Analysis of fluid system fed by infinite servers queue subject to catastrophes servers and reparable

by

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Abstract:

This paper analyzes a fluid queue driven by an M/M/∞ background system subject to catastrophic breakdowns and exponential repairs. The model extends classical fluid queues by incorporating catastrophic failures that simultaneously remove all customers, along with repair dynamics that restore the system to full capacity. Using Laplace transforms and continued-fraction techniques linked to confluent hypergeometric functions, we derive the stationary distribution of the buffer content and the joint law of buffer level and background population. The theoretical novelty lies in the explicit continued-fraction representation, the closed-form characterization of stability, and the tractable formulas for key measures such as non-empty probability, throughput, and mean buffer level. Beyond methodological advances, the model has applications in telecommunication networks with mass call failures, data centers exposed to blackouts, production systems with common-cause machine breakdowns, and service systems under disaster recovery regimes. Numerical illustrations highlight how arrival, service, failure, and repair parameters shape long-run performance, offering insights into both reliability planning and capacity design.

Keywords: Infinite queue, Stationary analysis, Confluent Hypergeometric Function, Catastrophes, Repairable servers.

1-**Introduction**:

Fluid queues are a well-established framework for studying systems in which workload evolves continuously under the influence of random environments. They have been applied in telecommunications, computer networks, production, and service operations where arrival and service processes interact with dynamic input and output rates. A number of studies have examined infinite-server systems in this context. For instance, Linton [5], Bura Gulab Singh [2], Baykel-Gursoy [1], Sophia [8], and Gulab Singh [3] have analyzed infinite-server queueing models, often employing continued-fraction techniques to address queueing problems involving disasters and repairable servers in the classical (non-fluid) setting.

Fluid queues themselves have been studied under various structural assumptions and solution methods. Xu et al. [10], for example, investigated a fluid queue model controlled by an M/M/c queue with working vacation using the matrix-analytic approach. Krishna Kumar et al. [4] applied the continued-fraction method to infinite-server fluid queues, deriving the expected fluid content and stationary buffer distributions. Parthasarathy and Vijayashree [7] considered a fluid queue driven by a discouraging-arrivals queue, showing how modified input dynamics affect performance. These studies demonstrate the flexibility of fluid queue modeling and the range of analytic techniques available for equilibrium analysis.

Despite this progress, the literature has typically treated catastrophes, repairable servers, and infinite-server environments separately. To the best of our knowledge, no prior work has integrated an $M/M/\infty$ background with both catastrophic failures and exponential repairs to analyze fluid queues. This integration is important because many practical systems are exposed to sudden large-scale service collapses followed by repair and recovery. Examples include mass call failures in telecommunication networks, data center outages, and coordinated machine breakdowns in manufacturing systems.

The present study fills this gap by introducing and analyzing a fluid queue controlled by an infinite-server system with both catastrophic failures and repairable servers. Customers arrive according to a Poisson process and receive exponential service in the M/M/∞ background. Catastrophes simultaneously remove all customers, while repair times are exponentially distributed, restoring the system to full operation. The buffer receives fluid input during active phases and depletes during downtime. Our theoretical contributions are as follows. First, we derive a precise stability condition based on the stationary availability of the catastrophe–repair process. Second, we obtain explicit stationary distributions of buffer content and the joint law of buffer level and background population using Laplace transforms and continued-fraction methods linked with confluent hypergeometric functions. Third, we present closed-form expressions for performance measures such as the probability of a non-empty buffer, throughput, and expected buffer content. On the applied side, the model is relevant to telecommunication networks with mass call failures, data centers subject to blackouts, and production systems experiencing collective breakdowns. The results provide analytical tools for capacity planning, reliability assessment, and disaster recovery design. By combining rigorous theoretical results with clear practical applications, the study contributes both to the advancement of fluid queue theory and to its use in modern reliability and operations management.

2-Description of the model and stability condition

Consider a Markovian queueing system that uses an endless number of servers to drive a fluid queue with limitless buffer content. The server is fed into the ongoing breakdown failures and repairs. While the servers are in an up/active state, arrivals happen one to one in a Poisson process with a rate of λ. With parameter μ, service times are dispersed exponentially. The service facility experiences breakdowns (catastrophes) based on a Poisson process of rate ν when the system is nonempty. Every time a disaster strikes, all of the server's malfunction. The repair time distribution is exponential with rate η, and the repair procedure is initiated immediately. The server instantly resumes its active status upon the completion of a repair. Now, the server is prepared to offer services if a new arrival occurs.

Let $\{N(t); t \geq 0\}$ be the number of customers in the system at time t with state $\Omega = \{0,1,2,.....\}$ and

$$J(t) = \begin{cases} 1, \textit{ if the server is active state} \\ 0, \textit{ if the server is under repair state} \end{cases}$$

Clearly, the two-dimensional process $\{N(t), J(t), t \geq 0\}$ constitute a continuous time Markov chain whose state space is given by $S = \{(0.0) \cup (n,1); n = 0,1,2,....\}$.

