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Effect of coupled rotational and transverse vibration on the vortex structures around a rectangular cross-section

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Abstract. Rectangular structures subjected to turbulent flows are subject to vortex shedding, which induces dynamic forces that depend on the flow conditions and structural motion. Structures with rotational and transverse structural modes may be excited by vortex shedding to oscillate, inducing vibration and fatigue problems. This work investigates the coupled torsional and transverse vibration of a rectangular cross-section with an aspect ratio of 4. To illustrate the effect of mode coupling, three cases are considered: rotation only, rotation and transverse vibration with the same natural frequencies, and rotation and transverse vibration with different natural frequencies. Results show that allowing the structure to vibrate with the two modes with the same natural frequency results in higher vibration amplitudes. Flow fields indicate that higher vibration amplitudes are associated with a wider wake, a distinctive vortex shedding pattern, a widespread turbulent field, and a longer vortex reattachment length on the side of the structure.
Keywords: Rectangular cylinder, vortex-induced vibration, coupled modes

1. Introduction

When fluid flows over a structure, periodic vortex shedding generates forces dependent on the cross-section geometry, angle of incidence of flow, and velocity. If the structure is flexible, it may undergo vortex-induced vibration (VIV), which may induce cyclic stresses leading to fatigue failures [1,2]. The vortex shedding frequency is governed by the Strouhal number, which is a function of upstream velocity and characteristic length. On the other hand, structural vibration has modal frequencies, which are a function of the stiffness-to-inertia ratio. When the flow velocity is increased, the vortex shedding frequency approaches the natural frequency of the structure, and a resonance-like "lock-in" phenomenon occurs. This leads to an increase in the vibration amplitude, generating large dynamic forces and cyclic stresses, and can potentially accelerate fatigue and reduce structural strength.

While the VIV of cylindrical and slender elastic structures has been extensively studied, less work has addressed rectangular cross-sections. Rectangular structures under fluid flow can encounter large dynamic forces due to vortex shedding and its interaction with the body [3,4]. Several studies considered the transverse vibratory motion of solid bodies with bluff cross-sections, where the movement is normal to the structure axis and the main direction of the flow due to its applications in wind engineering [5,6], ocean engineering [7], energy harvesting [8,9], and others. The vibration response is a function of several parameters, including the mass ratio, i.e., the ratio of the structure mass to the displaced fluid mass, the damping ratio compared with its critical value, and the flow velocity [10].

Unlike circular structures, where the motion is predominantly transverse, rectangular sections also experience rotational vibration modes. Matsumoto et al. [11] experimentally observed that both the torsional and transverse vibration of a rectangular cylinder can be excited by both vortex-induced forces and shear layer excitation. In addition, Daniels et al. [12] have shown that flow turbulence intensity and aspect ratios are significant parameters in both transverse and torsional excitation mechanisms. Mohany et al. [13] showed that the shedding pattern downstream of a rectangular cylinder can be influenced by coupling with incident harmonic oscillations.

Notably, studies on the transverse and torsional vibration of rectangular structures report them independently. However, the two modes are expected to coexist in many structures and may interact with each other if their frequencies are sufficiently close [14]. In this work, the coupled torsional and transverse vibration of a rectangular cross-section of an aspect ratio of 4 under flow excitation is numerically modeled. The dynamic forces applied to the structure are predicted by a computational fluid dynamic model, and the motion of the structure is predicted by a fully coupled rigid body motion solver. The effect of changing the flow velocity and the frequency ratio of the two coupled modes on the flow structures is investigated to explore the aerodynamic characteristics of the flow field under different excitation conditions.

