the population. In this paper we give a test of the association based on Kendall's τ. The test is similar to that of Meena and Sathe and so its properties are easly studied. It is assumed that the underlying distribution is continuous.

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PROPOSED TEST

Let (X_i,Y_i), $i=1,2,\ldots$ be from a bivariate continuous distribution F. We want to test the hypothesis

$$H_o : \tau = \tau_o \qquad \text{Vs.} \quad H_1 \ \tau=\tau_1 \quad \ldots\ldots \qquad (1)$$

where τ is Kendall's correlation coefficient τ defind as the probability $\pi^{(c)}$ that two random pairs (X_1,Y_1), (x_2,Y_2) from F are concordant minus the probability $\pi^{(d)}$ that they are discordant. We take two pairs of observations at each step. At the nth step we define

$$U_i = \begin{cases} 1 & \text{if } (X_{2i-1},Y_{2i-1}) \text{ and } (X_{2i},Y_{2i}) \text{ are concordant} \\ 0 & \text{otherwise} \quad , i=1,2,\ldots. \end{cases}$$

Since F is continuous then $\pi^{(c)} + \pi^{(d)} = 1$ and it follows that U_i has a binomial distribution with

$$Pr(U_i = 1) = p = \frac{1+\tau}{2} .$$

Hence (1) can be reduced to

$$H_o : P = P_o \qquad \text{vs} \qquad H_1 : P = P_1$$

and Wald's (1947) SPRT for the binomial parameter can be applied. That is, we continue sampling (taking two pairs of observations each time) until one of the two inequalities .

$$\log \frac{\beta}{1-\alpha} < \left(\sum_{i=1}^{n} U_i\right) \log \frac{P_1}{P_o} + \left(n - \sum_{i=1}^{n} U_i\right) \log \frac{1-P_1}{1-P_o} < \log \frac{1-\beta}{\alpha}$$

is violated, where (α,β) is the desired strength of the test. We accept H_o if the left in equality is violated and we reject H_o if the right inequality is violated .

The AOC and ASN

It is important here to notic that the proposed test has the same formula as the test suggested by Meena and Sathe with his ρ replaced by τ. So his numerical results apply here and we have the following table when $\tau_o = 0$.

ASN for proposed Test

$H_o : \tau = 0$		$\alpha = \beta = .05$		
	$\lvert\tau\rvert=0.10$	$\lvert\tau\rvert=.50$	$\lvert\tau\rvert=0.75$	$\lvert\tau\rvert=.95$
ASN H_o	1056.7	38.8	14.6	6.6
ASN H_1	1060.2	42.5	18.8	11.2

However the exact and approximate distributions of U_i have complicated formulae and then it is difficult to obtain the ASN for the fixed sample to compare with results given here.

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