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Transition from decelerating to accelerating universe with quadratic equation of state in *f(R,T)* gravity

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ABSTRACT

In this paper, we have constructed a flat Friedmann-Robertson-Walker (FRW) cosmological model in $f(R,T)$ gravity. The solution of Friedmann equations has been obtained assuming the quadratic equation of state $p = \alpha\rho + \beta\rho^2$, where α and β are parameters. The model describes an accelerating universe with positive energy density, negative pressure and negative cosmological constant. The behaviour of the deceleration parameter shows that the universe accelerates after an epoch of deceleration in a good agreement with recent observations. The non-conventional scenario for an accelerating universe with negative cosmological constant has been discussed.

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1. Introduction and motivation

According to observations, the present universe is flat and expanding at an accelerating rate (Perlmutter et al. 1999; Percival et al. 2001; Stern et al. 2010). The general assumption to explain the accelerating expansion is the existence of dark energy with negative pressure which acts as a repulsive gravity. To understand the nature of such exotic form of energy, several theoretical models have been constructed including quintessence (Tsujikawa 2013), Chaplygin gas (Kamenshchik et al. 2001), phantom energy (Caldwell 2002), k-essence (Chiba et al. 2000), tachyon (Sen 2002), ghost condensate (Arkani-Hamed et al. 2004; Ahmed and Moss 2008, 2010) and holographic models (Wei 2009; Ahmed and Rafat 2018; Ahmed and Alamri 2019a). In addition to scalar field models, modified gravity (Nojiri et al. 2017) represents another important explanation approach. It has been shown that modified gravity can explain the galactic rotation curves without the need to dark matter assumption (Nojiri and Odintsov 2006a; De Felice and Tsujikawa 2010). Examples of such modified gravity theories are $f(R)$ gravity (Nojiri and Odintsov 2006b) where R is the Ricci scalar, Gauss-Bonnet gravity (Nojiri et al. 2008) and f(T) gravity (Ferraro and Fiorini 2007) where T is the torsion scalar. Driven by the torsion effects, $f(T)$ gravity can explain the accelerated expansion without assuming a new form of energy (Bengochea and Ferraro 2009). $f(R)$ gravity has been generalised to $f(R,T)$ gravity (Harko et al. 2011), where T is the trace of the energy-momentum tensor. Some cosmological aspects of $f(R,T)$ gravity have been investigated in (Shabani and Farhoudi 2013; Pradhan et al. 2015; Xu et al. 2016; Nasr Ahmed et al. 2016; Ahmed and Alamri 2018, 2019b).

The current work is dedicated to the study of flat FRW cosmological solutions in $f(R,T)$ gravity using a quadratic equation of state. In the context of general relativity, the effects of a quadratic equation of state have been investigated at early times in homogenous and inhomogeneous anisotropic cosmology (Ananda and Bruni 2006). This was motivated by brane-world scenarios where the quadratic density corrections dominate at early times and help exploring new gravitational physics at high energies (Olmo and Rubiera-Garciab 2015). Quadratic deviation from the standard FRW cosmology is an interesting feature of brane-world cosmology where the effect of quadratic terms becomes important at the very early time (Binetruy et al. 2000). The implications of quadratic density corrections in brane-world scenarios on the inflationary paradigm have been studied in (Coley 2005). The effect of quadratic density term on inflation in the brane-world scenario has been studied in (Maartens et al. 2000). The quadratic energy density term also appears in modified Friedman equations resulting from Loop quantum gravity corrections (Vandersloot 2005). (Nojiri and Odintsov 2005) and (Capozziello et al. 2006) studied dark energy universe with generalised equations of state. In (Nojiri and Odintsov 2005), it has been pointed out that using such phenomenological equations of state is an easy way to produce dark epoch of the universe and may help describing the phantom era. In (Capozziello et al. 2006), observational constraints have been set on dark energy with generalised equations of state. So, investigating FRW cosmology through a quadratic equation of state in different $f(R,T)$ gravity

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reconstructions is an interesting topic. The general form of the quadratic equation of state can be expressed as

$$p = p_o + \alpha\rho + \beta\rho^2 \tag{1}$$

Where p_o, α and β are parameters. This form comes from Taylor expansion of arbitrary barotropic equation of state, $p = p(\rho)$. In the current work we consider $p_o = 0$.