2. Methodology

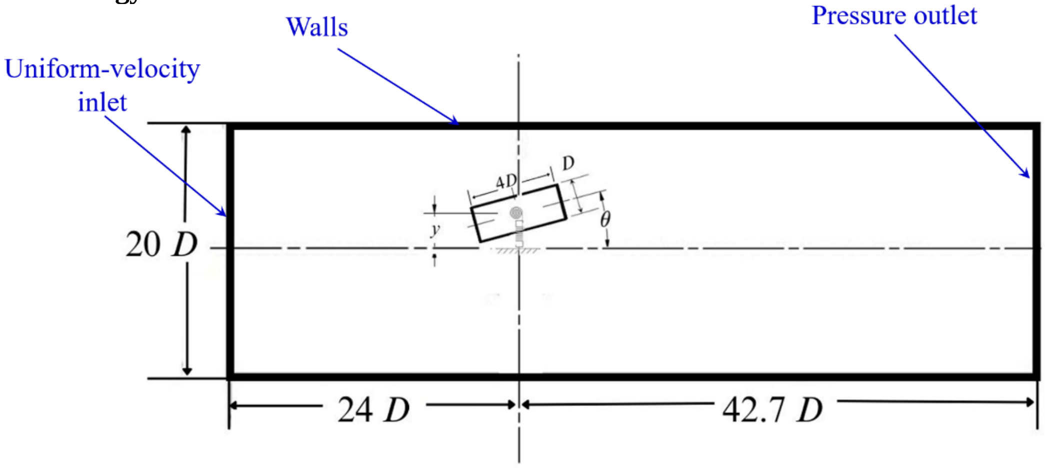

Figure 1. Numerical domain configuration and nomenclature

The computational model incorporates a rectangular cross-section with a crosswise edge length of 0.075 m and a streamwise edge length of 0.3 m, the same configurations previously utilized by Daniels et al. [12]. It models an incompressible two-dimensional turbulent flow around the structure utilizing the Reynolds-Averaged Navier-Stokes (RANS) equations. The incompressible Navier-Stokes equations are represented as Equations 1 and 2.

$$\nabla \cdot \mathbf{u} = 0 \tag{1}$$

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho(\mathbf{u} \cdot \nabla)\mathbf{u} = -\nabla p + \mu \nabla^2 \mathbf{u} \tag{2}$$

Where, **u** is the normalized upstream flow velocity vector, ρ is the fluid density, μ is the fluid viscosity, and p is the normalized pressure. The equations of motion for the structure are defined in Equations 3 and 4, indicating that the two vibration modes operate independently from a structural perspective [12].

$$F_y = my + k_y\ y \tag{3}$$

$$T_z = I\theta + k_\theta\ \theta \tag{4}$$

Where, F_y is the transverse fluid force, T_z is the fluid-induced torque, m is the mass of the structure per unit length, I is the cross-section moment of inertia per unit length, k_y is the transverse stiffness, k_θ is the rotational stiffness, y is the transverse displacement, and θ is the angular displacement.

Fluid forces are coupled through the vortex-shedding process, which may lead to mode coupling due to fluid-structure interactions. The model omits gravitational forces in both fluid dynamics and structural analyses. The k−ω SST model is used to model turbulence effects. The cross-section is permitted only linear motion in the y-direction, perpendicular to the flow and the cylinder z-axis. No motion is allowed in the streamwise or axial directions. A uniform stiffness is applied in the torsional and transverse orientations. An iterative solver from the OpenFOAM open-source library is employed for calculations, updating pressure and velocity fields to attain convergence to absolute residuals of 10^{-6}. A second-order numerical method for convection and flux terms guarantees the model's stability and accuracy. The model forecasts the vibrational response of a structure, including a uniform rectangular cross-section with an aspect ratio of 4 and a longer edge aligned along the flow direction. The Grid Convergence Index method [15,16] was used to evaluate the spatial convergence of the computational grid. Three mesh configurations were used, with a refinement ratio of r_{21} = 1.4. The M2 mesh configuration showed an acceptable mesh convernge with a convergence index GCI_{21}= 1.4%.

Table 1 presents the vibration response of a rectangular cross-section with an aspect ratio of 4 for three cases, namely rotation vibration only, rotation and transverse vibration with the same natural frequencies, and rotation and transverse vibration with different natural frequencies at to flow velocities 9 m/s and 16 m/s. At these two velocities, conditions of peak resonance and no resonance have been observed. Table 1 presents the natural frequencies of the transverse and rotational vibration modes for the three cases considered in this study.

Table 1. Structural mode frequencies for the three considered cases

	Rotation only	Rotation and transverse vibration	
		$f_y = f_\theta$	$f_y \neq f_\theta$
f_y [Hz]	-	21.5	13.3
f_θ [Hz]	21.5	21.5	21.5

3. Results

The model prediction for the pure rotational motion was compared with the reported values in the studies of Matsumoto et al. [11] and Daniels et al. [12]. Results in Table 2 show that the root mean square of the torsional vibration angle is in good agreement with the values in the two studies. The model predicted that the torsional vibration reaches a root mean square θ = 2.49° at a reduced velocity of 5.6. This is the peak vibration within the lock-in range between normalized velocities of 4 and 7, indicating its reliability in predicting the vibration amplitude of this structural configuration.