For this queueing system, with λ, μ, ν and η are positive , the stationary probabilities of the system size $\{p_n; n \geq 0\}$ when the server is in active state and the failure state probability G, of the server can be obtained from Gulab Singh [3] as follows

$$G = \frac{1}{1+\eta\nu^{-1}}(1-\eta\rho_1), \; p_n = \eta\,\rho_1\,\rho_n, \; p_0 = \eta\,\rho_1 \qquad \textbf{(1)}$$

where $$\rho_n = (\rho)^n \frac{{}_1F_1(n+1, \nu\mu^{-1}+n+1; -\rho)}{\prod_{i=1}^{n}(\nu\mu^{-1}+i)\,{}_1F_1(1, \nu\mu^{-1}+1; -\rho)} \quad , \quad \rho = \frac{\lambda}{\mu} \qquad \textbf{(2)}$$

and $$\rho_1 = \sum_{j=0}^{\infty} \frac{(-1)^j}{(\lambda+\eta)^{j+1}} \left[\sum_{n=1}^{\infty} (\eta + \delta_{n1}\,\mu)\,\rho_n \right]^j < 1. \qquad \textbf{(3)}$$

Now assume a fluid queueing model with an infinite servers credit buffer, where the queueing system previously described feeds the input and output rates of a fluid commodity. The fluid commodity is processed first in, first out, and then it builds up in the buffer.

Only when every fluid commodity that arrived prior to time t has been removed will the fluid commodity coming at time t be removed from the buffer.

Let $\{X(t); t \geq 0\}$ represent the amount of fluid commodity residing in the credit buffer at time t which is regulated by the two –dimensional Markovian queueing process $\{X(t), J(t); t \geq 0\}$. The fluid commodities in the buffer credit at a constant rate $\sigma > 0$ at active state ($J(t) = 1$) whereas the fluid commodity depletes during the repair period , ($J(t) = 0$) of the server at a constant rate $\sigma_0 < 0$ as long as the buffer is non-empty.

Obviously, $\{X(t); t \geq 0\}$ is a non-negative random process and its dynamics is defined by the equation

$$\frac{d\,X(t)}{dt}=\begin{cases}0 & for\ X(t)=0,\ J(t)=0\\ \sigma_0 & for\ X(t)>0,\ J(t)=0\\ \sigma & for\ X(t)>0,\ J(t)=1,\ N(t)=0,1,2,...\end{cases} \tag{4}$$

Clearly, the 3-dimensional process $\{N(t), J(t), X(t); t \geq 0\}$ constitutes a Markov process that, given an appropriate stability condition, processes a unique stationary distribution.

The mean drift of the fluid commodity in the credit buffer can be compute as follows

$$\begin{aligned}d &= \sigma_0\,G+\sigma\sum_{n=0}^{\infty}p_n\\ &= \sigma_0\,G+\sigma(1-G)\\ &= \sigma+(\sigma_0-\sigma)G\\ &= \sigma+(\sigma_0-\sigma)\left\{\left(\frac{1}{1+\eta\,\nu^{-1}}\right)(1-\eta\,\rho_1)\right\}<0\end{aligned}$$

This, the normalizing condition of the fluid queue is

$$\rho_1<1, \text{ and } d<0. \tag{5}$$

3- Stationary distribution of fluid queue

Let

$$R(x)=P(N=0,J=0,X<x)$$

$$F_n(x)=pr.\{N=n,J=1,X\leq x\},n\in\Omega,x\geq 0$$

with boundary condition

$$R(0)=\frac{d}{\sigma_0} \quad \text{and } F_n(0)=0,n=0,1,2,\dots.$$

The ordinary differential equations of the stationary distribution fluid queue are:

$$\sigma_0 \frac{dR(x)}{dx} = -\eta R(x) + \nu \sum_{n=1}^{\infty} F_n(x)$$

(6)

$$\sigma \frac{dF_0(x)}{dx} = -\lambda F_0(x) + \mu F_1(x) + \eta R(x)$$

(7)

$$\sigma \frac{dF_n(x)}{dx} = -(\lambda + n\mu + \nu) F_n(x) + (n+1)\mu F_{n+1}(x) + \lambda F_{n-1}(x); n = 1, 2, ...$$

(8)

with boundary condition

$R(0) = \frac{d}{\sigma_0} = b.$ where b is unknown constant

and $F_n(0) = 0,\ n = 0, 1, 2, ...$

(9)

The condition $R(0) = b$, refer to the probability b, $0 < b < 1$, the buffer content of fluid commodity is empty and the net input rate is zero , when the server is under repair in the organization queueing model.

In the following complement, let $g^*(s) = \int_0^\infty e^{-sx} g(x)\, dx$ be the Laplace transformation (LT) of the function $g(x)$.

