



Statistical distance determination of the Camelopardalis area

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ABSTRACT

In this paper, we determine the distances to Camelopardalis area and generates the mean absolute magnitudes and the dispersions for the spectral types and subtypes. The method of calculation depends on the assumption that absolute magnitudes and apparent magnitudes follow a Gaussian distribution function. The effect of Malmquist bias has been studied to show what extent bias is effective in comparison. We estimate the distances and generate the mean absolute magnitude and dispersions of all spectral types and subtypes. The nonsystematic difference between the calculated distances for different spectral types are remarkable, this may be attributed to the different chemical compositions and evolution scenarios of each spectral type.

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1. Introduction

Camelopardalis (also known as the giraffe) is situated in the northern sky, this large but faint constellation is the eighteenth biggest in the night sky. It has a place with the Ursa Major family of constellations and is bordered by Draco, Ursa Minor, Cepheus, Cassiopeia, Perseus, Auriga, Lynx and Ursa Major and ought to be viewed as circumpolar.

The constellation was made by Petrus Plancius and recorded by the German astronomer Jakob Bartsch in 1624. Camelopardalis is occupying an area of 757 square degrees and seen at latitudes between +90° and -10°. Some of the stars in this constellation were used by William Crowell to form the constellation Scirus Volans in 1810. However, this did not catch on with later cartographers. Today, Camelopardis is one of the 88 constellations utilized by the IAU.

Trigonometric, spectroscopic and dynamical parallaxes are methods that can be used to determine distances to objects similar to somewhere in the range of some tens of parsec to some hundreds (Jenkins 1952; Wilson and Bappu 1957; Gleise 1978; Blitz 1980; Mihalas and Binney 1981). Zero age main sequence fitting and Moving star clusters are likewise two different techniques for distance determination (Blaauw 1973; Heck 1978).

Most important is the standard candle procedure, which used to estimate distances to nearby objects as well as to for remote galaxies and clusters of galaxies (Sandage and Tammann 1971; Gascoigne 1974; Iben and Tuggle 1975; Hartwick and Hutchings 1978; Martin et al. 1979; Vaucoulers

1979). The size distribution functions of the dark clouds, H II- region radii and globular clusters were used to determine distances of the astronomical objects (Issa 1980, 1981, 1982, 1985).

Calculating distance to the astronomical objects using statistical distributions is performed by many authors. Examples are, Sharaf et al. (2003) used the Gaussian distribution function to estimate cosmological distance, Abdel-Rahman et al. (2009) modified the method of Sharaf et al. (2003) by change the limits of the integral and derive the distance equation and, Abdel-Rahman et al. (2012) used the exponential distribution function to estimate the new distance equation.

In the present paper, we are going to estimate the distances to individual stars of different spectral types and subtypes included in Camelopardalis, depending on a self-generation of the mean absolute magnitude and dispersions. Also, we estimate some physical properties for Camelopardalis.

2. Observational data and method of analysis

We used the Gaussian approach (hereafter G_B) as suggested by Abdel-Rahman et al. (2009) to model the distribution of the absolute magnitude, therefore, the distance d could be determined from the following relation

$$d = 10^{1 + \frac{(m_1 - M_0 - \sigma y_B)}{5}} \quad (1)$$

where y_B is a solution of the following transcendental equation

$$G(y_B) = y_B - \left\{ e^{-\frac{z_1^2}{2}} + e^{-z_1 y_B} \right\} \left\{ \sqrt{\frac{\pi}{2}} \times \left[\operatorname{Erf} \left(\frac{y_B}{\sqrt{2}} \right) + \operatorname{Erf} \left(\frac{z_B}{\sqrt{2}} \right) \right] \right\} - \alpha_B = 0, \quad (2)$$

$$\alpha_B = \frac{m_1 - \bar{m}_{GB}}{\sigma}, \quad z_B = z_1 + y_B \quad \text{and} \quad z_1 = \frac{m_1 - m_g}{\sigma}.$$

where m_1 is the faintest apparent magnitude, m_g is the brightest apparent magnitude, σ is the dispersion, and \bar{m}_{GB} is the mean apparent magnitude given by:

$$\bar{m}_{GB} = \frac{\int_{m_g}^{m_1} m \psi(m) dm}{\int_{m_g}^{m_1} \psi(m) dm} = \frac{\int_{m_g}^{m_1} m e^{-\frac{(m+5-5\log d - M_0)^2}{2\sigma^2}} dm}{\int_{m_g}^{m_1} e^{-\frac{(m+5-5\log d - M_0)^2}{2\sigma^2}} dm} \quad (3)$$

The Erf is given by

$$\operatorname{Erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-\frac{x^2}{2}} dx \quad (4)$$

Malmquist (1924) have derived formula for the absolute magnitude of the form

$$M_0 = \bar{M} + 1.382 \sigma^2 \quad (5)$$

where \bar{M} is the average of the absolute magnitude of the sample.