Table 2. Model Verification

$\frac{U_\infty}{f_\theta D}$	θ_{rms}		
	Mutsumoto et al. [10] Experimental	Daniels et al [11] 3D LES	Current Study 2D RANS
5.0	2.38	2.25	2.00
5.6	2.31	2.53	2.49
6.8	0.48	0.34	0.30

Table 3 shows the flow-excited vibration response of three different configurations at two different upstream flow velocities. At U_∞ = 9 m/s, vortex shedding occurs at the same frequency as the natural torsional mode frequency. Therefore, the highest vibration amplitude is observed. The torsional vibration response is expressed in terms of the root mean square of the rotation angle. For an upstream velocity U_∞= 9 m/s, the structure oscillates around its centroid with an angle θ_{rms} = 2.5° when constrained to only rotate. If the object is allowed to rotate and oscillate normal to the flow, the vibration amplitude increases. The increase is small, about 7%, if the torsional mode frequency is different from the transverse vibration mode. If the two modes have the same natural frequencies, the vibration angle increases significantly by around 34%. This behavior is partially the same when the flow velocity is increased to 16 m/s. At this flow velocity, the vortex shedding frequency differs from the natural torsional vibration mode frequency. The vibration amplitude is low, with θ_{rms} = 0.13° if only rotation is allowed. If the structure is allowed to oscillate normal to the flow at the same time with the same natural frequency, the oscillation angle increases 13 times to 1.69°. However, the torsional vibration almost stops when the two natural frequencies are different. This shows that the difference in the mode frequencies works to divert energy from one mode to another and results in no rotational motion. This behavior could be useful if a reduction of vibration is desired.

Table 3. Vibration response characteristics

U_∞ [m/s]	θ_{rms} [deg]		
	Rotation only	$f_y = f_\theta$	$f_y = 0.65 f_\theta$
9	2.5	3.35	2.67
16	0.13	1.69	0.01

Figure 2 shows the flow velocity pattern around the structure at its peak rotation angle during the vibration cycle for the same cases in Table 3. The flow field is significantly affected by the vibration amplitude such that the reduced flow velocity region around the structure is wider for the higher flow velocity. The flow velocity is lower, and a wake region is observed at higher flow velocity when the structure is allowed to vibrate normal to the flow in addition to rotation. The flow field shows a composite shear layer pattern on the side of the structure when it is oriented at a higher angle, which results in the flow reattachment on the side, as seen for all three cases with U_∞ = 9 m/s. While the three cases have reattachment patterns, differences in the flow velocity and reattachment points can be attributed to the vertical component of the structure velocity. It is important to note that the transverse vibration amplitude is small, reaching only about 0.1 *D*.