By applying the LT for the equations (6) – (8), and using the boundary conditions (9), we obtain the following equations

$$R^*(s)=\frac{b}{(s+\eta'')}+\nu''\frac{\sum_{n=1}^{\infty}F_n^*(s)}{(s+\eta'')} \quad \textbf{(10)}$$

$$(\sigma s+\lambda)F_0^*(s)=\eta R^*(s)+\mu F_1^*(s) \quad \textbf{(11)}$$

$$(\sigma s+\lambda+n\mu+\nu)F_n^*(s)=\lambda F_{n-1}^*(s)+(n+1)\mu F_{n+1}^*(s), n=1,2,... \quad \textbf{(12)}$$

Eq. (12) can be expressed as

$$\frac{F_n^*(s)}{F_{n-1}^*(s)}=\frac{\lambda'}{s+\lambda'+\nu'+n\mu'-(n+1)\mu'\frac{F_{n+1}^*(s)}{F_n^*(s)}}$$

$$=\frac{\lambda'}{s+\lambda'+\nu'+n\mu'-\frac{(n+1)\lambda'\mu'}{\lambda'+\nu'+(n+1)\mu'-\frac{(n+2)\lambda'\mu'}{s+\lambda'+\nu'+(n+2)\mu'-...}}} \quad \textbf{(13)}$$

CFs can be used to express the fractions of confluent hypergeometric function. By using the identity given in Lorentzen and Waadeland ([6], (4.1.5), page 573)

$$\frac{{}_1F_1(e+1;c+1;u)}{{}_1F_1(e;c;u)}=\frac{c}{c-u+}\frac{(e+1)u}{c-u+1+}\frac{(e+2)u}{c-u+2+}...,$$

which can be expressed as

$$c\frac{{}_1F_1(e;c;u)}{{}_1F_1(e+1;c+1;u)}-(c-u)=\frac{(e+1)u}{c-u+1+}\frac{(e+2)u}{c-u+2+}..., \quad \textbf{(14)}$$

From Eq. (14) in (13), we obtain

$$\frac{F_n^*(s)}{F_{n-1}^*(s)}=\frac{\lambda'}{\mu'}\frac{{}_1F_1^*(n+1;(s+\nu')\mu'^{-1}+n+1;-\lambda'\mu'^{-1})}{((s+\nu')\mu'^{-1}+n)\,{}_1F_1^*(n;(s+\nu')\mu'^{-1}+n;-\lambda'\mu'^{-1})} \quad \textbf{(15)}$$

Iterating Eq. (15) for n=12, 3, …, results in

$$F_n^*(s) = (\frac{\lambda'}{\mu'})^n \frac{{}_1F_1^*(n+1;(s+\nu')\,\mu'^{-1}+n+1;-\lambda'\,\mu'^{-1})}{\prod_{i=1}^{n}((s+\nu')\,\mu'^{-1}+i)\,{}_1F_1^*(1;(s+\nu')\,\mu'^{-1}+1;-\lambda'\,\mu'^{-1})} F_0^*(s)$$

$$F_n^*(s) = \phi_n^*(s)\,F_0^*(s) \qquad \textbf{(16)}$$

where

$$\phi_n^*(s) = (\lambda'\,\mu'^{-1})^n \frac{{}_1F_1^*(n+1;(s+\nu')\,\mu'^{-1}+n+1;-\lambda'\,\mu'^{-1})}{\prod_{i=1}^{n}((s+\nu')\,\mu'^{-1}+i)\,{}_1F_1^*(1;(s+\nu')\,\mu'^{-1}+1;-\lambda'\,\mu'^{-1})} \qquad \textbf{(17)}$$

Clearly, for any $s>0$,

$$\sigma_0\,R^*(s) + \sigma\sum_{n=0}^{\infty} F_n^*(s) = \frac{b\sigma_0}{s} \text{ and } R^*(s) = \frac{b}{(s+\eta'')} + \nu''\frac{\sum_{n=1}^{\infty}\phi_n^*(s)}{(s+\eta'')} F_o^*(s)$$

From substituting from Eq. (16) in Eq. (11), it gives that

$$(\sigma s+\lambda)\,F_0^*(s) = \eta\left\{\frac{b}{s} - \frac{\sigma}{\sigma_0}\sum_{n=1}^{\infty}\phi_n^*(s)\,F_0^*(s) - \frac{\sigma}{\sigma_0}F_0(s)\right\} + \mu\,\phi_1^*(s)F_0^*(s)$$