Sharaf et al. (2005), used the percentage errors for the mean ($F_m(\sigma) = \left| \frac{r_s(\sigma) - \bar{r}}{\bar{r}} \right| * 100$) to select the optimum dispersion, where $r_s(\sigma)$ and \bar{r} are the statistical distances corresponding to dispersion interval and the average distance of the individual stars respectively. The optimum dispersion occurs at a minimum value of percentage errors.

We used the CCD observations of 1376 stars by, Zdanavičius and Zdanavičius (2005), covering an area of about 1.5 square degrees, centered at $\alpha(2000) = 3^h 55^m 55^s$, $\delta(2000) = +56^\circ 57' 05''$, ($l = 146^\circ$, $b = +2.6^\circ$). The observations were carried out with Maksutov-type 35/51 cm telescope of the Molėtai observatory in Lithuania. The data contains: α , δ , apparent magnitude, absolute magnitude, galactic longitude and latitude. Also, the parallax for single stars and the spectral and sub types and others parameters.

3. Results and discussions

3.1. Distances estimations and the generation of M_0 and σ

We estimate the distances and generate the mean absolute magnitudes and dispersions for the spectral types and subtypes whenever the number of stars is sufficient statistically.

All stars of early and late spectral types O, B, A, F, G, and M were used to derive the statistical distances. Whenever the number is sufficient, distances were determined in each association spectral subtypes.

The statistical approach described in section 2 is applied to the Camelopardalis area. The results are given in Tables 1–5.

In Table 1: column 1 is devoted for the Spectral type, column 2 for the limiting magnitude m_L , column 3 for m_g , column 4 for the mean apparent magnitude \bar{m} , column 5 includes the unbiased dispersion σ , column 6 gives the unbiased mean absolute magnitude M_0 , column 7 contains the parameter α , column 8 gives the solution of Equation (2) (y) and column 9 for the statistical distance r_s computed by the present method.

Table 1. The parameters and distances r_s of all – spectral types for Camelopardalis.

Spectral type	m_L	m_g	\bar{m}	σ	M_0	α	y	$r_s(\text{pc})$
O	10.82	10.05	10.38	1.57	-4.23	0.28	-0.71	3569 ± 293.5
B	15.97	9.14	14.17	1.58	-0.57	1.14	0.06	3971 ± 113.5
A	15.85	6.96	14.19	1.44	1.12	1.15	0.05	2274 ± 69
F	15.46	9.31	14.05	1.30	3.21	1.08	0.05	934 ± 30.7
G	14.47	10.58	12.99	1.34	1.81	1.11	0.07	1049 ± 91
K	14.73	10.67	12.97	1.34	1.84	1.32	0.11	1094 ± 77
M	13.49	11.11	12.41	1.38	-0.95	0.78	-0.09	2416 ± 525.75

Table 2. The parameters and distances r_s of A – spectral sub-types for Camelopardalis.

Spectral sub-types	m_1	m_g	\bar{m}	σ	M_0	α	y	$r_s(\text{pc})$
A0	15.47	10.8	13.75	1.45	1.86	1.07	0.06	2816.53 ± 235
A1	15.85	10.67	14.15	1.52	0.38	1.12	0.06	2720.59 ± 137.4
A2	15.55	6.96	14.10	1.40	0.77	1.04	0.03	2551.08 ± 188.5
A3	15.7	11.48	14.45	1.39	1.19	0.90	0.02	2306.46 ± 156.5
A4	15.62	11.95	14.37	1.40	1.37	0.90	0.02	2033.4 ± 137
A5	15.56	11.58	14.09	1.62	0.49	0.91	0.02	1920.49 ± 251.7
A6	15.56	10.79	14.14	1.36	1.38	1.05	0.05	2050.78 ± 365
A7	15.4	12.8	14.44	1.34	1.43	0.71	-0.08	2077.74 ± 210
A8	15.64	10.49	13.99	1.33	1.65	1.24	0.08	1960 ± 262
A9	15.47	10.8	13.75	1.4	1.86	1.23	0.09	1435.77 ± 226.7

Table 3. The parameters and distances of B – spectral sub-types for Camelopardalis.