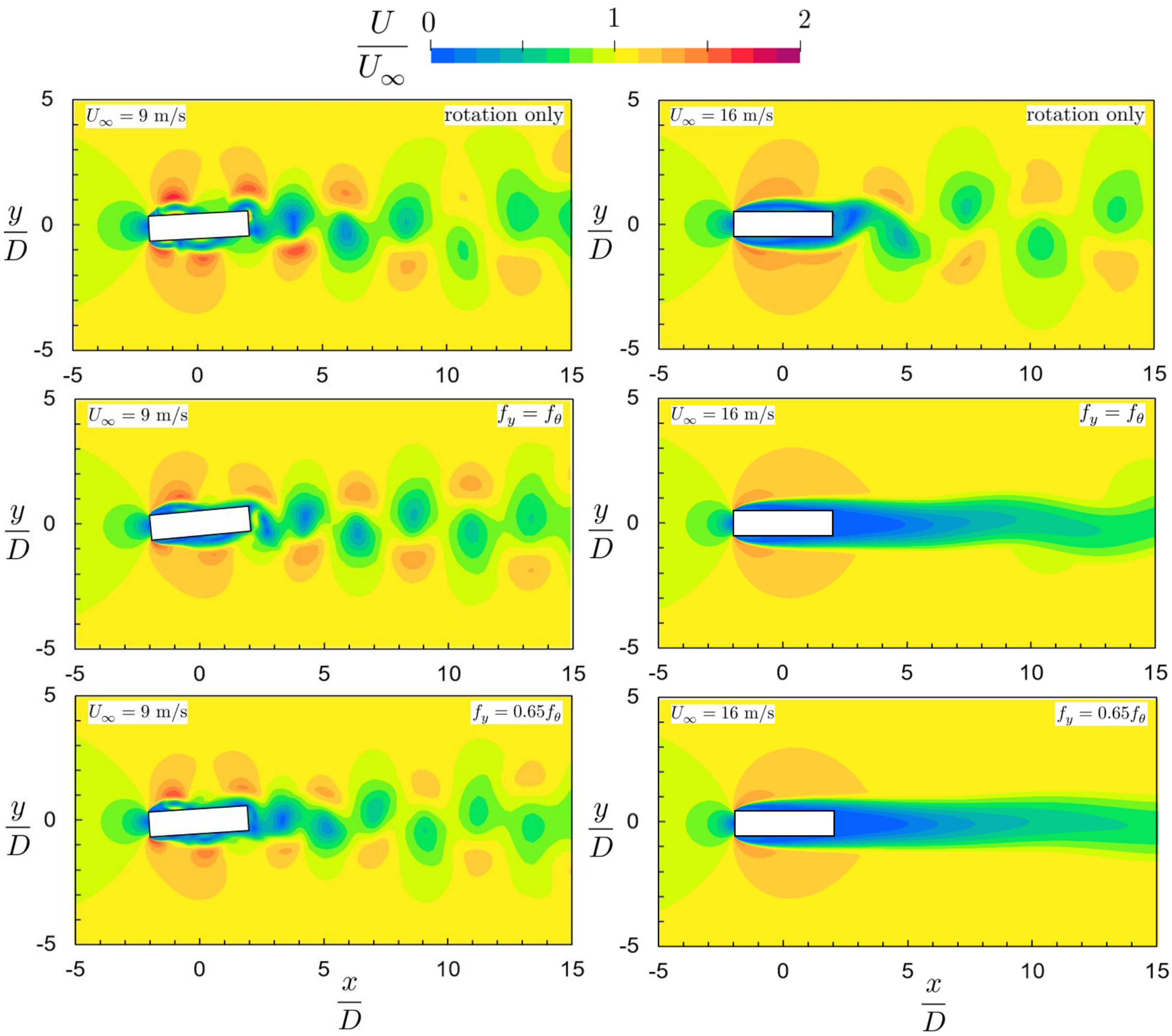

Figure 2. The flow velocity pattern at two different flow velocities for the three considered configurations.

Moreover, the case with two different natural frequencies has a wake that is more entrained around the structure than the two other cases. On the other hand, the lower vibration amplitudes are associated with a narrow wake of a width in the order of $1.5D$. Even as the structure oscillates slightly, the wake only starts to oscillate at a downstream distance of $10D$ from the centroid. This wake oscillation is similar to that observed downstream of a bluff body or a series of tandem bluff bodies, indicating that the von-Karman instability is inhibited. This behavior suggests that passive obstacles that inhibit the generation of von-Karman vortices can effectively reduce the vibration amplitude.

Figure 3 presents the vortex shedding patterns downstream of the three configurations at the same flow velocities reported in Table 3. The vorticity is normalized by the flow velocity and the cross-section height. The figure shows that the main difference between the two flow velocities is the reattachment of the shear layers separated at the leading edges. For the lower flow velocity, the shear layers shed into vortices that interact with the side of the structure. Other vortices are shed at the downstream edge of the structure. At the higher flow velocity, the structure may shed vortices as a single bluff body just at the leading edge. Moreover, Figure 3 shows that at the lower flow velocity, the vibration amplitude is associated with the reattachment length of the vortices. The case where the structure is only allowed to rotate shows smaller vortices at the leading edge.