Also, knowing that

$$F_0^*(s) = \frac{\eta' b}{s}\left[s+\lambda'+\eta''-\mu'\phi_1(s)+\eta''\sum_{i=1}^{\infty}\phi_i(s)\right]^{-1} \qquad \textbf{(18)}$$

then simplification of Eq. (18) reduces to

$$F_0^*(s) = \frac{\eta' b}{s}\sum_{j=1}^{\infty}\frac{(-1)^j}{(s+\lambda'+\eta'')^{j+1}}\left[\sum_{k=1}^{\infty}(\eta''-\delta_{k1}\mu')\phi_k^*(s)\right]^j \qquad \textbf{(19)}$$

On the other side, we get

$$R(x) = b\,e^{-\eta'' x} + \nu''\int_0^x\left(F_0(z)\sum_{i=1}^{\infty}\phi_i(z)\right)e^{-\eta''(x-z)}dz \quad , \qquad \textbf{(20)}$$

$$F_0(x) = \eta' b\sum_{j=0}^{\infty}(-1)^j\int_0^x e^{-(\lambda'+\eta'')z}\,\frac{z^j}{j!}\left[\sum_{i=1}^{\infty}\left(\eta''-\delta_{i1}\mu'\right)\phi_i(z)\right]^{*j} dz\,. \qquad \textbf{(21)}$$

Also, for $F_n(x)$ inversion Eq. (16), it gives

$$F_n(x) = \phi_n(x) * F_0(x) \quad \textbf{(22)}$$

where $\lambda' = \frac{\lambda}{\sigma}$, $\mu' = \frac{\mu}{\sigma}$, $\nu' = \frac{\nu}{\sigma}, \nu'' = \frac{\nu}{\sigma_0}$, $\eta' = \frac{\eta}{\sigma}, \eta'' = \frac{\eta}{\sigma_0}$, δ_{i1} is the Kronecker Delta, the symbol $*$ refers to the convolution , $*j$ refers to $j-$fold convolution and $F_0(x)$ given in Eq. (20).

Also, $\phi_n(x)$ is the inverse Laplace transform of $\phi_n^*(s)$ (see the Appendix).

$$\phi_n(x) = (\lambda')^n \sum_{j=0}^{\infty} (-\lambda')^j \binom{n+j}{j} f_{n+j}(x) * \sum_{l=1}^{\infty} (\lambda')^l b_l(x) \quad \textbf{(23)}$$

for

$$f_n(x) = \frac{1}{(\mu')^{n-1}} \sum_{r=1}^{n} \frac{(-1)^{r-1}}{(r-1)!(n-r)!} e^{-(\nu'+r\mu')x}, n = 1,2,3,...$$

$$b_n(x) = \sum_{i=1}^{n} (-1)^{i-1} f_i(x) * b_{n-i}(x), n = 2,3,4,...; b_1(x) = f_1(x).$$

4- Stationary distribution of buffer content.

Under equilibrium conditions, the structure of the stationary probability distribution of the buffer's fluid level is examined.

Let $H(x) = \lim_{t \to \infty} P(X(t) \le x), x \ge 0$ denote the stationary cumulative distribution function of the buffer content. Hence the stationary distribution of the buffer content is given by

$$F(x) = R(x) + \sum_{i=0}^{\infty} F_i(x)$$

Or $$F(x) = \frac{\sigma_0 b}{\sigma} + (1 - \frac{\sigma_0}{\sigma}) R(x) \quad \textbf{(24)}$$

where $R(x)$ is given by (20).

5- Performance analysis

Some particular performance measures of the model pertaining to the fluid level in the buffer are used such as the buffer nonempty probability, the throughput, and the expected buffer content are studied

(i)The credit buffer non-empty probability (P_{NEB}) is given by

$$P_{NEB} = P(X > 0) = 1 - F(0) = 1 - b \quad \textbf{(25)}$$

(ii)The throughput, (T_{Fluid}) of the fluid commodity in the fluid system is determined by

$$T_{Fluid} = output\,rate \times P(X > 0)$$

$$= -\sigma_0 P(X > 0) = -\sigma_0 [1 - F(0)] = -\sigma_0 (1 - b) \quad \textbf{(26)}$$

(iii) The expected buffer content (X) can be written as:

$$E(X) = \int_0^\infty [1 - F(x)] dx = \int_0^\infty \left[1 - \frac{b\sigma_0}{\sigma} - (1 - \frac{\sigma_0}{\sigma}) R(x) \right] dx \quad \textbf{(27)}$$

6- Numerical analysis

In this section, graphical findings are shown to examine how the probability P_{NEB} behaves. The expected buffer content (X), say $E(X)$ with various parameters i.e. arrival rate λ and service rate (μ) for the service times of the server.

Using the results developed earlier and study numerically the effect of the system parameters on various performance measures of the fluid queue such as the buffer non-empty probability, the mean flow transfer time of fluid commodity, and the cumulative distribution function of the fluid commodity in the stationary situation.