Spectral sub-types	m_l	m_g	\bar{m}	σ	M_0	α	y	r_s (pc)
B0-B3	15.22	9.14	12.79	1.71	-1.85	1.43	0.11	3716.18 \pm 278
B4	15.97	14.1	14.78	1.56	-0.76	0.77	-0.22	5522.57 \pm 345.7
B5	15.74	11.42	14.15	1.59	-0.76	1.00	0.04	3879.57 \pm 335
B6	15.66	12.95	14.44	1.52	-0.68	0.80	-0.07	4488.11 \pm 285.8
B7	15.67	11.23	14.46	1.45	-0.33	0.83	0.01	4137.86 \pm 377.6
B8	15.88	10.43	14.64	1.49	-0.08	0.84	0.01	3793.24 \pm 184
B9	15.93	10.62	14.46	1.45	0.23	1.02	0.04	3522.55 \pm 261.4

Table 4. The parameters and distances r_s of F – spectral sub-types for Camelopardalis.

Spectral sub-types	m_l	m_g	\bar{m}	σ	M_0	α	y	r_s (pc)
F0	15.46	10.68	13.9	1.32	1.99	1.18	0.07	1556.41 \pm 182.3
F1	15.45	12.73	14.36	1.30	1.94	0.84	-0.02	1738.04 \pm 206.4
F2	15.43	9.31	13.95	1.34	2.68	1.11	0.05	1106.78 \pm 101.4
F3	15.28	11.12	14.22	1.29	2.86	0.83	0.01	1055.41 \pm 103.6
F4	15.36	10.77	14.22	1.27	3.23	0.89	0.02	941.08 \pm 74
F5	15.32	10.2	13.99	1.3	3.35	1.03	0.04	827.53 \pm 52.4
F6	15.34	9.83	13.95	1.33	3.66	1.04	0.05	679.24 \pm 36.7
F7	15.18	11.6	14.06	1.28	3.58	0.9	0.02	733.02 \pm 52.3
F8	15.36	9.49	14.19	1.27	3.96	0.92	0.02	679.65 \pm 37
F9	14.93	11.31	13.86	1.24	3.79	0.86	0.01	632.57 \pm 50.6

Table 5. The GB parameters and distances r_s to a Camelopardalis area according to spectral sub-types of G.

Spectral sub-types	m_l	m_g	\bar{m}	σ	M_0	α	y	r_s (pc) – GB
G0	15.12	12.09	14.14	1.21	3.644	0.81	-0.01	780.83 \pm 113.6
G1	15.05	12.02	14.18	1.2	3.787	0.73	-0.03	730.98 \pm 72.8
G2-G3	15.15	10.47	14.02	0.9	3.201	1.26	0.07	1432.01 \pm 413
G4	14.92	11.68	13.62	1.09	3.682	1.19	0.08	797.59 \pm 166
G5	14.93	12.33	13.84	1.24	2.171	0.88	-0.01	1353.41 \pm 242
G6	14.93	11.33	13.63	1.34	1.47	0.9723	0.04	1529.55 \pm 195
G7-G8	14.47	10.58	12.99	1.34	1.81	1.1097	0.07	1049.17 \pm 161

The significant difference between distances of the different spectral types indicates the difference in the chemical structure of the material in the Camelopardalis complex. The majority of stars closer than 1 kpc are F, G, and K main sequence stars while the majority of O and B types stars are at distances larger than 3 kpc and these distances coincide with Zdanavičius and Zdanavičius (2005). The majority of A and M stars are at the distances larger than 2 kpc and smaller than 2.5 kpc. This may be attributed to differences in the chemical constitution of the original cloud (Abdel-Rahman 2006).

Table 2 shows the results for A- spectral subtypes of Camelopardalis area. the headings of the columns are self-explained but column 1 is A- subtypes. We find that the distances of A0 and A1 are 2817 and 2721 pcs respectively, the difference in the distance is of about 100 pcs. Although both belong to the same association and spectral type, a 100 pcs difference is quite big to interpret. While the distances of A2 and A3 are 2551 and 2306, the difference about 200 pcs and for A4, A6, and A7 have nearly the same distances (the difference from 17 to 34 pcs), while the distance of A5 and A8 are 1920 and 1960, the difference is about 40 pcs, this means that in general a slight difference between their distances. This can be an indicator of some kind of differences in the chemical constituents of the original cloud. A9 falls at 1436 pcs, on the near side of the association and Earlier subtypes fall on the far edge of the association. Since our

distances are statistical, we believe that the difference between the average distance of all A-type stars and the distances to each class is significant. However, the difference in the distances of the spectral subtypes in one and the same subgroup may indicate another term of fine subgroupings. It is a quite big difference. If it is so, we expect fine subgroupings for each spectral subtype. The same can be deduced for the other spectral subtypes in different types (Abdel-Rahman 2006).