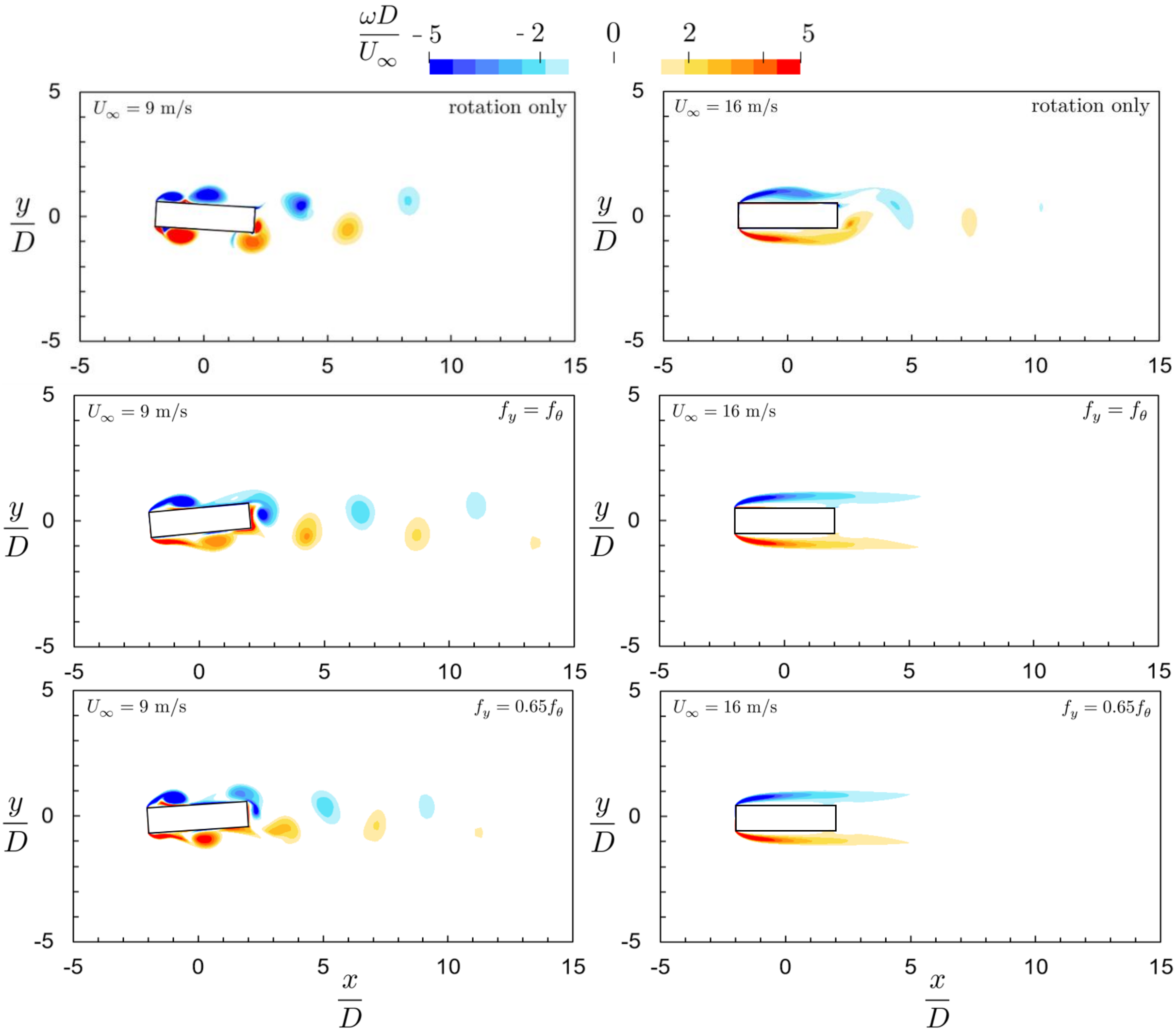

Figure 3: The vortex shedding pattern at two different flow velocities for the three considered configurations.

Figure 4 shows the fields of the turbulent kinetic energy k normalized by the upstream flow kinetic energy. The figure clearly shows that while higher amplitudes of vibration at the lower flow velocity result in a widespread turbulence over a wider wake due to the von-Karman instability, the other cases still induce significant turbulence levels in the narrower wake downstream of the cross-section despite the narrow wake.

Notably, the case where the structure is only allowed to rotate has shorter vortices comparable to the case with the case where the two mode frequencies are different. On the other hand, the higher flow velocity induces the two cases where translation is allowed to have a similar wave, strikingly different from the case where only rotation is allowed. It is important to note here that this significant effect of the translation mode comes in effect at really low amplitudes of the vertical motion, reaching only 1 mm.