We first examine the effects of the system parameters on the buffer non-empty probability, P_{NEB}, of the fluid commodity in Figures 1(a)-1(b). The behavior of the descriptor P_{NEB} is plotted as a function of the depletion rate σ_0 for the depletion rate σ_0 =-3, -4, -8 and -10 of the driven queue in Figure 1(a). The results indicate that the curves P_{NEB} increase while the absolute values of σ_0 decrease, whereas they decrease as the values of σ increase for a fixed value of σ_0.

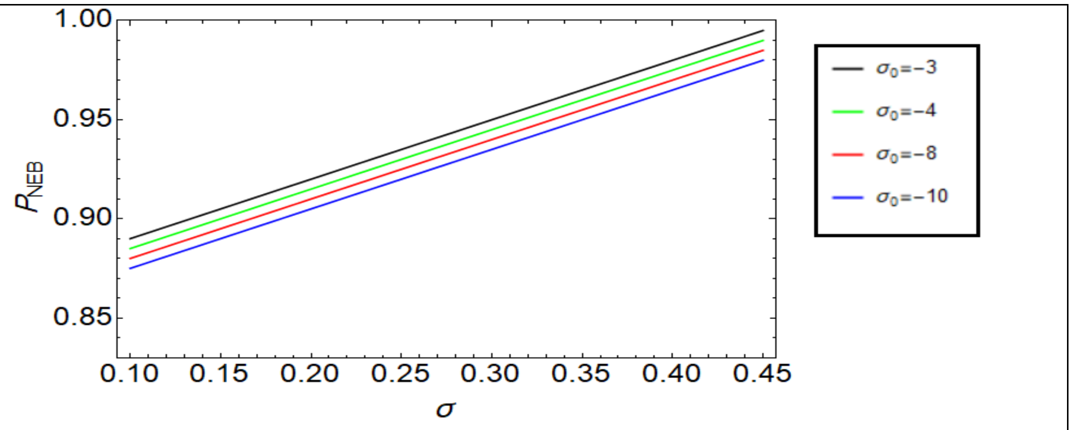

Fig.1(a) $\lambda = 2, \mu = 4, \nu = 1, \eta = 4$

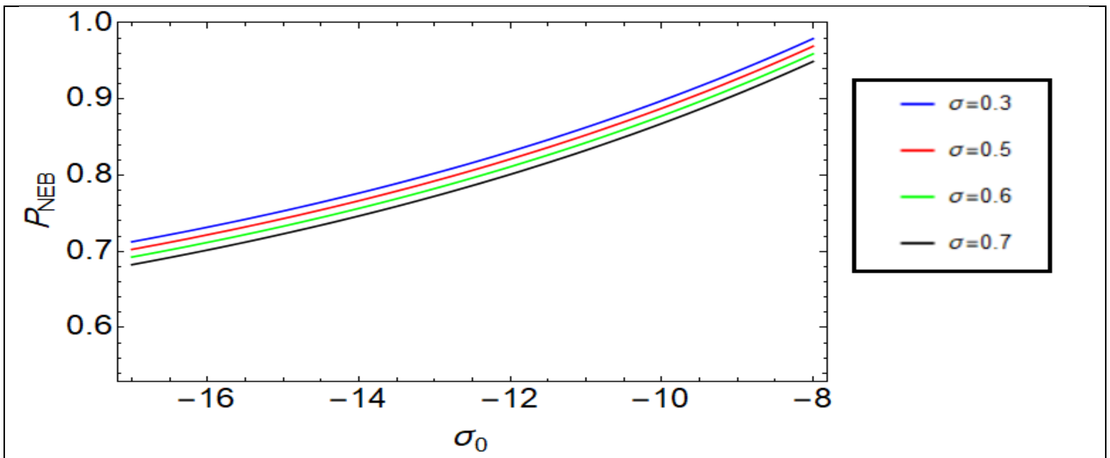

Fig.1(b) $\lambda = 2, \mu = 4, \nu = 1, \eta = 4$

As shown in figure 1(a) all four curves of P_{NEB} increase in a linear style with σ however, they decease while the absolute values of σ_0 increase for given σ where in figure 1(b) all the carves of the description P_{NEB} concerning to σ increase when the absolute values of σ_0 are decreasing, and they increase further with increasing values of σ for a taken value of σ_0