In Table 3 we give the statistical distances for B- spectral subtypes stars in Camelopardalis area. We found that the distance of B4 is greater than all distances of spectral subtypes. The difference in distances between B0-B3, B8, and B5 is about 77 to 164 pcs and B6, B7 is about 351 pcs. The distance of B9 is 3523 pcs which is lower than all distances of spectral subtypes.

We introduce in Table 4 the results for Camelopardalis area. The distances of F0 to F3 are 1556, 1738, 1107 and 1055 pcs and the differences in distances between F0 and F1 are approximately 182 pcs while F2 and F3 are approximately 52 pcs. The differences between F4 and F5 is 113 pcs and F6, F8, and F7 approximately 54 pcs while the distance of F9 is the smallest distance in F- subtypes and is equal to 633 pcs. If these distances are correct, then this may be evolutionary and can be attributed to differences in the chemical and physical constitution of the original interstellar cloud. i.e. F-spectral type is situated near the side of clouds.

In Table 5, The distances of G0 and G1 are 781, 731 pcs and the difference in the distance about 50 pcs while the difference between G2 – G3, G5, and G6 in the range of 100 and 200 pcs. The distances difference between G4 and G7-G8 are 798 and 1049 pcs respectively. That means that the G-spectral type is situated near the side of clouds. Again if these distances are correct, then this may be evolutionary and can be attributed to differences in the chemical and physical constitution of the original inter-stellar cloud.

3.2. Statistics of some parameters: the distances d , σ and M_0

For A spectral subtypes: we found that the minimum value of spectral subtypes is 1436 pcs for A9 and the maximum 2817 for A0 with range 1381 pcs and the mean distance is $2187 \text{ pcs} \pm 132 \text{ pcs}$, while the dispersion of spectral subtypes near the mean dispersion of A spectral type in the Table 1 has standard error ± 0.028 . The minimum mean absolute magnitude for spectral subtypes is 0.381 for A1 and the maximum is at 1.864 and the average is 1.24 ± 0.168 for A1. We note that the range of these distances is very big. Again, we expect that this can be an indicator of some kind of differences in the chemical constituents of the original cloud.

For B subtypes, we note that the minimum distance occurs at B9 while the maximum occurs at B4 and the range is 2000 pcs, is very big, although these spectral subtypes within the same clouds. The mean distance for all subtypes is 4151 ± 257 . The mean absolute magnitude and its dispersions near the mean dispersion in the Table 1.

The distances of F spectral subtypes are between 632 and 1738 and the mean distance is $995 \text{ pcs} \pm 121 \text{ pcs}$. The range in mean absolute magnitude is 2 and the average is 3.1 ± 0.23 while the range of dispersion is 0.1 and the average is 1.3 ± 0.01 .

The range of distance of G spectral subtypes is 800 pcs approximately and the mean distance for F spectral subtypes is $1096 \pm 128 \text{ pcs}$, and the range of mean absolute is 2.3 and the average is 2.8 ± 0.37 while the range of dispersion is 0.446 and the average is 1.19 ± 0.06 .

The above statistics and distance calculations show that there are large differences in spectral distances between each other and that the range between them is very large. We think that the difference is due to the difference in the chemical composition of the cloud or that these stars fall in different groups in the Camelopardalis area as the size is very large (757 square degrees and seen at latitudes between $+90^\circ$ and -10°) and the observation took only a small sector ($l = 146$, $b = +2.6$) and the stars may be different in composition to the difference of chemistry of the region and that there is overlap between other stellar groups. Zdanavičius and Zdanavičius (2005) computed the

distances of spectral types and subtypes and found that the distance up to 3 kpc are at in the inter arm and the Perseus arm regions. However, the distance from 3–5 kpc which may be related to the outer spiral arm which is in a good agreement with our results.

4. Conclusion

In the present work we implemented the Gaussian distribution function of the absolute magnitudes and apparent magnitudes to determine the distance to Camelopardalis area. We draw the results reached through the following points:

- There are nonsystematic and remarkable differences between the calculated distances for different spectral types, this may be attributed to the different chemical composition and evolution scenario of each spectral type.
- From Table 1, The distance of the far side of Camelopardalis is $\sim 3971 \text{ pc}$ while the distance of the near side is $\sim 934 \text{ pc}$. The statistical methods smear all these factors; the smearing out means that stars from the far side are used in the distribution to determine the distance as well as stars of the near side i.e. they were used with the same weights.
- The calculated distances of the spectral subtypes appeared in Tables 2–5 have similar behavior as the calculated distances of the spectral types.
- According to the calculations of Zdanavičius, Zdanavičius, and Straizys (2005), the sample of stars with distances up to 3 kpc are at in the inter arm and the Perseus arm regions and that having distances from 3 to 5 kpc may be related to the outer spiral arm.

Disclosure statement

No potential conflict of interest was reported by the authors.

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