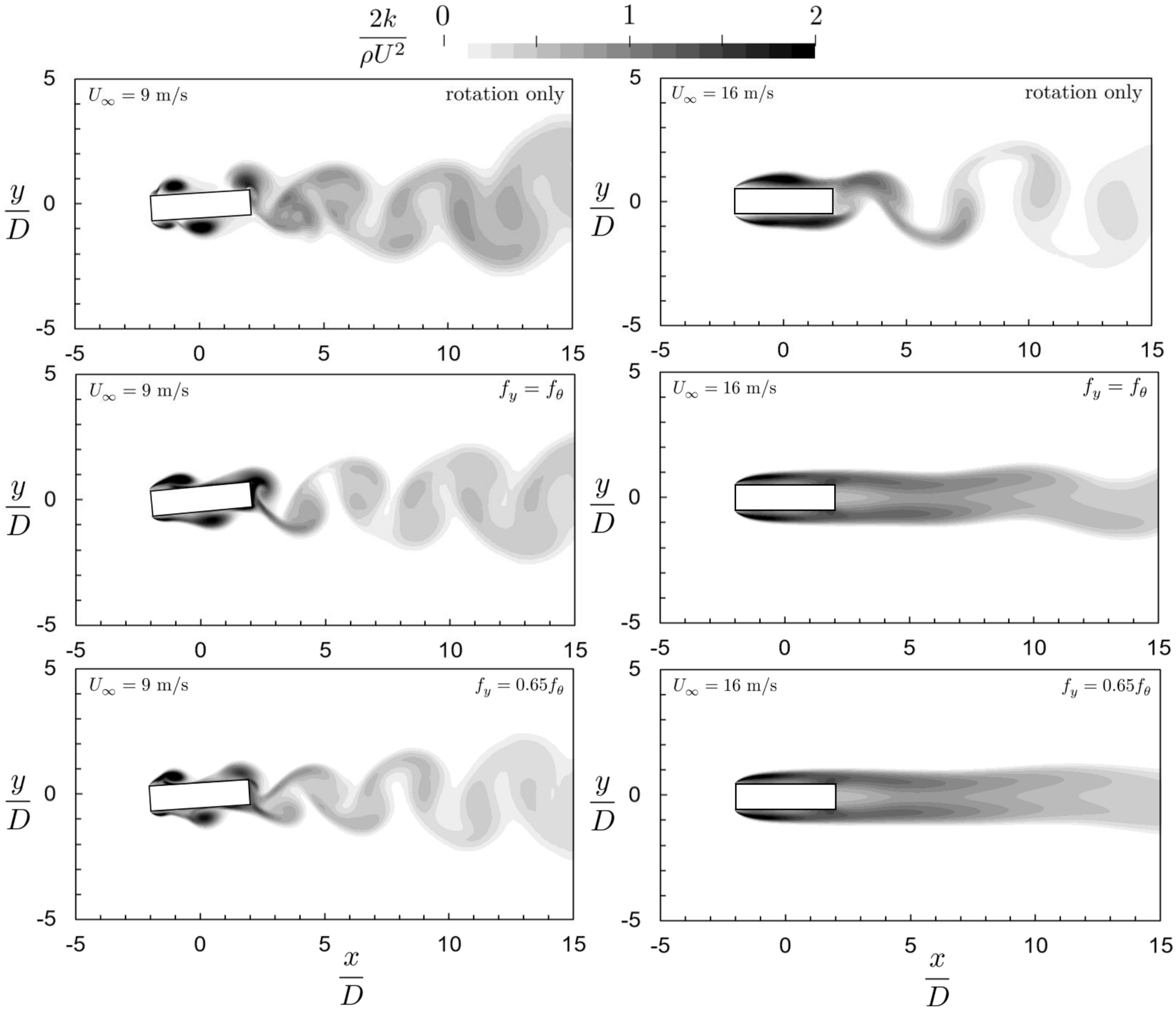

Figure 4. The turbulent kinetic energy pattern at two different flow velocities for the three considered configurations.

4. Conclusion

In this work, a coupled solver was used to predict the fluid dynamics and structural vibration of a rectangular cross-section with a length-to-height ratio of 4 when subject to smooth upstream flow. The structure was allowed to vibrate in three configurations: rotation only, coupled rotation, and transverse vibration with the same natural frequency, and coupled rotation and transverse vibration with different natural frequencies. Results show that allowing the structure to vibrate with the two modes with the same natural frequency resulted in higher vibration amplitudes both at the peak of the vortex-induced vibration instability and at higher flow velocities with lower vibration amplitudes. Flow fields indicate that higher vibration amplitudes are associated with a wider wake, a distinctive vortex shedding pattern, a widespread turbulent field, and a longer vortex reattachment length on the side of the structure. Observations in the current study indicate that shifting the structural vibration modes can be beneficial to reduce the vibration of a structure with a rectangular cross-section when subject to upstream flows. Consequently, vortex inhibition might offer a passive method to reduce vibration amplitudes.

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