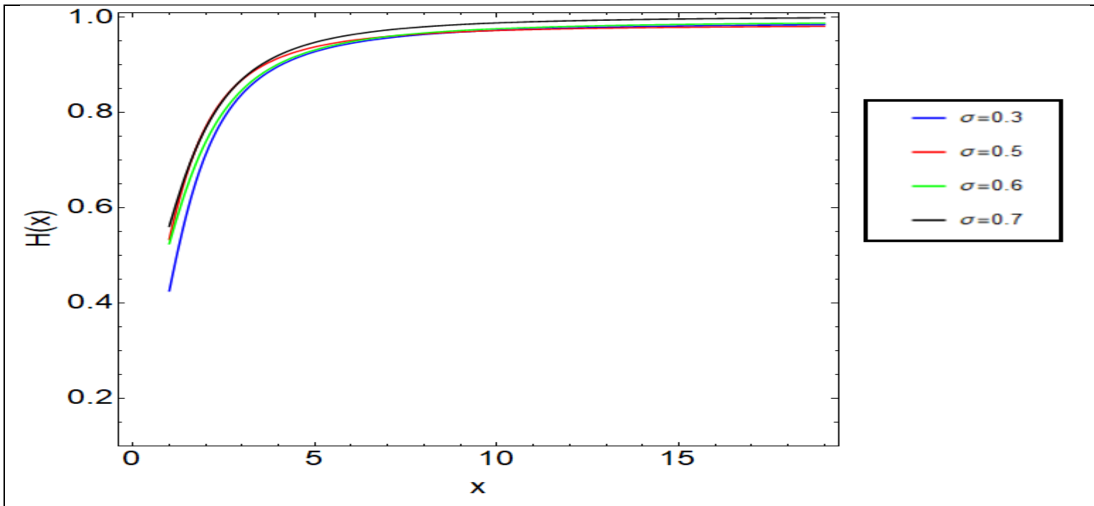

Fig.2(a) $\lambda=2, \mu=4, \nu=1, \eta=4, \sigma_0=-3$

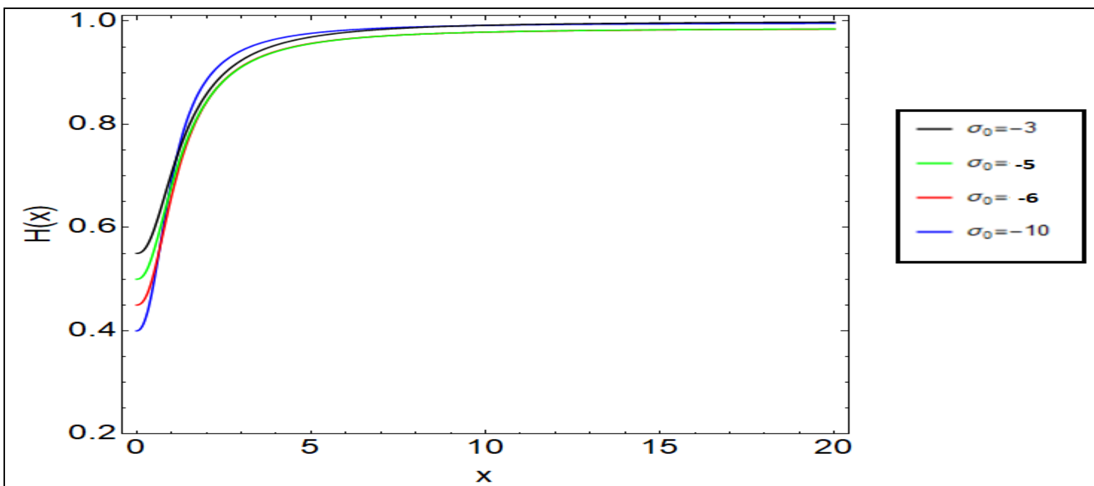

Fig.2(b) $\lambda=2, \mu=4, \nu=1, \eta=4, \sigma=0.5$

It is also noted in Figures 2(a) and 2(b) that $H(x)$ increases monotonically with the increases of buffer content x. Furthermore, observed in Figures 2(a), 2(b) that the buffer content distribution increase with the increase of the depletion rate σ_0 and input rate σ.

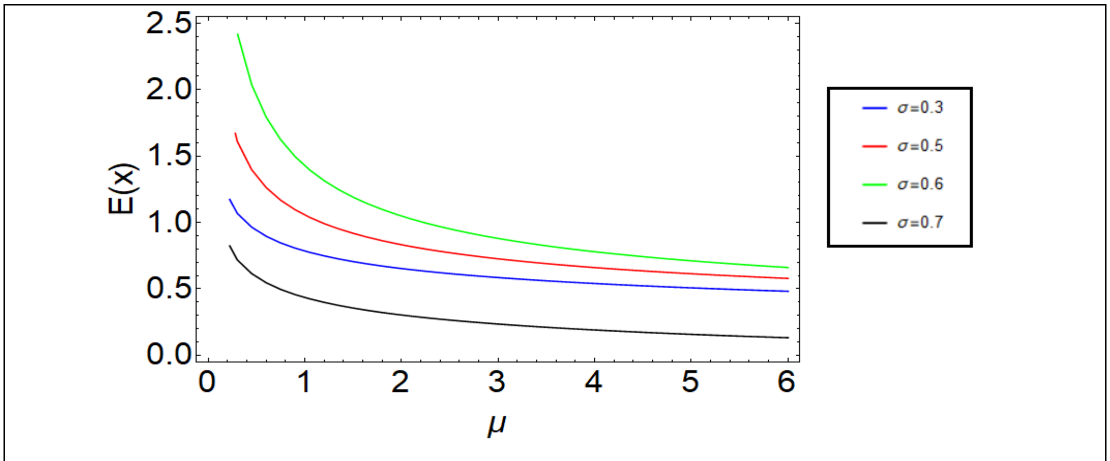

Fig.3(a) $\lambda = 2, \nu = 1, \eta = 4, \sigma_0 = -3$.