The paper is organised as follows: The introduction and motivation behind the current work is included in section 1. The derivation of the modified Friedmann equations with variable cosmological constant in a specific $f(R,T)$ gravity reconstruction is included in section 2. The analytical solution of the cosmological equations is given in section 3. The final conclusion is included in section 4.

2. Field equations

The $f(R,T)$ gravity action is given by (Harko et al. 2011)

$$S = \frac{1}{16\pi}\int f(R,T)\sqrt{-g}d^4x + \int L_m\sqrt{-g}d^4x, \tag{2}$$

where L_m is the matter Lagrangian density. By varying the action S with respect to $g_{\mu\nu}$, we obtain the field equations of $f(R,T)$ gravity as

$$f_R(R,T)R_{\mu\nu} - \frac{1}{2}f(R,T)g_{\mu\nu} + (g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)f_R(R,T) = 8\pi T_{\mu\nu} - f_T(R,T)T_{\mu\nu} - f_T(R,T)\Theta_{\mu\nu}. \tag{3}$$

where $\Box = \nabla^\mu\nabla_\mu$, $f_R(R,T) = \frac{\partial f(R,T)}{\partial R}$, $f_T(R,T) = \frac{\partial f(R,T)}{\partial T}$ and ∇_μ denotes the covariant derivative. $\Theta_{\mu\nu}$ and the stress-energy tensor $T_{\mu\nu}$ are given by

$$\Theta_{\mu\nu} = -2T_{\mu\nu} - pg_{\mu\nu}, T_{\mu\nu} = (\rho + p)u_\mu u_\nu - pg_{\mu\nu}. \tag{4}$$

The four-velocity u_μ satisfies the conditions $u_\mu u^\mu = 1$ and $u^\mu\nabla_\nu u_\mu = 0$. ρ and p are the energy density and pressure of the fluid, respectively. The divergence of the energy-momentum tensor is given by (Barrientos and Rubilar 2014)

$$\nabla^\mu T_{\mu\nu} = \frac{f_T(R,T)}{8\pi - f_T(R,T)}[(T_{\mu\nu} + \Theta_{\mu\nu})\nabla^\mu \ln f_T(R,T) + \nabla^\mu\Theta_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\nabla^\mu T]. \tag{5}$$

Which represents the violation of energy-momentum conservation in $f(R,T)$ gravity. Different choices of the function $f(R,T)$ lead to different theoretical models. Taking $f(R,T) = f_1(R) + f_2(T)$, the gravitational field equations (3) becomes

$$f_1'(R)R_{\mu\nu} - \frac{1}{2}f_1(R)g_{\mu\nu} + (g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)f_1'(R) = 8\pi T_{\mu\nu} + f_2'(T)T_{\mu\nu} + \left(f_2'(T)p + \frac{1}{2}f_2(T)\right)g_{\mu\nu}. \tag{6}$$

We simply take $f_1(R) = R$ and $f_2(T) = T$, so $f(R,T) = (R+T)$. In the case $f_2(T)\equiv 0$, we re-obtain the field equations of General Relativity. Now, equation (6) becomes

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi T_{\mu\nu} + T_{\mu\nu} + (p + \frac{1}{2}T)g_{\mu\nu}. \tag{7}$$

This could be rearranged as

$$G_{\mu\nu} - \left(p + \frac{1}{2}T\right)g_{\mu\nu} = (8\pi + 1)T_{\mu\nu}. \tag{8}$$

Where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$ is the Einstein tensor. Comparing with Einstein equations