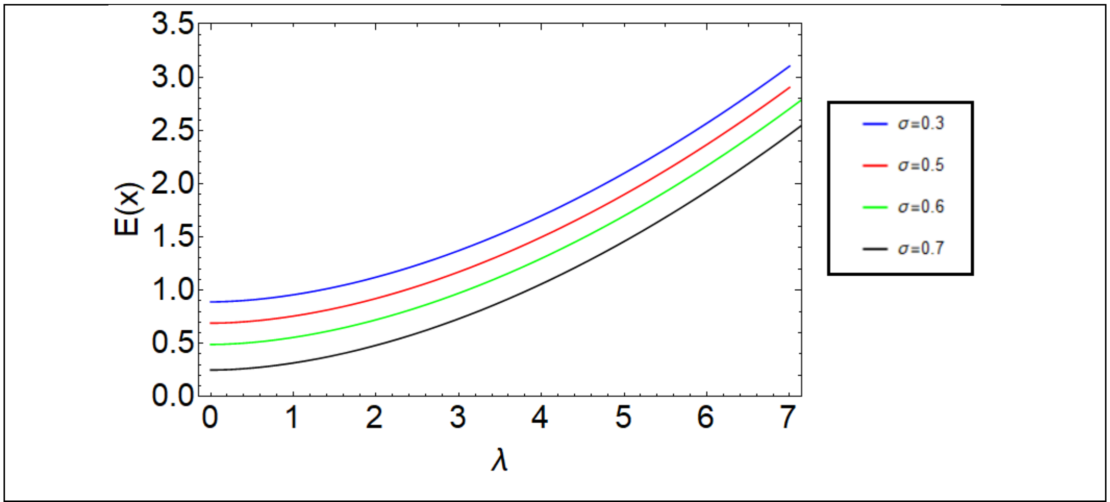

Fig.3(b) $\mu = 10, \nu = 1, \eta = 4, \sigma_0 = -3$

The expected puffer content , $E(x)$ is increase with the increase of arrival rate λ and decrease with of the service rate μ and input rate σ.

7-Conclusions

We investigated a fluid buffer modulated by an infinite-server system with catastrophic breakdowns and repairable servers. By combining Laplace transforms with continued-fraction techniques, we obtained exact stationary results for the buffer content distribution and operational measures. The main theoretical contributions are: (i) formulation of a new class of fluid queues that integrates catastrophes and repairs into the M/M/∞ framework, (ii) development of a continued-fraction solution connected to confluent hypergeometric functions, and (iii) explicit stability criteria and closed-form performance metrics. These additions extend existing fluid queue theory and demonstrate the analytical tractability of models with large-scale failures. The results are not only of mathematical interest but also directly applicable to domains where sudden service collapses occur, such as call centers subject to mass call drop, cloud and edge computing platforms vulnerable to outages, and production-inventory systems experiencing collective machine breakdowns. Future work may adapt the methodology to networks of fluid queues, heavy-tailed failure distributions, and control policies for disaster resilience.

Appendix: derive the expression for $\phi_n(x)$ (see Gulab Singh Bura [3] and Sudhesh R. [9].

From Eq. (22)

$$\phi_n^*(s) = (\frac{\lambda'}{\mu'})^n \frac{{}_1F_1^*(n+1;(s+\nu')\mu'^{-1}+n+1;-\lambda'\,\mu'^{-1})}{\prod_{i=1}^{n}((s+\nu')\,\mu'^{-1}+i)\,{}_1F_1^*(1;(s+\nu')\,\mu'^{-1}+1;-\lambda'\,\mu'^{-1})}$$

It is known that

$${}_1F_1^*(n+1;(s+\nu')\,\mu'^{-1}+n+1;-\lambda'\,\mu'^{-1}) = \sum_{k=0}^{\infty} \frac{(n+1)_k}{\left((s+\nu')\,\mu'^{-1}+n+1\right)_k} \frac{(-\lambda'\,\mu'^{-1})^k}{k!}$$

Where $(\Phi)_k$ is known as the Pochhammer symbol, defined as

$$(\Phi)_k = \begin{cases} 1, & k=0 \\ \Phi(\Phi+1)(\Phi+2)...(\Phi+k-1), & k=1,2,3,... \end{cases}$$

Therefore,

$$\frac{{}_1F_1^*(n+1;(s+\nu')\,\mu'^{-1}+n+1;-\lambda'\,\mu'^{-1})}{\prod_{i=1}^{n}\left((s+\nu')\,\mu'^{-1}+i\right)} = (\mu')^n \sum_{k=0}^{\infty} \frac{\binom{n+k}{k}(-\lambda')^k}{\prod_{i=1}^{n+k}(s+\nu'+i\mu')}$$

By resolving into partial fractions, given that

$$\frac{{}_1F_1^*(n+1;(s+\nu')\,\mu'^{-1}+n+1;-\lambda'\,\mu'^{-1})}{\prod_{i=1}^{n}\left((s+\nu')\,\mu'^{-1}+i\right)} = \mu' \sum_{k=0}^{\infty} \binom{n+k}{k} (-\lambda'\,\mu'^{-1})^k \sum_{i=1}^{n+k} \frac{(-1)^{i-1}}{(i-1)!(n+k-i)!} \frac{1}{s+\nu'+i\mu'}$$

Also,

$${}_1F_1^*(1;(s+\nu')\,\mu'^{-1}+1;-\lambda'\,\mu'^{-1}) = \sum_{1=0}^{\infty} \frac{(-\lambda')^1}{\prod_{i=1}^{1}(s+\nu'+i\mu')} = \sum_{1=0}^{\infty}(-\lambda')^1\; f_1^*(s), \quad f_0^*(s)=1.$$

$$\left[{}_1F_1^*(1;(s+\nu')\,\mu'^{-1}+1;-\lambda'\,\mu'^{-1})\right]^{-1} = \sum_{1=0}^{\infty}(\lambda')^1\, b_1^*(s).$$

where $b_0^*(s)=1$ and for $k=1,2,3,...$

$$b_k^*(s) = \begin{vmatrix} f_1^*(s) & 1 & & & & \\ f_2^*(s) & f_1^*(s) & 1 & & & \\ \cdot & \cdot & \cdot & & & \\ \cdot & \cdot & \cdot & & & \\ \cdot & \cdot & \cdot & & & \\ \cdot & \cdot & \cdot & & & \\ f_{k-1}^*(s) & f_{k-2}^*(s) & f_{k-3}^*(s) & \cdot\;\cdot\;\cdot & f_1^*(s) & 1 \\ f_k^*(s) & f_{k-1}^*(s) & f_{k-2}^*(s) & \cdot\;\cdot\;\cdot & f_2^*(s) & f_1^*(s) \end{vmatrix} = \sum_{l=1}^{\infty} (-1)^{l-1} b_{k-l}^*(s)\, f_l^*(s).$$

From all the above formulas, hence

$$\phi_n^*(s) = (\lambda')^n \sum_{j=0}^{\infty} (-\lambda')^j \binom{n+j}{j} f_{n+j}(s) * \sum_{l=1}^{\infty} (\lambda')^l\, b_l^*(s)$$

On inversion,

$$\phi_n(x) = (\lambda')^n \sum_{j=0}^{\infty} (-\lambda')^j \binom{n+j}{j} f_{n+j}(x) * \sum_{l=1}^{\infty} (\lambda')^l\, b_l(x)$$

Where,

$$f_l(x) = \frac{1}{(\mu')^{l-1}} \sum_{r=1}^{l} \frac{(-1)^{r-1}}{(r-1)!(l-r)!} e^{-(\nu' + r\mu')x},\ l = 1, 2, 3, \ldots$$

$$b_l(x) = \sum_{i=1}^{l} (-1)^{i-1} f_i(x) * b_{l-i}(x),\ l = 2, 3, 4, \ldots; b_1(x) = f_1(x).$$

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Analysis of fluid system fed by an infinite servers queue subject to catastrophes servers and reparable

الملخص:

تبحث هذه الدراسة في نموذج طوابير انتظار سائلة يتضمن طوابير انتظار ماركوفية لانهائي الخدم، تخضع لآليات الأعطال والإصلاح. يُشتق التوزيع المستقر لمحتوى المخزن المؤقت باستخدام دوال هندسية فائقة الترابط، مما يُظهر أهميته في تحليل أنظمة الطوابير. يُقدم هذا التحليل المستقر رؤى قيّمة وتطبيقات عملية في شبكات الاتصالات والحاسوب المعاصرة. تُفيد نماذج طوابير الانتظار السائلة في تقييم نقل حزم البيانات، وديناميكيات تدفق البيانات في الشبكات عالية السرعة، وإدارة الطاقة في أنظمة البطاريات القابلة لإعادة الشحن. يتم الحصول على حلول صريحة الشكل لاحتمالات الحالة المستقرة المشتركة ومؤشرات الأداء الرئيسية، مع تقديم أمثلة عددية للتحقق من صحة النتائج التحليلية.

الكلمات المفتاحية: الطوابير لا نهائية الخدم، التحليل المنتظم، دوال فوق هندسية فائقة، أعطال مفاجئة، اعادة اصلاح الخدم.