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi T_{\mu\nu}. \tag{9}$$

The term $\left(p + \frac{1}{2}T\right)$ is now related to the cosmological constant by $p + \frac{1}{2}T \equiv -\Lambda$. So, our choice of $f(R,T)$ gives a specific formula for the varying cosmological constant. For the perfect fluid energy-momentum tensor we have $T_1^1 = T_2^2 = T_3^3 = -p(t)$ and $T_4^4 = \rho(t)$ and then the trace $T = \rho(t) - 3p(t)$. So, (8) can be written as (Ahmed and Pradhan 2014)

$$G_{\mu\nu} - \frac{1}{2}(\rho - p)g_{\mu\nu} = (8\pi + 1)T_{\mu\nu}. \tag{10}$$

And the cosmological constant is written as:

$$\Lambda(t) = -\frac{1}{2}(\rho(t) - p(t)) \tag{11}$$

The non-conservation of energy-momentum tensor (5) in the current model is given by

$$\nabla^\mu T_{\mu\nu} = -\frac{1}{8\pi + 1}\left[\nabla^\mu(pg_{\mu\nu}) + \frac{1}{2}g_{\mu\nu}\nabla^\mu T\right]. \tag{12}$$

The FRW metric is given by

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1 - Kr^2} + r^2d\theta^2 + r^2\sin^2\theta d\phi^2\right] \tag{13}$$

where K is either 0, -1 or $+1$ for flat, open and closed universe, respectively. Applying equation (8) to the metric (13) and taking (11) into account, we get the cosmological equations as

$$\frac{\dot{a}^2 + K}{a^2} = \frac{(8\pi + 1)\rho + \Lambda(t)}{3}. \tag{14}$$

$$\frac{\ddot{a}}{a} = -\frac{(8\pi + 1)}{6}(\rho + 3p) + \frac{\Lambda(t)}{3}. \tag{15}$$

In this paper, we consider only the flat case ($K = 0$) supported by observations (de Bernardis et al. 2000;

Bennett et al. 2003; Spergel et al. 2003a). Equations (14) and (15) are two differential equations in three unknown functions $a(t)$, $p(t)$ and $\rho(t)$. In the following section, we provide an exact solution making use of the quadratic equation of state (1) with $p_o = 0$.

3. Solutions

Substituting (11) and (1) in (14), we get:

$$\dot{a} = a\sqrt{\frac{\rho(1+16\pi+\alpha+\beta\rho)}{6}} \tag{16}$$

Where C_1 and C_2 are arbitrary constants. We choose C_1 and C_2 such that $\lim_{\rho\to\infty} t(\rho) = 0$. This condition gives:

$$C_1 = \frac{\sqrt{\beta}\left(-4\sqrt{\pi(1+\alpha)} + (1+\alpha-16\pi)\arctan\left(4\sqrt{\frac{\pi}{1+\alpha}}\right)\right)}{2(1+8\pi)\sqrt{6\pi(1+\alpha)^3}}, \quad \alpha \neq -1. \tag{19}$$

$$C_2 = -\frac{\sqrt{\beta}}{6\sqrt{6}(\pi+8\pi^2)}, \qquad \alpha = -1. \tag{20}$$

Then, we get for $\alpha \neq -1$ and $\alpha = -1$ respectively

$$t(\rho) = \begin{cases} \dfrac{-4\sqrt{\pi(1+\alpha)(\rho\beta)}+4\sqrt{\pi(1+\alpha)(1+16\pi+\alpha+\beta\rho)}+(16\pi-\alpha-1)\sqrt{\rho\beta}\left(\arctan\left(\frac{4\sqrt{\pi\rho\beta}}{\sqrt{1+\alpha}\sqrt{(1+16\pi+\alpha+\beta\rho)}}\right)-\arctan\left(4\sqrt{\frac{\pi}{1+\alpha}}\right)\right)}{2(1+8\pi)\sqrt{6\pi\rho(1+\alpha)^3}} \\ \frac{1}{6\sqrt{6}}\left(\frac{(4\pi+\beta\rho)\sqrt{16\pi+\beta\rho}}{\pi(\beta+8\pi\beta)\sqrt{\rho^3}} - \frac{\sqrt{\beta}}{\pi+8\pi^2}\right) \end{cases}$$

Substituting (16) in (15), we get:

$$\dot{\rho} = -\frac{(1+8\pi)(1+\alpha+\beta\rho)\sqrt{6\rho^3(1+16\pi+\alpha+\beta\rho)}}{1+16\pi+\alpha+2\beta\rho} \tag{17}$$

Then, using (16) and (17), we can get

$$a(\rho) = \begin{cases} \rho^{-\frac{1+\alpha+16\pi}{6(1+\alpha)(1+8\pi)}}(1+\alpha+\beta\rho)^{-\frac{1+\alpha-16\pi}{6(1+\alpha)(1+8\pi)}}, & \alpha \neq -1. \\ e^{\frac{8\pi}{3\rho\beta(1+8\pi)}}\rho^{-\frac{1}{3(1+8\pi)}}, & \alpha = -1 \end{cases}$$

While we found it difficult to write down an explicit expression for ρ in terms of the cosmic time t for general α and β, we can get an explicit expression for t in terms of ρ. From (17) we get

$$t(\dot{\rho}) = \frac{1}{\dot{\rho}(t)} = -\frac{1+16\pi+\alpha+2\beta\rho}{(1+8\pi)(1+\alpha+\beta\rho)\sqrt{6\rho^3(1+16\pi+\alpha+\beta\rho)}} \tag{18}$$

Which has the solution

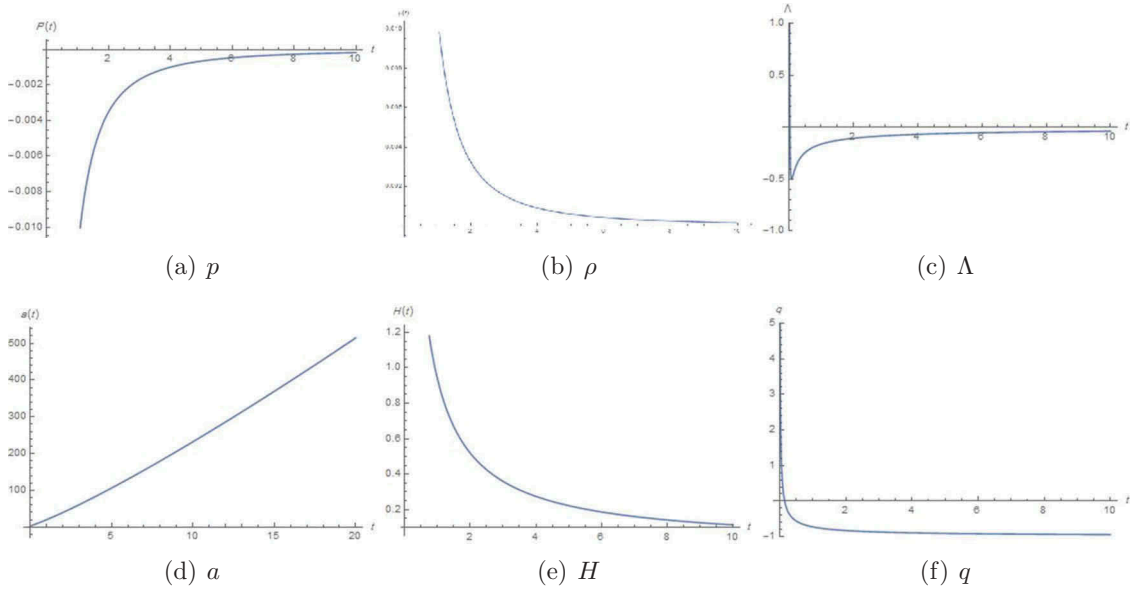
We can easily verify that $\lim_{\rho\to\infty} t(\rho) = 0$ and $\lim_{\rho\to 0} t(\rho) = \infty$. Now $\rho(t)$ is the inverse function of $t(\rho)$ and we can simply use Mathematica software to plot $\rho(t)$ for given values of β and α. Consequently, we can express all cosmological parameters in terms of ρ and plot them. The Hubble parameter H and the deceleration parameter q can now be written in terms of ρ as:

$$H = \frac{\dot{a(t)}}{a(t)} = \frac{\sqrt{\rho(1+16\pi+\alpha+\beta\rho)}}{\sqrt{6}} \tag{21}$$

$$q = -\frac{\ddot{a(t)}a(t)}{\dot{a}(t)^2} = 2\left(1+\frac{12\pi(\beta\rho+\alpha-1)}{\beta\rho+\alpha+16\pi+1}\right). \tag{22}$$

Figure 1(a–c) shows a variation of pressure p, density ρ and cosmological constant Λ versus cosmic time. The model predicts a negative pressure in an agreement with the dark energy assumption. The energy density is positive and tends to zero when $t \to \infty$ as

$$t(\rho) = \begin{cases} \frac{1}{\sqrt{6}(1+8\pi)}\left(-\frac{2\sqrt{1+16\pi+\alpha+\beta\rho}}{\sqrt{\rho}(1+\alpha)} + \frac{(1+16\pi+\alpha)\sqrt{\beta}\arctan\left[\frac{4\sqrt{\pi\beta\rho}}{\sqrt{(1+\alpha)(1+16\pi+\alpha+\beta\rho)}}\right]}{2\sqrt{\pi(1+\alpha)^3}}\right) + C_1, & \alpha \neq -1. \\ \frac{(4\pi+\beta\rho)\sqrt{16\pi+\beta\rho}}{6\pi(\beta+8\pi\beta)\sqrt{6\rho^3}} + C_2, & \alpha = -1 \end{cases}$$



(a) p (b) ρ (c) Λ

(d) a (e) H (f) q

Figure 1. The behaviour of a, H, q, p, ρ and Λ verses cosmic time for $\alpha = -1$ and $\beta = 1$.

expected. The evolution of the cosmological constant Λ (Figure1(c)) shows that it reaches a very small negative value at late-time. While observations suggest a very small positive Λ (Perlmutter et al. 1999; Tonry et al. 2003; Clocchiatti et al. 2006), negative Λ is also possible by other observations and can present a solution to the eternal acceleration problem which has been shown to be a consequence of the positive Λ (Vincenzo et al. 2008). The negative Λ approach has been studied by many authors (Cardenas et al. 2003; Grande et al. 2006; Gong and Wang 2007; Vincenzo et al. 2008; Prokopec 2011; Landry et al. 2012; Maeda and Ohta 2014; Baier et al. 2015; Ahmed and Alamri 2018; ChruÅ›ciel et al. 2018), and it is shown to be supported by the AdS/CFT correspondence (Aharony et al. 2000). The possibility of observationally viable FRW cosmologies with negative Λ has been shown by (Prokopec 2011). A stable de Sitter solution with negative Λ has been found in Gauss-Bonnet gravity by (Maeda and Ohta 2014). Solutions of Einstein-complex scalar field equations with negative Λ has been presented in (ChruÅ›ciel et al. 2018). So, the current solution with negative Λ has a solid theoretical and observational ground.

The deceleration parameter is negative ($q < 0$) for an accelerating universe and positive ($q > 0$) for a decelerating universe. Figure 1(f) shows a sign flipping of the deceleration parameter from positive to negative, it lies in the range $-1 \leq q \leq 0$ which matches with the observations made by (Riess et al. 1998) and (Perlmutter et al. 1999). So, the model predicts a deceleration to acceleration transition in a good agreement with observations. The behaviour of the Hubble parameter (Figure 1(e)) shows that it tends to infinity as time goes to zero.

4. Conclusion

A flat FRW cosmological model in which a deceleration-to-acceleration transition happens has been presented in a specific $f(R, T)$ gravity reconstruction. An exact solution to Friedmann equations has been obtained with a quadratic equation of state describing a universe with negative cosmological constant and negative pressure. The model is in a good agreement with observations, we have also shown that such a universe with negative varying cosmological constant has a solid theoretical base.

Disclosure statement

No potential conflict of interest was reported by the authors.

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